Supplementary Appendix to "Exchange Rate Regimes and Wage Comovements in a Ricardian Model with Money"

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This supplementary appendix derives the propositions in Kurokawa, Pang and Tang (forthcoming) and the related lemmas. For convenience in reference, we briefly restate the model and key conditions here. For country $j = H, F$, the period preference of the infinitely lived representative consumer is

$$U_t^j = \frac{\left(C_t^j\right)^{1-\zeta}}{1-\zeta} - \kappa \frac{\left(L_t^j\right)^{1+\gamma}}{1+\gamma} + \chi h\left(\frac{M_t^j}{P_t^j}\right),$$

where

$$C_t^j = \left[\left(\int_0^1 C_t^j\left(i\right)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}\cdot\epsilon} \cdot C_t^j\left(z\right)^{1-\epsilon}\right],$$

$\zeta, \kappa, \gamma, \chi$ and $\eta > 0$ and $0 < \epsilon < 1$. The period budget constraint in each country is

$$\begin{aligned}
&\int_0^1 P_{Ht}\left(i\right) C_t^H\left(i\right) di + P_t^H\left(z\right) C_t^H\left(z\right) + M_t^H + \sum_{s_{t+1}} q_{t+1|t}^H B_{t+1}^H \\
&= W_t^H L_t^H + M_{t-1}^H + B_t^H + \Pi_t^H + T_t^H, \\
&\int_0^1 P_{Ft}\left(i\right) C_t^F\left(i\right) di + P_t^F\left(z\right) C_t^F\left(z\right) + M_t^F + \sum_{s_{t+1}} q_{t+1|t}^H B_{t+1}^F / e_t \\
&= W_t^F L_t^F + M_{t-1}^F + B_t^F / e_t + \Pi_t^F + T_t^F.
\end{aligned}$$

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There is also a borrowing constraint in each country

$$B_{t+1}^{H} \geq -P_t^H \bar{b}^H,$$
$$B_{t+1}^{F}/e_t \geq -P_t^F \bar{b}^F.$$

Given the prices $P_{jt}(i)$ and $P_t^j(z)$, the minimization of the cost of C_t^j yields the following unit cost of C_t^j, which we refer to as the price of C_t^j

$$P_t^j = \left[\epsilon^{-\epsilon}(1-\epsilon)^{\epsilon-1}\right]\left[\int_0^1 (P_{jt}(i))^{1-\eta}\, di\right]^{\frac{1}{1-\eta}\epsilon} \left(P_t^j(z)\right)^{1-\epsilon}. \tag{A.1}$$

Hence, the budget constraint in each country can be written as

$$P_t^H C_t^H + M_t^H + \sum_{s_{t+1}} q_{t+1|t}^H B_{t+1}^H = W_t^H L_t^H + M_{t-1}^H + B_t^H + \Pi_t^H + T_t^H, \tag{A.2}$$
$$P_t^F C_t^F + M_t^F + \sum_{s_{t+1}} q_{t+1|t}^H B_{t+1}^F/e_t = W_t^F L_t^F + M_{t-1}^F + B_t^F/e_t + \Pi_t^F + T_t^F. \tag{A.3}$$

The production technology for tradable and nontradable goods is

$$Y_t^j(i) = A_t^j(i)\, L_t^j(i), \tag{A.4}$$
$$Y_t^j(z) = A_t^j(z)\, L_t^j(z), \tag{A.5}$$

where $A_t^j(i)$ and $A_t^j(z)$ are the stochastic productivities.

Under perfect competition, the domestic price of good i posted by the firms in country j in the local currency is

$$P_t^j(i) = W_t^j / A_t^j(i),$$

but the prevailing market prices that consumers in the home and foreign countries actually pay are now given by

$$P_{Ht}(i) = min\left\{P_t^H(i), (1+\tau)\, P_t^F(i)\, e_t\right\}, \tag{A.6}$$
$$P_{Ft}(i) = min\left\{(1+\tau)\, P_t^H(i)/e_t, P_t^F(i)\right\}. \tag{A.7}$$

