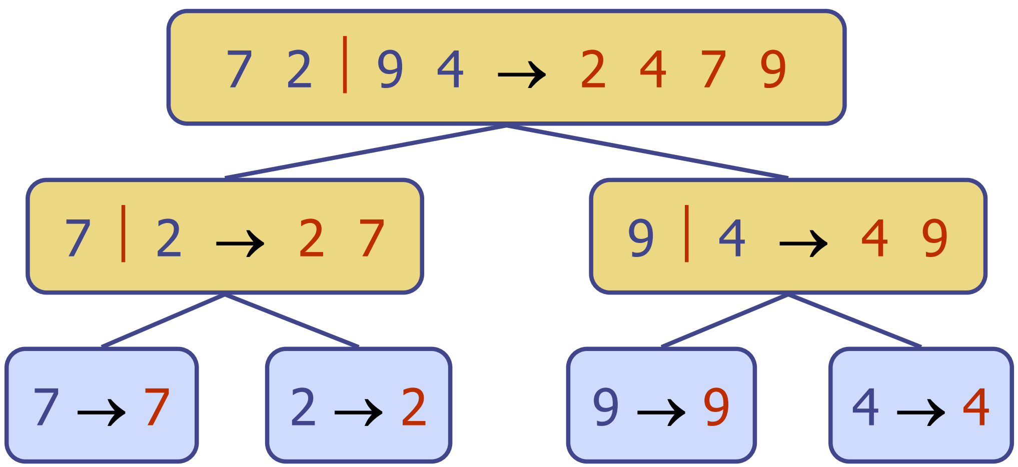
Recursive Sorting



Divide-and-Conquer

- Divide-and conquer is a general algorithm design paradigm:
  - Divide: divide the input data S in two disjoint subsets S_1 and S_2
  - Recurse: solve the subproblems associated with S_1 and S_2
  - Conquer: combine the solutions for S_1 and S_2 into a solution for S
- The base case for the recursion are subproblems of size 0 or 1
- Merge-sort is a sorting algorithm based on the divide-and-conquer paradigm

Better Sorting Through Recursion

- Selection Sort → Quick Sort
- Insertion Sort → Merge Sort

Merge-Sort

- Merge-sort on an input sequence S with n elements consists of three steps:
  - Divide: partition S into two sequences S_1 and S_2 of about $n/2$ elements each
  - Recurse: recursively sort S_1 and S_2
  - Conquer: merge S_1 and S_2 into sorted sequence

```
mergeSort(S)
    if S.size() <= 1
        return

    (S1, S2) = partition(S, 2)
    mergeSort(S1)
    mergeSort(S2)
    S = merge(S1, S2)
```

Merging Two Sorted Sequences

- The conquer step of merge-sort consists of merging two sorted sequences *A* and *B* into a sorted sequence *S* containing the union of the elements of *A* and *B*
- Merging two sorted sequences, each with $n/2$ elements takes *?* time

```
merge(A, B)
    S = array of size A.length + B.length
    sIndex = 0
    aIndex = 0
    bIndex = 0

    while aIndex < A.length and bIndex < B.length
        if A[aIndex ] < B[bIndex ]
            S[sIndex++] = A[aIndex++]
        else
            S[sIndex++] = B[bIndex++]

    while aIndex < A.length
        S[sIndex++] = A[aIndex++]

    while bIndex < B.length
        S[sIndex++] = B[bIndex++]
```

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            S[sIndex++] = B[bIndex++]

    while aIndex < A.length
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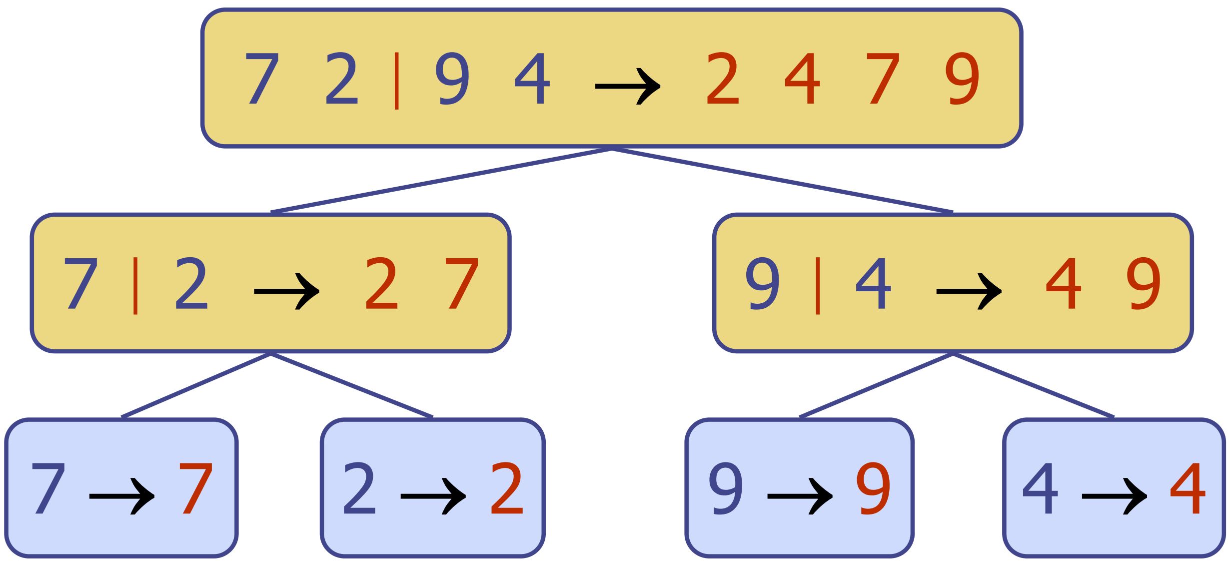
```
merge(A, B)
    S = ArrayList of size A.size() + B.size()
    while A.isEmpty() == false and  B.isEmpty() == false
        if A.get(0) < B.get(0)
            S.add(A.remove(0))
        else
            S.add(B.remove(0))
    while A.isEmpty() == false
        S.add(A.remove(0))
    while B.isEmpty() == false
        S.add(B.remove(0))
    return S
```

