Bowdoin College

Math 2020: Introduction to Mathematical Reasoning
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Homework 8

1. Pages 127-129 #3, 4(a-e), 9

2. Decide whether the following functions are injective and surjective. To receive credit, you must justify your answer using the definition of a injective and surjective functions from class.

$$f : \mathbb{R} \to \mathbb{R} \text{ defined by } f(x) = 8x - \frac{1}{3} \qquad \text{and} \qquad f : \mathbb{R} \to \mathbb{R} \text{ defined by } f(x) = x^3$$

3. Decide whether the following functions are injective. To receive credit, you must justify your answer using the definition of an injective function from class.

$f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^x$ and $f : \mathbb{N} \to \mathbb{N}$ defined by $f(n)$ is the sum of the digits of n.

4. Bijections are functions that are both injective and surjective. For parts (a) and (b) of this problem you do not need to prove that your function is a bijection, but you should talk it through with your group. **For part (c) make sure to prove that your function is a bijection.**

   (1) Find a function from the interval $[0, 1]$ to the interval $[0, 3]$ which is a bijection.
   (2) Find a function from the interval $[1, 3]$ to the interval $[5, 7]$ which is a bijection.
   (3) Find a function from the interval $[3, 4]$ to the interval $[4, 7]$ which is a bijection.

Hint: Parts (a) and (b) should give you some hints as to how to construct the function for part(c). What if part (c) had asked for a bijection from $[3, 4]$ *to* $[5, 6]$*? How can you combine the ideas from parts (a) and (b) into a solution for part (c)?*

5. Now you get to make up some functions! You do not have to prove that your functions have or do not have the desired properties.

   (1) Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ which is injective but not surjective.
   (2) Give an example of a function $g : \mathbb{Z} \to \mathbb{Z}$ which is surjective but not injective.