The market for nontradable goods is also perfectly competitive. Consequently, the local-currency price for the nontradable goods is

$$P_t^j(z) = W_t^j / A_t^j(z). \tag{A.8}$$

The market clearing conditions are

$$L_t^H = \int_0^{k_t^F} L_t^H(i)\, di + L_t^H(z), \tag{A.9}$$

$$L_t^F = \int_{k_t^H}^{1} L_t^F(i)\, di + L_t^F(z), \tag{A.10}$$

$$Y_t^H(i) = C_t^H(i) + C_t^F(i)(1+\tau) \quad \forall i < k_t^H, \tag{A.11}$$

$$Y_t^F(i) = C_t^H(i)(1+\tau) + C_t^F(i) \quad \forall i > k_t^F, \tag{A.12}$$

$$Y_t^j(i) = C_t^j(i) \quad \forall k_t^H \leq i \leq k_t^F, \tag{A.13}$$

$$Y_t^j(z) = C_t^j(z), \tag{A.14}$$

$$B_t^H + B_t^F = 0. \tag{A.15}$$

For the consumer's maximization problem, the first order conditions with respect to $C_t^j(i)$, $C_t^j(z)$, L_t^j, M_t^j, and B_{t+1}^j are

$$\left(C_t^j\right)^{1-\zeta} \frac{\epsilon \left(C_t^j(i)\right)^{\frac{-1}{\eta}}}{\int_0^1 \left(C_t^j(i)\right)^{\frac{\eta-1}{\eta}} di} = P_{jt}(i)\,\lambda_t^j, \tag{A.16}$$

$$\left(C_t^j\right)^{1-\zeta} \frac{1-\epsilon}{C_t^j(z)} = P_t^j(z)\,\lambda_t^j, \tag{A.17}$$

$$\kappa \left(L_t^j\right)^{\gamma} = W_t^j \lambda_t^j, \tag{A.18}$$

$$\frac{\chi h'\left(M_t^j / P_t^j\right)}{P_t^j} = \lambda_t^j - \beta E_t\left(\lambda_{t+1}^j\right), \tag{A.19}$$

$$\beta \pi_{t+1|t} \lambda_{t+1}^H = q_{t+1|t}^H \lambda_t^H, \tag{A.20}$$

$$\beta \pi_{t+1|t} \lambda_{t+1}^F / e_{t+1} = q_{t+1|t}^H \lambda_t^F / e_t, \tag{A.21}$$

where λ_t^j is the Lagrange multiplier for the budget constraint, β is the time discount factor,

$E_t(\lambda^j_{t+1}) = \sum_{s_{t+1}} \pi_{t+1|t}\lambda^j_{t+1}$, and $\pi_{t+1|t} = \pi_{t+1}/\pi_t$ is the conditional probability of s^{t+1} given s^t.

An equivalent approach is to maximize the utility of the home and foreign consumers subject to the budget constraints (A.2) and (A.3), respectively. The first order condition with respect to the aggregate consumption C^j_t is

$$\left(C^j_t\right)^{-\zeta} = P^j_t \lambda^j_t, \tag{A.22}$$

while other first order conditions are identical.

As in Chari, Kehoe and McGrattan (2002), we use equations (A.20)-(A.22) to obtain the equilibrium exchange rate as

$$e_t = \frac{P^H_t \left(C^H_t\right)^{\zeta}}{P^F_t \left(C^F_t\right)^{\zeta}} \delta = \frac{\lambda^F_t}{\lambda^H_t} \delta, \tag{A.23}$$

where δ is a constant depending on the state of the economy in the initial period, and is the marginal utility of consumption per the home currency in the home country relative to that in the foreign country in the initial period.

A.1 Proofs of Proposition 1 and a related lemma

First, we prove a lemma useful for the proof of Proposition 1.