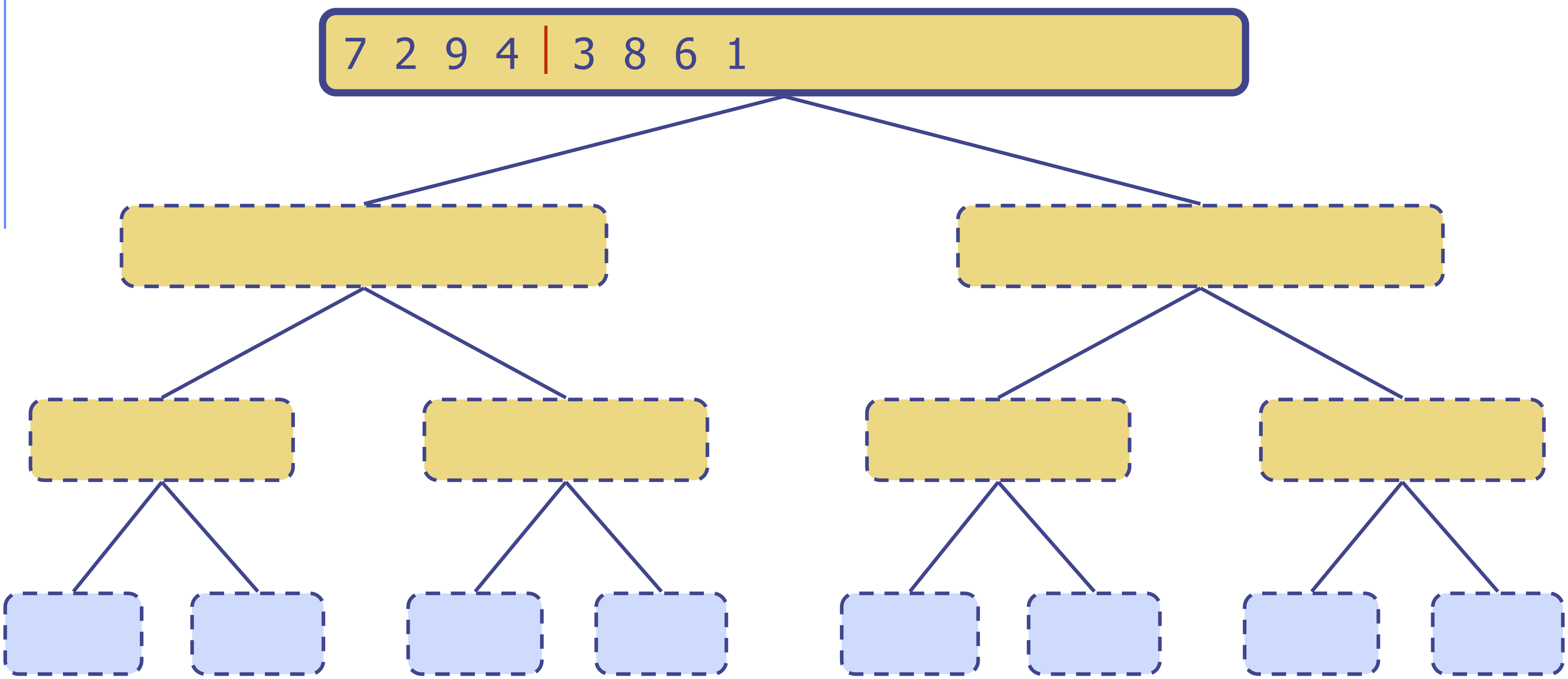
Merge-Sort Tree

- An execution of Merge-Sort can be depicted by a binary tree
  - each node represents a recursive call of Merge-Sort and stores
    - unsorted sequence before the execution and its partition
    - sorted sequence at the end of the execution
  - the root is the initial call
  - the leaves are calls on subsequences of size 0 or 1



