Bowdoin College

Math 2020: Introduction to Mathematical Reasoning
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Homework 6

1. Later this semester we will prove the following lemma:

> **Lemma:** Let $a, b \in \mathbb{Z}$ and p be a prime. If p divides the product ab then p must divide a or p must divide b.

Assume the lemma is true, and use it to prove the following corollary:

> **Corollary.** Let $a_1, a_2, \cdots, a_n \in \mathbb{Z}$ and p a prime. If p divides the product $a_1 \times a_2 \times \cdots \times a_n$ then p divides a_i for some i.

Hint: Since the result asks you to prove something is true for all $n \in \mathbb{N}, n \geq 2$, what method of proof should you use?

2. Pages 299 and 300: problems 1, 2 and 8. No justification is necessary for number 2.

3. Page 301 #9

Here is a way to start problem 9.

For a number to be divisible by 6, you have to show it has a factor of 2 and a factor of 3.

(a) Can you convince yourself that one of the factors of n is even? Can you justify your answer?

(b) Now you just have to show that one of the factors of n is a multiple of 3. Try doing this in cases. If k is a multiple of 3, we can write $k = 3m$ for some $m \in \mathbb{Z}$. But what if k is not a multiple of 3? What if when you divide k by 3, you get a remainder?

–What can that remainder be? If the remainder is 1, for instance, how can you write k?

This gives you 3 possibilities for how to write k, depending on what the remainder is when you divide by 3. In each case, make an argument that one of the factors of n is a multiple of 3.

4. Use the lemma from class on greatest common divisors to compute:

(a) (56,104)

(b) (462,3003)

Show your work until you reach a problem with very small numbers!

Divisibility Lemmas

For you convenience, here are the divisibility lemmas from class.

Divisibility Lemma 1. Let $a, b \in \mathbb{Z}$ with $a \neq 0$. If $a|b$ then $a|bk$ for any $k \in \mathbb{Z}$.

Divisibility Lemma 2. Let $a, b, c \in \mathbb{Z}$ with $a \neq 0$. If $a|b$ and $a|c$ then $a|(b + c)$ and $a|(b - c)$.

Divisibility Lemma 3. Let $a, b, c \in \mathbb{Z}$ with $a, b \neq 0$. If $a|b$ and $b|c$ then $a|c$.

Divisibility Lemma 4. Let $a, b \in \mathbb{Z} - \{0\}$. If $a|b$ then $|a| \leq |b|$.