Lemma 1. *The real exchange rate is*

$$e_t \frac{P^F_t}{P^H_t} = \left(\frac{e_t W^F_t}{W^H_t}\right)^{1-\epsilon} \left(\frac{A^H_t(z)}{A^F_t(z)}\right)^{1-\epsilon} D_t,$$

where

$$D_t = \left[\frac{\int_0^{k^H_t} \left((1+\tau) P^H_t(i)\right)^{1-\eta} di + \int_{k^H_t}^{k^F_t} \left(P^F_t(i)\, e_t\right)^{1-\eta} di + \int_{k^F_t}^{1} \left(P^F_t(i)\, e_t\right)^{1-\eta} di}{\int_0^{k^H_t} \left(P^H_t(i)\right)^{1-\eta} di + \int_{k^H_t}^{k^F_t} \left(P^H_t(i)\right)^{1-\eta} di + \int_{k^F_t}^{1} \left((1+\tau) P^F_t(i)\, e_t\right)^{1-\eta} di}\right]^{\frac{1}{1-\eta}\epsilon}.$$

Proof: Substituting equation (A.1) into the definition of real exchange rate, we have

$$e_t \frac{P_t^F}{P_t^H} = e_t \frac{\left(\int_0^1 \left(P_{Ft}(i)\right)^{1-\eta} di\right)^{\frac{1}{1-\eta}\epsilon}}{\left(\int_0^1 \left(P_{Ht}(i)\right)^{1-\eta} di\right)^{\frac{1}{1-\eta}\epsilon}} \frac{\left(P_t^F(z)\right)^{1-\epsilon}}{\left(P_t^H(z)\right)^{1-\epsilon}}.$$

Therefore, by using equations (A.6), (A.7), and (A.8), we can rewrite the expression for the real exchange rate as

$$\begin{aligned} e_t \frac{P_t^F}{P_t^H} &= e_t \left[\frac{\int_0^{k_t^H} \left(\frac{(1+\tau)P_t^H(i)}{e_t}\right)^{1-\eta} di + \int_{k_t^H}^{k_t^F} \left(P_t^F(i)\right)^{1-\eta} di + \int_{k_t^F}^1 \left(P_t^F(i)\right)^{1-\eta} di}{\int_0^{k_t^H} \left(P_t^H(i)\right)^{1-\eta} di + \int_{k_t^H}^{k_t^F} \left(P_t^H(i)\right)^{1-\eta} di + \int_{k_t^F}^1 \left((1+\tau) P_t^F(i)\, e_t\right)^{1-\eta} di}\right]^{\frac{1}{1-\eta}\epsilon} \frac{\left(\frac{W_t^F}{A_t^F(z)}\right)^{1-\epsilon}}{\left(\frac{W_t^H}{A_t^H(z)}\right)^{1-\epsilon}} \\ &= e_t^{1-\epsilon} \left(\frac{W_t^F}{W_t^H}\right)^{1-\epsilon} \left(\frac{A_t^H(z)}{A_t^F(z)}\right)^{1-\epsilon} \times \\ &\left[\frac{\int_0^{k_t^H} \left((1+\tau) P_t^H(i)\right)^{1-\eta} di + \int_{k_t^H}^{k_t^F} \left(P_t^F(i)\, e_t\right)^{1-\eta} di + \int_{k_t^F}^1 \left(P_t^F(i)\, e_t\right)^{1-\eta} di}{\int_0^{k_t^H} \left(P_t^H(i)\right)^{1-\eta} di + \int_{k_t^H}^{k_t^F} \left(P_t^H(i)\right)^{1-\eta} di + \int_{k_t^F}^1 \left((1+\tau) P_t^F(i)\, e_t\right)^{1-\eta} di}\right]^{\frac{1}{1-\eta}\epsilon}. \end{aligned} \tag{A.24}$$

Defining

$$D_t = \left[\frac{\int_0^{k_t^H} \left((1+\tau) P_t^H(i)\right)^{1-\eta} di + \int_{k_t^H}^{k_t^F} \left(P_t^F(i)\, e_t\right)^{1-\eta} di + \int_{k_t^F}^1 \left(P_t^F(i)\, e_t\right)^{1-\eta} di}{\int_0^{k_t^H} \left(P_t^H(i)\right)^{1-\eta} di + \int_{k_t^H}^{k_t^F} \left(P_t^H(i)\right)^{1-\eta} di + \int_{k_t^F}^1 \left((1+\tau) P_t^F(i)\, e_t\right)^{1-\eta} di}\right]^{\frac{1}{1-\eta}\epsilon}, \tag{A.25}$$

we can rewrite equation (A.24) as

$$e_t \frac{P_t^F}{P_t^H} = \left(\frac{e_t W_t^F}{W_t^H}\right)^{1-\epsilon} \left(\frac{A_t^H(z)}{A_t^F(z)}\right)^{1-\epsilon} D_t. \tag{A.26}$$