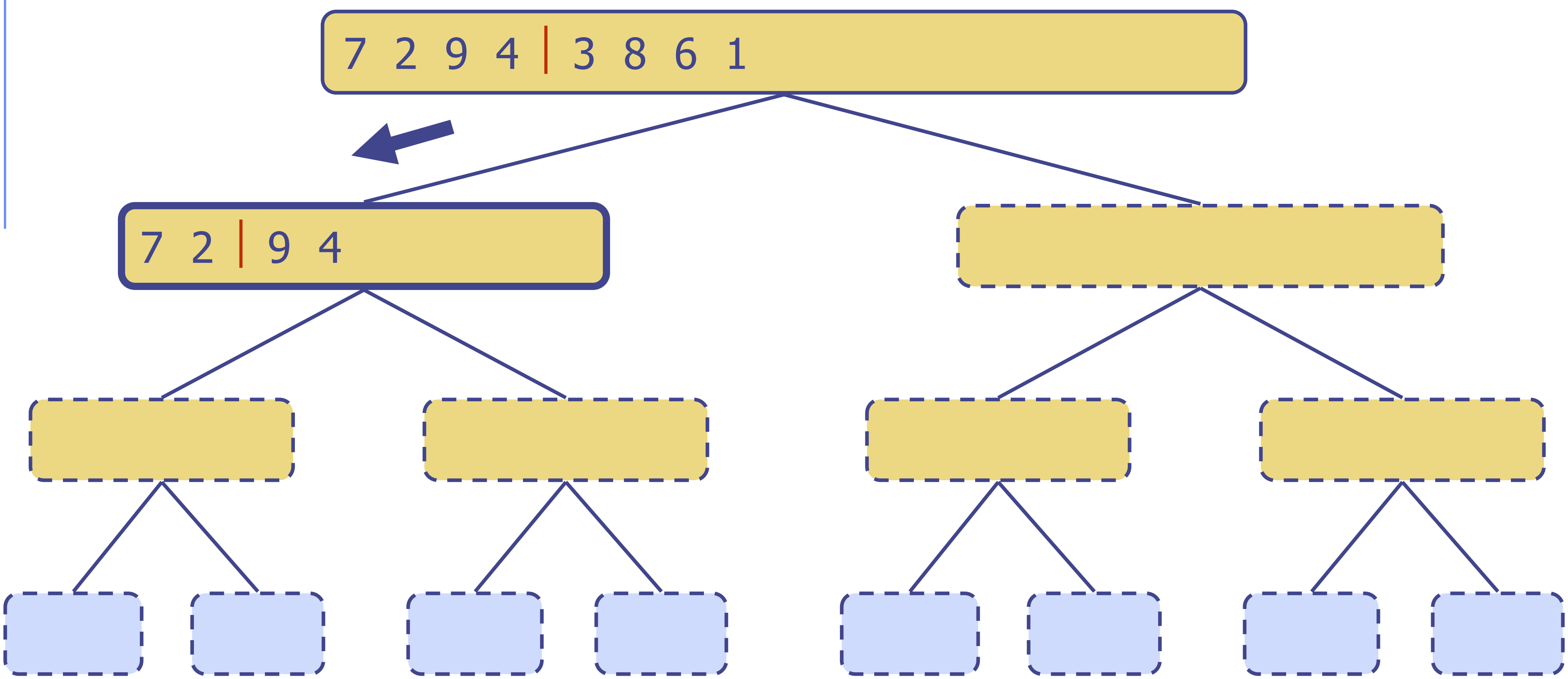
Execution Example

- Partition



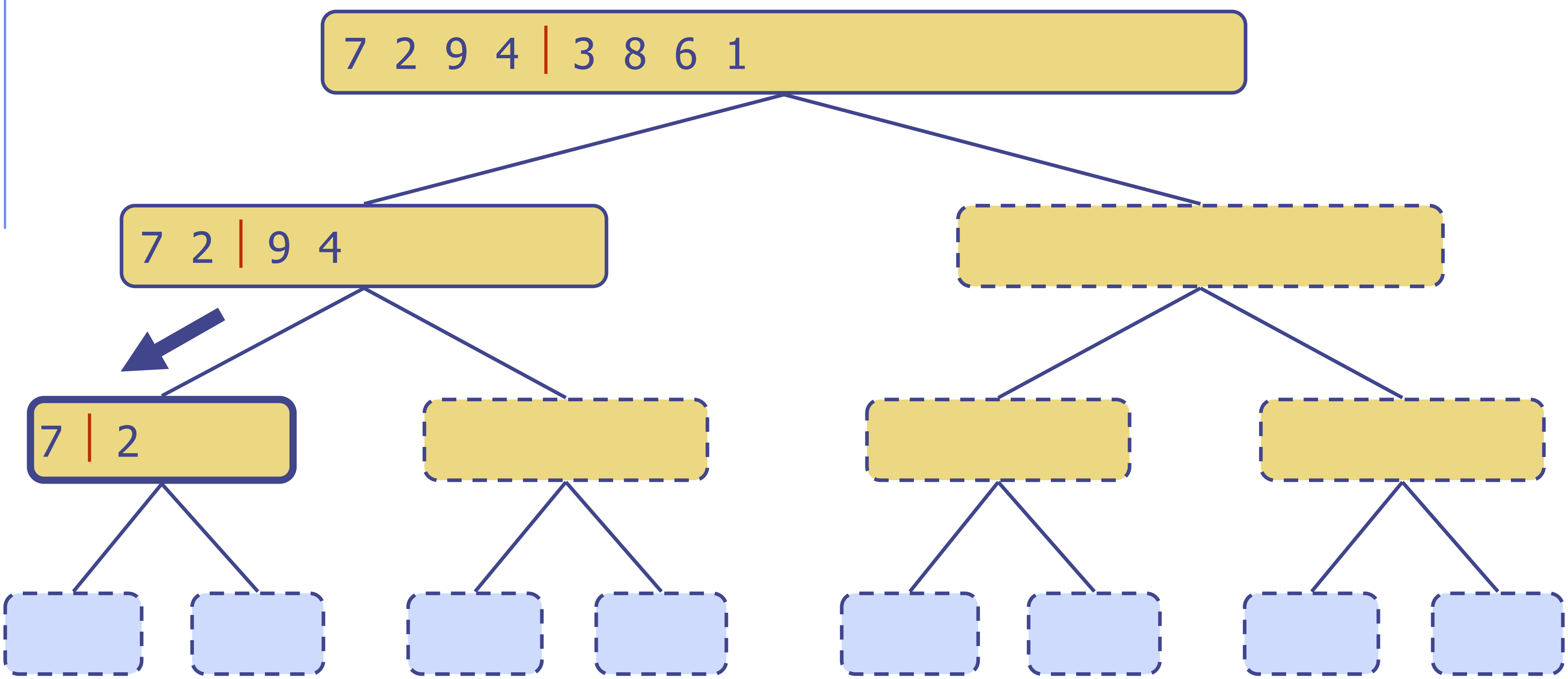
Execution Example (cont.)

