Bowdoin College

Math 2020: Introduction to Mathematical Reasoning
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Homework 7

1. In the problems below, find the least residue of the given powers using the method from class. Show enough work so that it's clear what pattern you have found!

$$13^{297}\ (mod\ 5) \text{ and } 10^{1237}\ (mod\ 7).$$

2. Book problems: pages 307-308 #1,3,4

Hints:

Here's how to think about problem 3, which asks you to show that "If n is an odd integer, then $n^2 \equiv 1\ (mod\ 8)$."

First, try a few examples to make sure you believe this!

To solve the problem, first figure out what the possible remainders are when you divide an odd number by 8. If n is odd, and n=8k+r, what are the possibilities for the remainder r?

In our new language of congruence, you are really saying that $n \equiv r\ (mod\ 8)$. Now square r in mod 8. So, for example, if $n \equiv 3\ (mod\ 8)$, then $n^2 \equiv 3^2\ (mod 8)$. Since $9 \equiv 1\ (mod\ 8)$, you can use transitivity to conclude that when $n \equiv 3\ (mod\ 8)$ then $n^2 \equiv 1\ (mod\ 8)$.

Can you adapt this to the other possibilities for the remainder when you divide n by 8?

A similar approach with different cases will work for problem 4. (But using mod 6, not mod 8.)

3. Prove that if $n > 4$ and n is not prime, then $(n-1)! \equiv 0(mod\ n)$. Before you think about how to prove this, do some numerical examples to figure out why it is true!

4. Page 307 #12,13(a)

In problem 13(a), if there is a solution in $\mathbb{Z}$ to the equation, then if you look at both sides mod 8, you get a solution in $\mathbb{Z}_8$....what does problem 12 say about a solution to this equation modulo 8?