Therefore, the real exchange rate is determined by the relative wage $e_t W_t^F / W_t^H$, productivities in nontradable goods, and the term D_t that can be viewed as the ratio of the price index for tradable goods in the foreign country to that in the home country. The presence of the term D_t is due to the trade costs τ. When τ is zero, $D_t = 1$. Note that the relationship between D_t and τ can be complex, depending on the distribution of tradable productivities in the two countries. For instance, for positive values of τ, if distributions of tradable productivities in the two countries are mirror images to each other (i.e., $A_t^H(i) = A_t^F(1-i)$ for all i), then D_t is also 1. ■

Proposition 1. *The relationship between growth in home nominal wages and the foreign counterpart is*

$$\frac{W_t^H}{W_{t-1}^H} = \frac{W_t^F}{W_{t-1}^F} \left(\frac{e_t}{e_{t-1}}\right)^{\frac{\epsilon}{\epsilon-1}} \left(\frac{D_t}{D_{t-1}}\right)^{\frac{1}{1-\epsilon}} \left(\frac{P_t^H}{P_{t-1}^H} \frac{P_{t-1}^F}{P_t^F}\right)^{\frac{1}{1-\epsilon}} \frac{A_t^H(z)}{A_{t-1}^H(z)} \frac{A_{t-1}^F(z)}{A_t^F(z)}. \tag{A.27}$$

Proof: Rewriting equation (A.26) in Lemma 1 yields an expression for W_t^H

$$W_t^H = W_t^F e_t^{\frac{\epsilon}{\epsilon-1}} D_t^{\frac{1}{1-\epsilon}} \left(\frac{P_t^H}{P_t^F} \right)^{\frac{1}{1-\epsilon}} \frac{A_t^H(z)}{A_t^F(z)}.$$

We can immediately obtain (A.27) by dividing the above expression for W_t^H by the corresponding expression for W_{t-1}^H. ■

A.2 Proofs of Proposition 2 and a related lemma

In order to derive Proposition 2 regarding nominal wage comovements, we first obtain expressions for the marginal utilities of nominal wealth, λ_t^H and λ_t^F, as Lemma 2 under assumptions (a) and (b1).

Lemma 2. *Under assumptions (a) and (b1), the marginal utility of nominal wealth λ_t^j is $\lambda_t^j = \frac{\chi \psi^j}{M_t^j}$, where ψ^j is a constant.*

Proof: From equation (A.19) and assumptions (a) and (b1), we have

$$\begin{aligned}
\lambda_t^j &= \frac{\chi}{M_t^j} + \beta E_t \left(\lambda_{t+1}^j \right) \\
&= \frac{\chi}{M_t^j} + \beta E_t \left(\frac{\chi}{M_{t+1}^j} \right) + \beta^2 E_t \left[E_{t+1} \left(\frac{\chi}{M_{t+2}^j} \right) \right] + \beta^3 E_t \left\{ E_{t+1} \left[E_{t+2} \left(\frac{\chi}{M_{t+3}^j} \right) \right] \right\} + \cdots \\
&= \frac{\chi}{M_t^j} \left\{ 1 + \frac{\beta}{1+g^j} E_t \left(\frac{1}{exp\left(\mu_{t+1}^j\right)} \right) + \left(\frac{\beta}{1+g^j} \right)^2 E_t \left[E_{t+1} \left(\frac{1}{exp\left(\mu_{t+1}^j\right) exp\left(\mu_{t+2}^j\right)} \right) \right] \right. \\
&\quad \left. + \left(\frac{\beta}{1+g^j} \right)^3 E_t \left\{ E_{t+1} \left[E_{t+2} \left(\frac{1}{exp\left(\mu_{t+1}^j\right) exp\left(\mu_{t+2}^j\right) exp\left(\mu_{t+3}^j\right)} \right) \right] \right\} + \cdots \right\}.
\end{aligned}$$