- Recursive call, partition



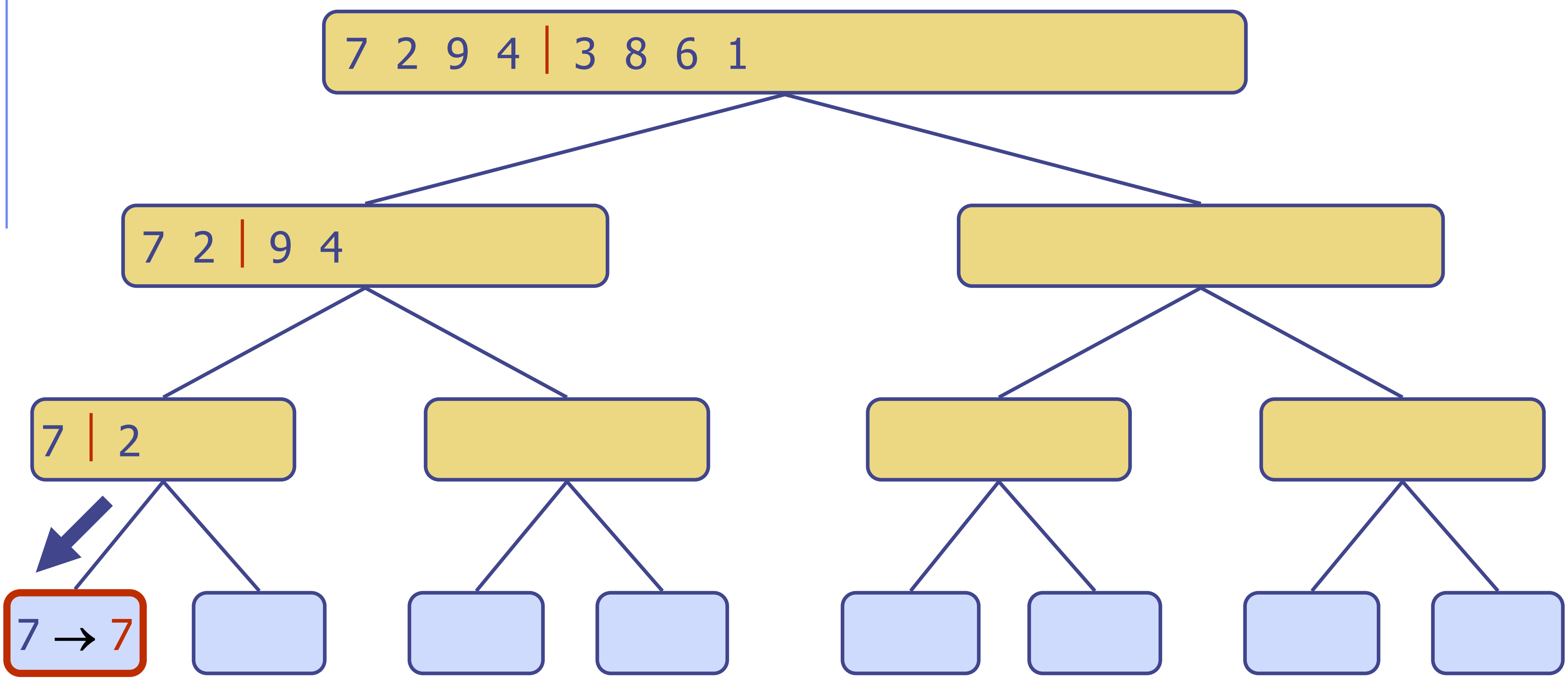
Execution Example (cont.)

