Bowdoin College

Homework 2A

For each induction proof you write, make sure to:

(1) Clearly state $P(n)$. Remember that "$n \geq a$" is not part of the statement of $P(n)$.
(2) Clearly state the smallest possible value of n for your base case.
(3) Do not equate expressions before you know they are equal.
(4) In the induction step, clearly state what you are trying to prove true, that is, $P(k+1)$.
(5) End your proof with a conclusion.

1. Prove the power rule for derivatives using induction, namely, that $\frac{d}{dx}x^n = nx^{n-1}$ for positive integers n. You may assume three things about differentiation:

(1) the product rule for derivatives.
(2) the derivative of the function $f(x) = x$ is 1.
(3) the derivative of a constant function $f(x) = c$ for any $c \in \mathbb{R}$ is zero.

2. (A more radical version of the problem from class!) Suppose we got rid of all the bills currently in use and replaced them with only \$3 bills and \$7 bills. Prove using induction that you can buy anything that costs \$12 or more with exact change.

3. Using induction, prove that the sum of the interior angles of an n-sided convex polygon is $(n-2) \times 180°$.

A polygon is called an n-gon if it has n sides (so a triangle is a 3-gon), and is convex if the following holds: any time you take two points on the polygon and draw the line segment between them, that line segment lies on or inside the polygon.

So a square would be convex, but the Star Trek logo would not be.

Make sure in your proof that you point out where you are using this assumption. It's fine to draw a picture to help explain your proof, but you do need to use induction. What is the smallest value for which the above problem makes sense? You may assume that the sum of the angles of a triangle is 180°.