The whole term after χ / M_t^j in the last equality is equal to a constant. Defining this constant as ψ^j, we have

$$\lambda_t^j = \frac{\chi \psi^j}{M_t^j}. \tag{A.28}$$

■

Combined with equation (A.23), an immediate corollary of Lemma 2 is that

$$e_t = \frac{M_t^H}{M_t^F} \frac{\delta \psi^F}{\psi^H}. \tag{A.29}$$

Using assumption (b1) about monetary shocks, under a flexible exchange rate regime the change in exchange rate is determined by

$$\frac{e_t}{e_{t-1}} = \frac{\left(1+g^H\right) exp\left(\mu_t^H\right)}{\left(1+g^F\right) exp\left(\mu_t^F\right)}. \tag{A.30}$$

We now derive Proposition 2 regarding nominal wage comovements under different exchange rate regimes.

Proposition 2. *Under assumptions (a) and (c), nominal wage comovements between the countries are more positive or less negative under the fixed exchange rate regime (assumption (b2)), compared to the flexible exchange rate regime (assumption (b1)). To be specific,*

$$corr^{FX}\left[ln\left(\frac{W_t^H}{W_{t-1}^H}\right), ln\left(\frac{W_t^F}{W_{t-1}^F}\right)\right] - corr^{FL}\left[ln\left(\frac{W_t^H}{W_{t-1}^H}\right), ln\left(\frac{W_t^F}{W_{t-1}^F}\right)\right] \geq 0,$$

where FX and FL denote the fixed and flexible exchange rate regimes, respectively. The strict equality holds only when monetary shocks μ_t^H and μ_t^F are perfectly correlated.

Proof: The first order condition for labor supply, equation (A.18), implies that

$$\frac{W_t^j}{W_{t-1}^j} = \frac{\lambda_{t-1}^j}{\lambda_t^j} \cdot \left(\frac{L_t^j}{L_{t-1}^j}\right)^\gamma. \tag{A.31}$$

Dividing the home version of equation (A.31) with the foreign version, we obtain

$$\frac{W_t^H}{W_{t-1}^H} = \frac{W_t^F}{W_{t-1}^F}\frac{e_t}{e_{t-1}}\left(\frac{L_t^H}{L_{t-1}^H}\frac{L_{t-1}^F}{L_t^F}\right)^\gamma.$$

Taking the log of the above equation gives

$$ln\left(\frac{W_t^H}{W_{t-1}^H}\right) = ln\left(\frac{W_t^F}{W_{t-1}^F}\right) + ln\left(\frac{e_t}{e_{t-1}}\right) + \gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\frac{L_{t-1}^F}{L_t^F}\right). \tag{A.32}$$

Using equation (A.28), the expression for marginal utility of nominal wealth in Lemma 2, and assumption (b1) about money supplies, under the flexible exchange rate regime equation (A.31) becomes

$$\frac{W_t^j}{W_{t-1}^j} = (1+g^j) exp(\mu_t^j)\left(\frac{L_t^j}{L_{t-1}^j}\right)^\gamma. \tag{A.33}$$

Using equation (A.33) to replace $ln\left(W_t^F/W_{t-1}^F\right)$ and equation (A.30) to replace e_t/e_{t-1} in equation (A.32), we obtain the expression for home wage growth under the flexible exchange rate regime

$$ln\left(\frac{W_t^H}{W_{t-1}^H}\right) = ln(1+g^H) + \mu_t^H + \gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\right). \tag{A.34}$$

Similarly, we obtain the expression for foreign wage growth under the flexible exchange rate regime

$$ln\left(\frac{W_t^F}{W_{t-1}^F}\right) = ln(1+g^F) + \mu_t^F + \gamma ln\left(\frac{L_t^F}{L_{t-1}^F}\right), \tag{A.35}$$

which also holds under the fixed exchange rate regime, because the term $ln(1+g^F)+\mu_t^F$ is common under both regimes due to assumptions (b1) and (b2) and the real term L_t^F/L_{t-1}^F is not affected by exchange rate regimes in our model with flexible prices and wages and assumption (c).