- Recursive call, partition



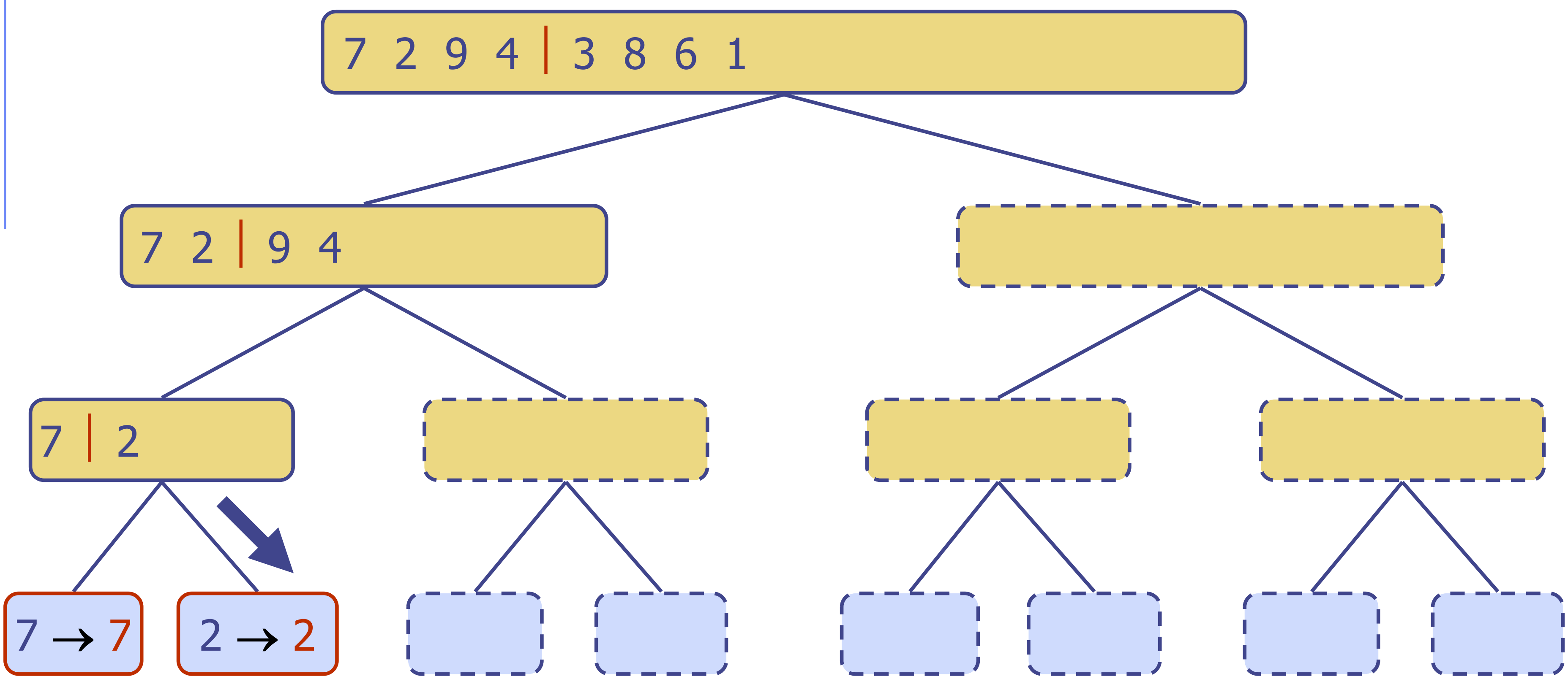
Execution Example (cont.)

- Recursive call, base case



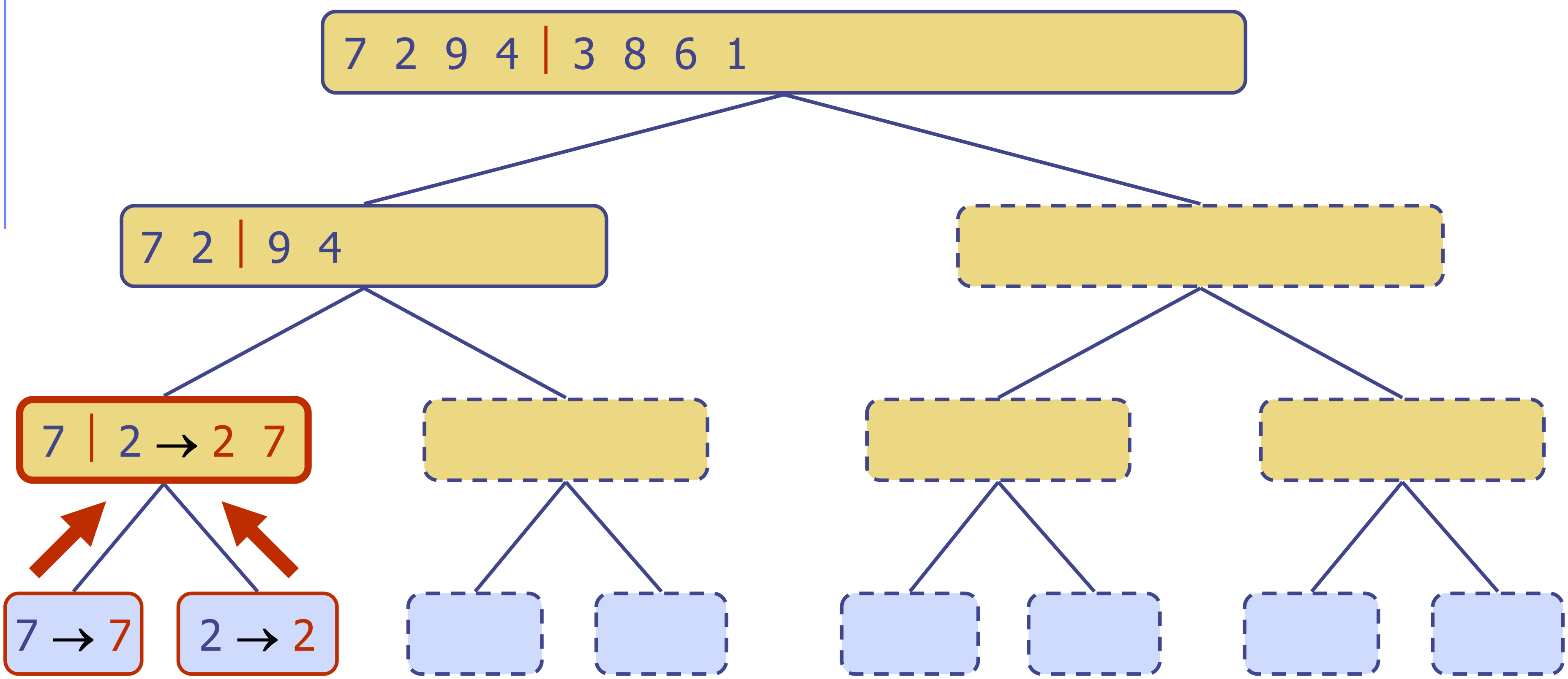
Execution Example (cont.)

