Bowdoin College

Math 2020: Introduction to Mathematical Reasoning
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Doubly Quantified Statements

For each of the quantified statements below, please do three things:

(1) Decide whether the statement is true or false.
(2) Justify the truth value.
(3) Write the negation, simplifying as much as possible, which means until there are no more negation signs in your answer.

You will need to watch the video on doubly quantified statements before working through the examples below.

Deal with x first. But you can't make a specific choice because it's a $\forall$.

1. Statement: $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ so that $2x + y = 3$.

Later, you will make a choice of y. It can depend on x because y comes after x in the statement.

Truth value: True.

Justification: Let x be any real number. Choose $y = 3 - 2x$. Then $y \in \mathbb{R}$ and
$2x + y = 2x + (3 - 2x) = 3$.

y is in the right set

y has the right property.

Negation: $\exists x \in \mathbb{R}$ so that $\forall y \in \mathbb{R}$, $2x + y \neq 3$.

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2. Statement: $\exists x \in \mathbb{R}$, so that $\forall y \in \mathbb{R}$, $2x + y = 3$.

Truth value: False.

Justification: Once you choose a value of x, you can solve for $y = 3 - 2x$. So there is only one value of y which will make the equation true, not $\forall y \in \mathbb{R}$.

Negation: $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$ so that $2x + y = 3$.

Tells me that I can't make a specific choice for z.

Tells me that I will have to make a specific choice for w. But it can depend on x, because x comes first in the statement.

3. Statement: $\forall z \in \mathbb{Q}, \exists w \in \mathbb{Z}$ so that $zw \in \mathbb{Z}$.

Truth value: True.

So next we have to use the definition of a rational #.

Justification: Let z be any rational number. So $z = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$ with $b \neq 0$. Choose $w = b$. Then $w \in \mathbb{Z}$ (because $b \in \mathbb{Z}$) and $zw = \frac{a}{b} \cdot b = a \in \mathbb{Z}$.

(Note: another approach is to choose $w=0$.)

Negation: $\exists z \in \mathbb{Q}$ so that $\forall w \in \mathbb{Z}$, $zw \notin \mathbb{Z}$.

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4. Statement: $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}$ so that $xy = 2$.

Truth value: False. (So you only have to give one value of x for which the rest of the statement fails.)

Justification: If $x = 4$ and $xy = 2$, we solve for $y = \frac{1}{2}$. But $\frac{1}{2} \notin \mathbb{N}$.

Negation: $\exists x \in \mathbb{N}$ so that $\forall y \in \mathbb{N}$, $xy \neq 2$.

Tells me that I won't be able to make a specific choice of b.

5. Statement: $\exists a \in \mathbb{Z}$ so that $\forall b \in \mathbb{Z},\ ab = b$.

Truth value: True. (So you only need to choose 1 value of a)

Justification: Choose $a = 1$. Then $a \in \mathbb{Z}$ and for any $b \in \mathbb{Z}$, $ab = (1)(b) = b$.

Negation: $\forall a \in \mathbb{Z}$, $\exists b \in \mathbb{Z}$ so that $ab \neq b$.

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6. Statement: $\exists a \in \mathbb{Z}$ so that $\forall b \in \mathbb{Z},\ ab = a$.

Truth value: True.

Justification: Choose $a = 0$. Then $a \in \mathbb{Z}$ and for any $b \in \mathbb{Z}$, $a \cdot b = 0 \cdot b = 0 = a$.

Negation: $\forall a \in \mathbb{Z}$, $\exists b \in \mathbb{Z}$ so that $ab \neq a$.