"Sex and the Single Statistician"
A Mathematician's Guide to Dating and Marriage

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Bowdoin College

Valentine's Day Aftermath, 2016

A Student Project

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$$10,409,396,852,733,332,453,861,621,760,000$$

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First posed by Martin Gardiner, but studied *much* earlier.

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Fortunately, the answer is simple.

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Clearly, I missed a good portion of the talk! But what goes around comes around and to help Harrison, I had to reconstruct the argument. Examining the first 7 out of $10,409,396,852,733,332,453,861,621,760,000$ possibilities is unlikely to give the right answer.

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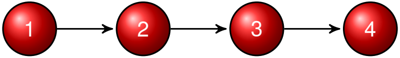
Sequence of eligible suitors ($n = 4$):



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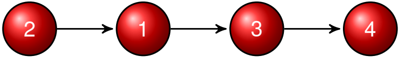

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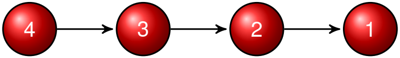

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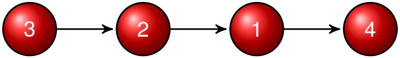

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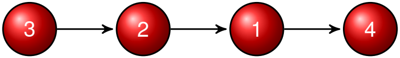

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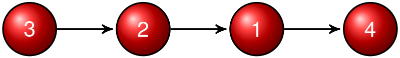

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No one had ever made him feel he was different from anybody else, until he went to Princeton. He took it out in boxing, and he came out of Princeton with painful self-consciousness and a flattened nose, and was married by the first girl who was nice to him.

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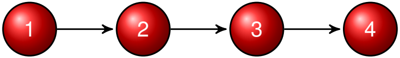

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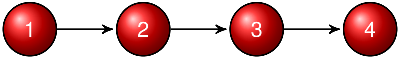

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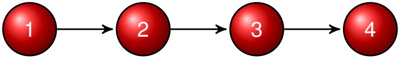

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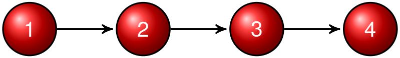

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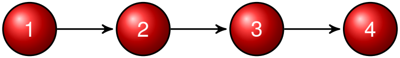

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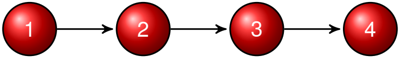

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Deeper Analysis

Let $P_n(k)$ be the probability of success by passing on the first k suitors. We have found that

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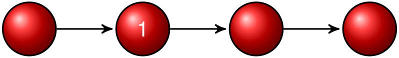

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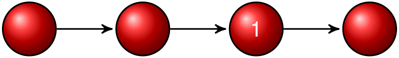

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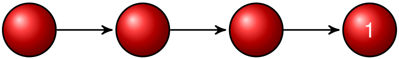

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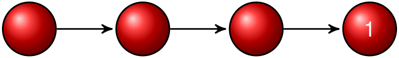

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$$P_4(2) = \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{3} = \frac{11}{24}$$

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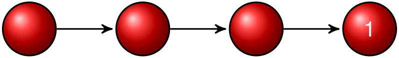

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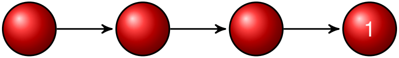

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Deeper Analysis

Let $P_n(k)$ be the probability of success. We have found that

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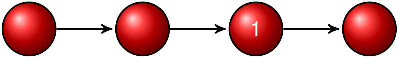

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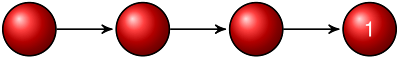

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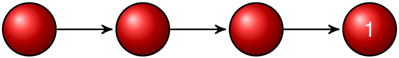

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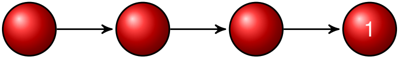

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Thus

$$P_4(3) = \frac{1}{4} + \frac{1}{4} \cdot \frac{2}{3} = \frac{5}{12}$$

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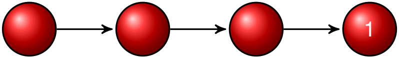

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Thus

$$P_n(3) = \frac{1}{n} + \frac{1}{n} \cdot \frac{2}{3} + \frac{1}{n} \cdot \frac{2}{4} + \ldots$$

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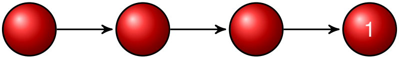

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General Case

Based on the above work, we can justify the following formula:

$$P_n(k) = \sum_{j=k}^{n} P(j\text{th suitor is best and is selected}) = \frac{1}{n} + \frac{1}{n} \cdot \frac{k-1}{k} + \frac{1}{n} \cdot \frac{k-1}{k+1} + \dots$$

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$n \backslash k$	1	2	3	4	5	6	7	8
1	1							
2	0.5	0.5						
3	0.33	0.5	0.33					
4	0.25	0.46	0.42	0.25				
5	0.2	0.42	0.43	0.35	0.2			
6	0.17	0.38	0.43	0.39	0.3	0.17		
7	0.14	0.35	0.42	0.40	0.35	0.26	0.14	
8	0.13	0.32	0.40	0.41	0.38	0.32	0.23	0.13
16	0.06	0.21	0.29	0.34	0.37	0.386	0.388	0.380

Asymptotics

Using the above formula, and having chosen an n, we can determine how many suitors to pass up before settling down. However, none of this will help Harrison. In his case:

$$n = 10,409,396,852,733,332,453,861,621,760,000$$

Effectively $n = \infty$. But let's look:

$$P_n(k) = \sum_{j=k}^{n} \frac{1}{n} \cdot \frac{k-1}{j-1} = \frac{k-1}{n} \sum_{j=k}^{n} \frac{1}{j-1} = \frac{k-1}{n} \sum_{j=k}^{n} \frac{n}{j-1} \frac{1}{n}$$