- Recursive call, base case



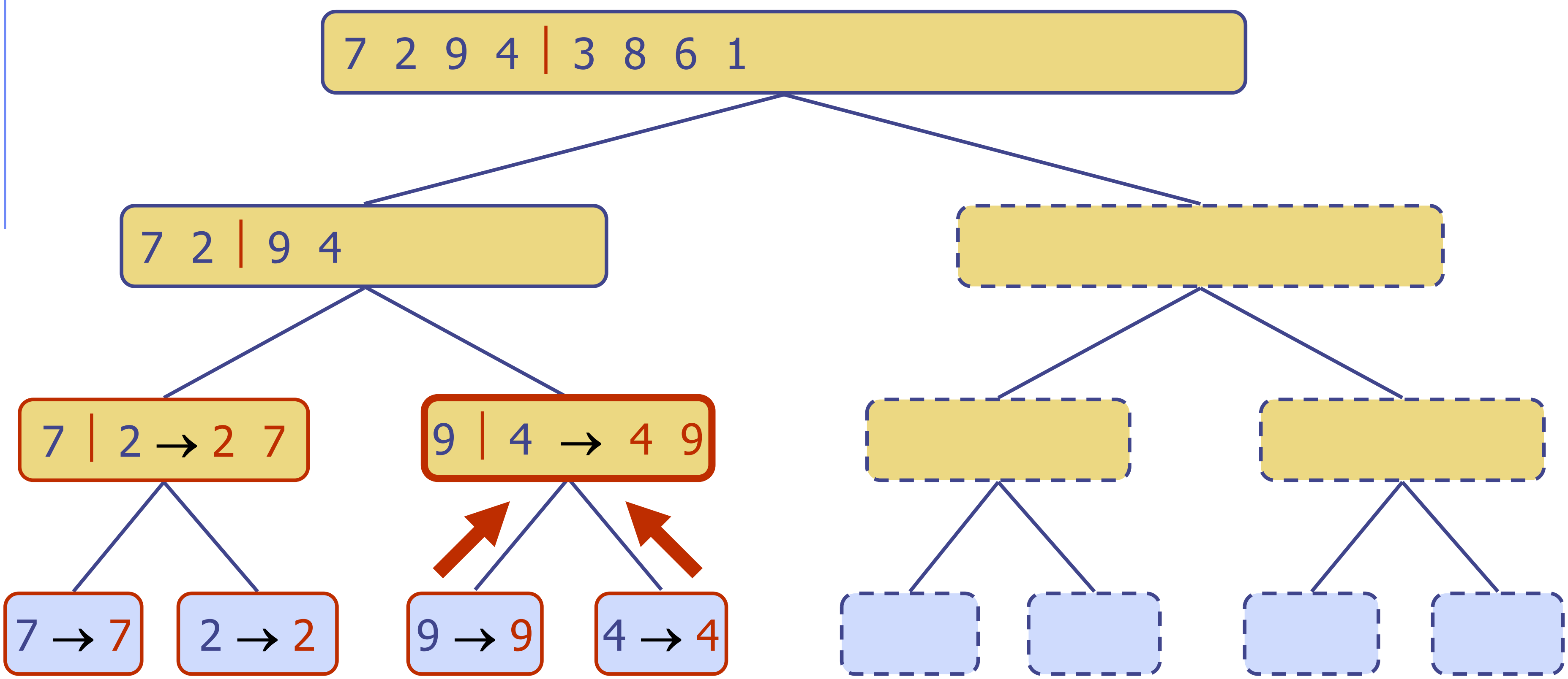
Execution Example (cont.)

