Bowdoin College

Math 2020: Introduction to Mathematical Reasoning
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Homework 3B

1. Page 75 #1,2

2. Let S be a nonempty set and $x \notin S$. Find two different types of sets in the power set $P(S \cup \{x\})$.

Hint: Is x in every element of the power set $P(S \cup \{x\})$? This is a set-up for the next problem.

You might try an example, say take $S = \{a, b\}$.

3. Let S be a nonempty set.

(a) Make up four sets, of sizes 1,2,3, and 4, write out the power set for each and count the number of elements in each power set.

(b) Based on your answers for part (a), make a guess at a formula for the number of elements in $P(S)$ and prove by induction that your formula is correct.

Hint for the induction: You can do this problem with weak induction. Start with a set $S_k = \{a_1, a_2, \cdots, a_k\}$ and add one more element to obtain a set of size $k+1$....Think about how you can use the previous homework problem in your induction proof.

1. Extra Credit

More induction with polygons!

A polygon is called an n-gon if it has n sides (so a triangle is a 3-gon), and is convex if the following holds: any time you take two points on the polygon and draw the line segment between them, that line segment lies on or inside the polygon.

Prove by induction that a convex n-gon has $\frac{n(n-3)}{2}$ diagonals.