Bowdoin College

Math 2020: Introduction to Mathematical Reasoning
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Homework 2B

1. Give a short proof of the following quantified statements, if they are true, and a counterexample if they are false. When saying that a number is rational, you must check all of the parts of the definition of a rational number.

(1) $\forall x \in \mathbb{Q},\ x^2 \geq 3$

(2) $\exists x \in \mathbb{Q},\ x + 1 = \frac{5}{2}$

(3) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Q}$ so that $x + y \in \mathbb{Q}$

2. Write out the negation of each expression in problem 1. Make sure to simplify completely! There should be no negation signs left in your final answer.

3. A chocolate bar consists of a number of squares of chocolate (say $n > 0$) arranged in a rectangular pattern. You split a bar into small squares by always breaking along the lines between squares. Prove using induction that it always takes $n - 1$ breaks to completely split the bar into small squares.

(Note: You do NOT know the arrangement of the squares! You can't assume that they are arranged into a long rectangle! That does not matter for the problem!)