- Merge



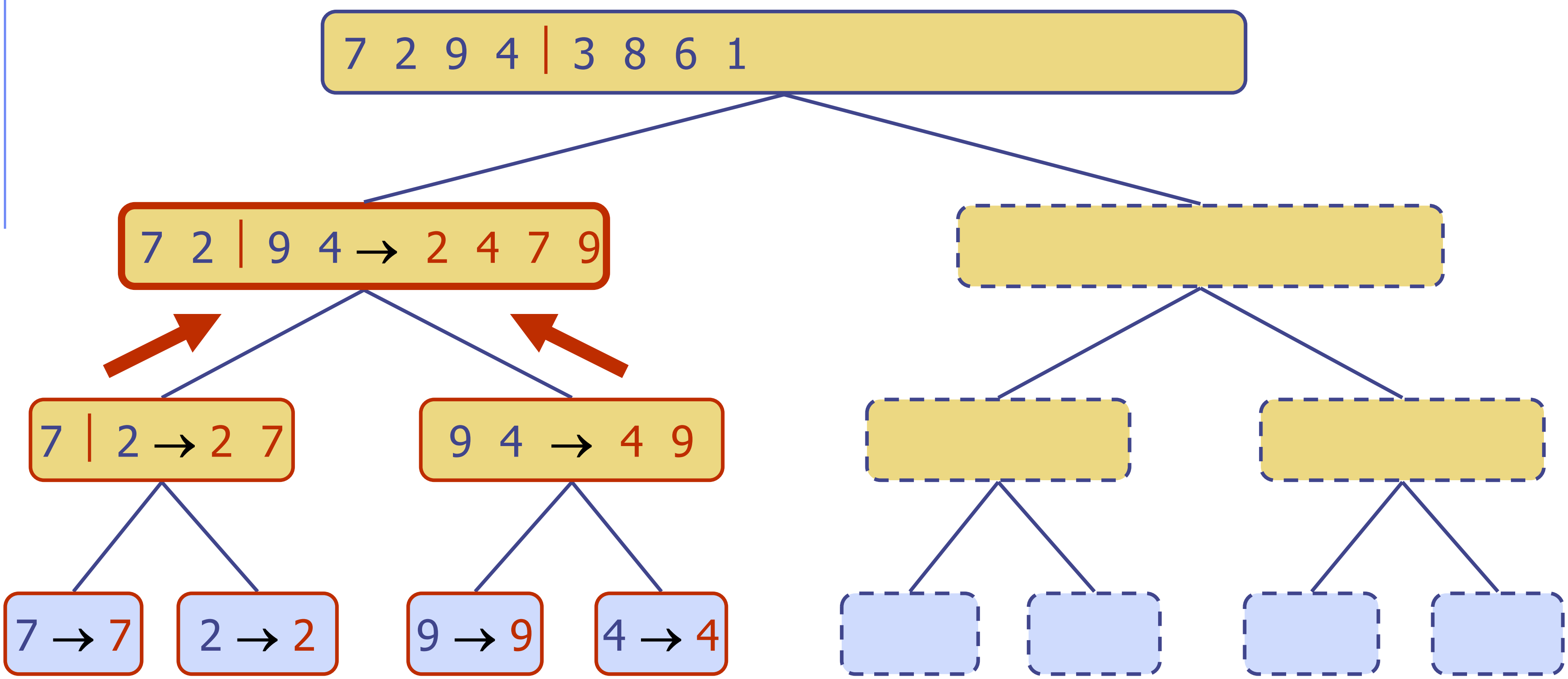
Execution Example (cont.)

- Recursive call, …, base case, merge



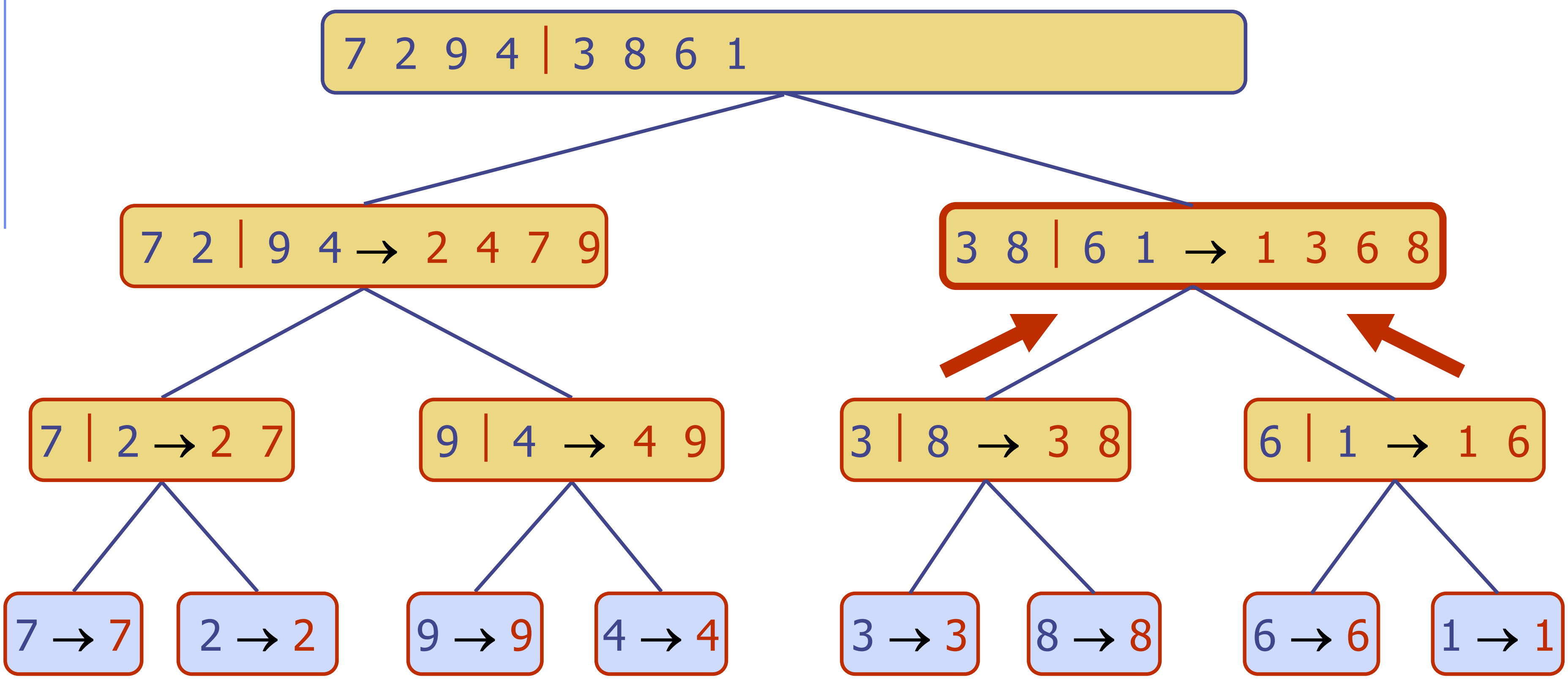
Execution Example (cont.)

