Bowdoin College

Math 2603: Introduction to Analysis
Prof. Thomas Pietraho

Homework 10

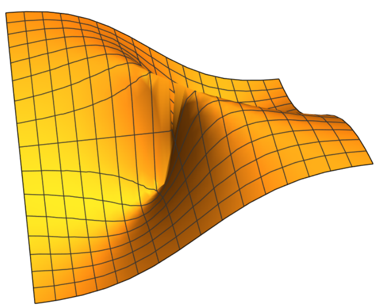
1. Consider a function $f : S_1 \to S_2$. Show that if x is not a limit point of S_1, then f must be continuous at x.

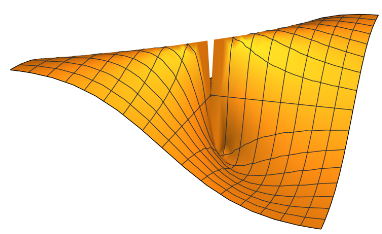
2. Show that every polynomial $p : \mathbb{R} \to \mathbb{R}$ is a continuous function.

3. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

   The plot of this function is somewhat interesting. Below are two perspectives:





   Determine whether $f(x,y)$ is continuous at the origin $(0,0)$ and justify your answer.

4. Consider a function $f : \mathbb{R} \to \mathbb{R}$ which satisfies $f(x) = 0$ for all $x \in \mathbb{Q}$. Show that if f is continuous, then in fact $f(x) = 0$ for all $x \in \mathbb{R}$!

5. This problem revisits the notion of an additive homomorphism, this time with a slightly enlarged domain. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is an additive homomorphism; that is, it satisfies

$$f(x+y) = f(x) + f(y) \text{ for all } x, y \in \mathbb{R}.$$

   Describe the set of all possibilities for f if we assume that it is continuous.

   Extra Credit: What are the possibilities for f if we don't assume that it is continuous?

6. Let (S, ρ) be a metric space endowed with the discrete metric. Describe all of its connected subsets.