Asymptotics

Thus

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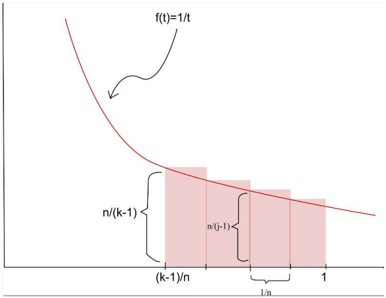

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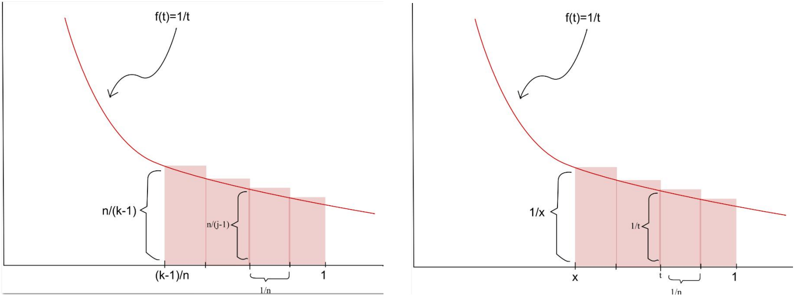

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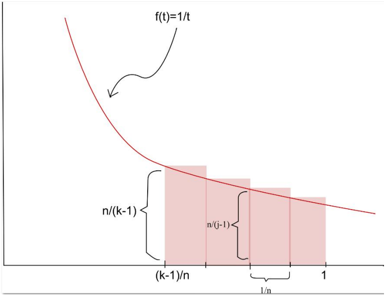
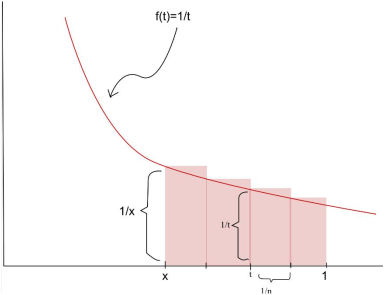
$$\lim_{n\to\infty}\sum_{j=k}^{n}\frac{1}{n}\cdot\frac{k-1}{j-1}=\lim_{n\to\infty}\frac{k-1}{n}\sum_{j=k}^{n}\frac{n}{j-1}\frac{1}{n}=x\int_{x}^{1}\frac{1}{t}\,dt=$$





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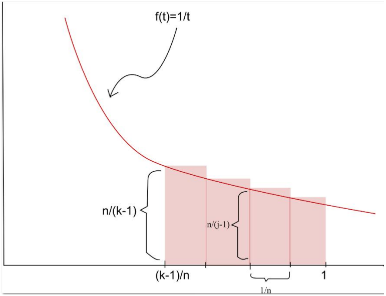

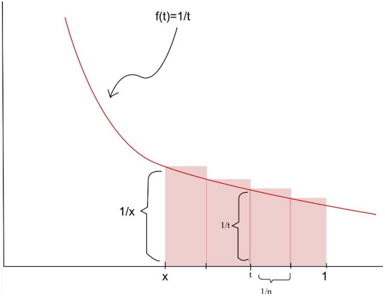

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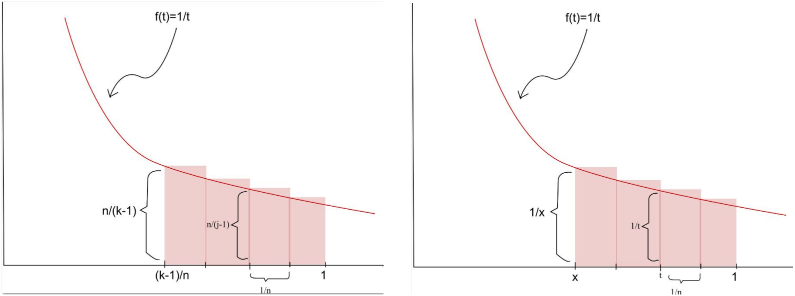

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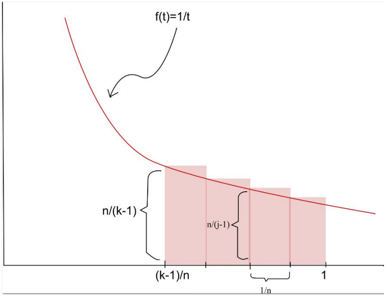

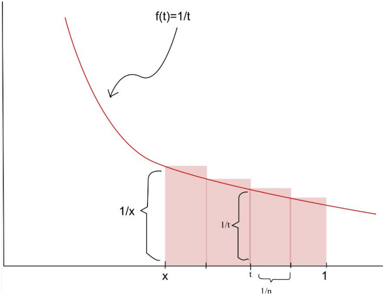

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Summary: Optimal Dating Strategy

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Question: Did any of this help Harrison?

Variations on a theme

- (Lindley) Add a utility function (some value of not choosing the "best").
- (Yang) Possiblity of Recall
- (Smith) Recall and unavailablity
- (Rasmussend and Pliska) Discounting
- (Pressman and Sonin) n unknown
- (Karlin) Random Arrivals
- (Sakaguchi) More than one is best.
- (A. Cayley) Proposed a lottery. A numbers on a slip of paper drawn from a hat. Last drawn number becomes your winnings.
- (Dynkin) Solution using Markov chains.