

BOWDOIN COLLEGE

MATH 2020: INTRODUCTION TO MATHEMATICAL REASONING
PROF. THOMAS PIETRAHO
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HOMEWORK 3A

1. Problems from the textbook:

- Page 44 #2,3,5
- Page 57 #1 *The problem is asking to you to list all possible subsets.*
- Page 61 #1,3 *For problem 3, just state true or false, no justification required.*

2. Consider an infinite family of sets, which I will call $A_1, A_2, A_3, A_4, \dots, A_n, \dots$ for all $n \in \mathbb{N}$. Define these sets by

$$A_n = \left(\frac{1}{2}, \frac{1}{2} + \frac{1}{n} \right)$$

for $n \in \mathbb{N}$. That is, each set is an open interval of the real line. (You might want to write out the endpoints of a few of these sets if you are having trouble understanding what they are.)

The notation $[a, b]$ denotes the set of x so that $a \leq x \leq b$ and (a, b) denotes the set of x so that $a < x < b$.

- (1) What is the union of all the A_n ?
- (2) What is the intersection of all the A_n ?
- (3) How do your answers change if A_n is defined as a closed interval (meaning that you include the endpoints of the interval)?

3. Let A , B and C be sets. The goal of this problem is to show that:

if $A \subset B$ and $B \subseteq C$ then $A \subset C$.

You will do that by following the steps below.

First notice that in class we stated the following lemma:

Lemma. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

So there must be something special about the difference between \subset and \subseteq which makes the statement in this problem require justification.

- (a) Write out the definition of $A \subset B$ and a sentence about how it is different than $A \subseteq B$.
- (b) In this problem, could $B - A$ be empty? Why or why not? Can you draw a Venn diagram (with circles) which illustrates $A \subset B$? Can you draw in a point $x \in B$ to illustrate that $A \subset B$ but $A \neq B$?
- (c) Since $A \subset B$, is it also true that $A \subseteq B$? (Yes or no)

(d) Now fill in the reasoning behind the following steps which will allow you to conclude that $A \subset C$.

–Explain how you know that $A \subseteq C$.

–Now you have figure out why $A \neq C$. Think about the Venn diagram you drew in part (b). Can you add a circle representing C to this diagram? You must express your reasoning for why $A \neq C$ in words, the picture is to help you, but is not a proof.

EXTRA CREDIT

Triominos are like dominos, but made out of three unit squares. There are two possible triominos, but they can be rotated in different ways, as seen in the picture below.

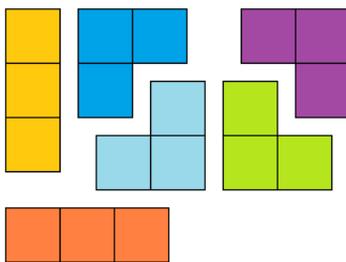


FIGURE 1. All possible triominos: two basic shapes which can be rotated. (*image from commons.wikimedia.org*)

Prove by induction that any $2^n \times 2^n$ checkerboard with one square missing can be completely covered by non-overlapping triominos.

Note: This has to work for ANY missing square. Here's a way to understand what the problem is saying. Take your $2^n \times 2^n$ checkerboard and color one square blue. The remaining squares can be completely covered with non-overlapping triominos.