Using equation (A.32) with $e_t/e_{t-1} = 1$ and equation (A.35), we obtain the expression for home wage growth under the fixed exchange rate regime

$$ln\left(\frac{W_t^H}{W_{t-1}^H}\right) = ln(1+g^F) + \mu_t^F + \gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\right). \tag{A.36}$$

Thus under the fixed regime, the correlation of nominal wage growth rates is

$$\begin{aligned} & corr^{FX}\left[ln\left(\frac{W_t^H}{W_{t-1}^H}\right), ln\left(\frac{W_t^F}{W_{t-1}^F}\right)\right] \\ &= \frac{cov\left[ln(1+g^F)+\mu_t^F+\gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\right), ln(1+g^F)+\mu_t^F+\gamma ln\left(\frac{L_t^F}{L_{t-1}^F}\right)\right]}{\sqrt{var\left[ln(1+g^F)+\mu_t^F+\gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\right)\right]}\sqrt{var\left[ln(1+g^F)+\mu_t^F+\gamma ln\left(\frac{L_t^F}{L_{t-1}^F}\right)\right]}} \\ &= \frac{var(\mu_t^F)+cov\left[\gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\right), \gamma ln\left(\frac{L_t^F}{L_{t-1}^F}\right)\right]}{\sqrt{var(\mu_t^F)+var\left[\gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\right)\right]}\sqrt{var(\mu_t^F)+var\left[\gamma ln\left(\frac{L_t^F}{L_{t-1}^F}\right)\right]}}. \end{aligned} \tag{A.37}$$

The last equality follows because of the independence of real variables from monetary shocks that is implied by assumption (c).

Similarly, under the flexible regime, the correlation of nominal wage growth rates is

$$\begin{aligned} & corr^{FL}\left[ln\left(\frac{W_t^H}{W_{t-1}^H}\right), ln\left(\frac{W_t^F}{W_{t-1}^F}\right)\right] \\ &= \frac{cov\left[ln(1+g^H)+\mu_t^H+\gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\right), ln(1+g^F)+\mu_t^F+\gamma ln\left(\frac{L_t^F}{L_{t-1}^F}\right)\right]}{\sqrt{var\left[ln(1+g^H)+\mu_t^H+\gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\right)\right]}\sqrt{var\left[ln(1+g^F)+\mu_t^F+\gamma ln\left(\frac{L_t^F}{L_{t-1}^F}\right)\right]}} \\ &= \frac{cov(\mu_t^H, \mu_t^F)+cov\left[\gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\right), \gamma ln\left(\frac{L_t^F}{L_{t-1}^F}\right)\right]}{\sqrt{var(\mu_t^F)+var\left[\gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\right)\right]}\sqrt{var(\mu_t^F)+var\left[\gamma ln\left(\frac{L_t^F}{L_{t-1}^F}\right)\right]}}. \end{aligned} \tag{A.38}$$

Note that in the last equality, we use assumption (b1), which states that the home and foreign monetary shocks have the same marginal distributions.

Because $var\left(\mu_t^H - \mu_t^F\right) \geq 0$ implies

$$var\left(\mu_t^H\right) + var\left(\mu_t^F\right) \geq 2 \cdot cov\left(\mu_t^H, \mu_t^F\right),$$

or

$$var\left(\mu_t^F\right) \geq cov\left(\mu_t^H, \mu_t^F\right),$$

it follows that

$$\begin{aligned} & corr^{FX}\left[ln\left(\frac{W_t^H}{W_{t-1}^H}\right), ln\left(\frac{W_t^F}{W_{t-1}^F}\right)\right] - corr^{FL}\left[ln\left(\frac{W_t^H}{W_{t-1}^H}\right), ln\left(\frac{W_t^F}{W_{t-1}^F}\right)\right] \\ = & \frac{var\left(\mu_t^F\right) - cov\left(\mu_t^H, \mu_t^F\right)}{\sqrt{var(\mu_t^F) + var\left[\gamma ln\left(\frac{L_t^H}{L_{t-1}^H}\right)\right]}\sqrt{var(\mu_t^F) + var\left[\gamma ln\left(\frac{L_t^F}{L_{t-1}^F}\right)\right]}} \geq 0, \end{aligned}$$

where the strict equality holds only when μ_t^H and μ_t^F are perfectly correlated. ■

References

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