Bowdoin College

Math 2020: Introduction to Mathematical Reasoning
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Homework 9

1. In the problems below, you are welcome to use piecewise-defined functions in your answer. Here is an example of a piecewise-defined function:

$$f(x) = \begin{cases} 3x & \text{if } x \geq 10 \\ x - 1 & \text{if } x < 10 \end{cases}$$

In this question you will have to make up some functions! So it's fine if everyone has different answers. (In fact, everyone in your group should not have exactly the same function!)

a. Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ which is injective but not surjective. You do not need to prove that your function is injective.

b. Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ which is NOT injective. State why your function is not injective.

c. Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ which is NOT injective but if we consider the same rule as a function $f : \mathbb{Z} - \{0\} \to \mathbb{Z} - \{0\}$ we do get an injective function. *Hint: Can you use a piecewise-defined function? So it has one rule on part of its domain, and another rule on another part? You don't have to do it this way!*

2. Decide whether the functions below are injective and/or surjective, and prove your answers.

a. $h : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ defined by $h(x, y) = 2x + 4y$

b. $g : \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by $g(a) = (3a, 4a)$

When are two ordered pairs equal? When their first coordinates are equal and their second coordinates are also equal!

Extra Credit

1. A two-digit number is divisible by 9 if the sum of it's digits is 9. Prove a more general divisibility test for 9:

If the sum of the digits of a positive integer n is divisible by 9 (so congruent to 0 in mod 9), then n is divisible by 9.

Hint: Think about place value – when we write 341, for example, what does the 3 represent? Does this give you a different way of thinking about/writing 341 if we write

$$341 = (3 \times 10^2) + (4 \times 10^1) + (1 \times 10^0)$$

2. Can you use the method above to conjecture and prove a divisibility test for 11?