- Merge



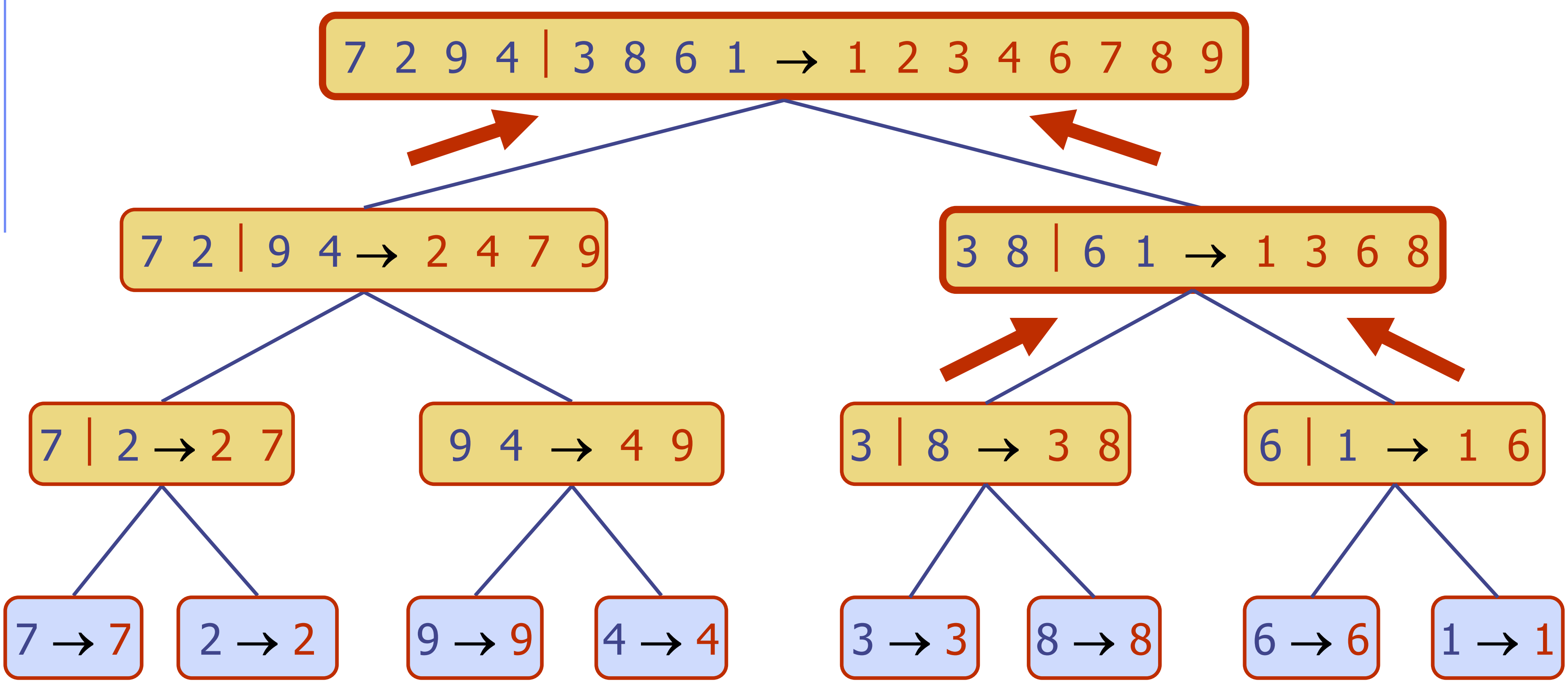
Execution Example (cont.)

- Recursive call, …, merge, merge



Execution Example (cont.)

- Merge



Non-Recursive Merge-Sort

merge runs of length 2, then 4, then 8, and so on

```
public static void mergeSort(Object[] orig, Comparator c) { // nonrecursive
  Object[] in = new Object[orig.length]; // make a new temporary array
  System.arraycopy(orig,0,in,0,in.length); // copy the input
  Object[] out = new Object[in.length]; // output array
  Object[] temp; // temp array reference used for swapping
  int n = in.length;
  for (int i=1; i < n; i*=2) { // each iteration sorts all length-2*i runs
    for (int j=0; j < n; j+=2*i)  // each iteration merges two length-i pairs
      merge(in,out,c,j,i); // merge from in to out two length-i runs at j
    temp = in; in = out; out = temp; // swap arrays for next iteration
  }
  // the "in" array contains the sorted array, so re-copy it
  System.arraycopy(in,0,orig,0,in.length);
}
```

merge two runs in the in array to the out array

```
protected static void merge(Object[] in, Object[] out, Comparator c, int start,
    int inc) { // merge in[start..start+inc-1] and in[start+inc..start+2*inc-1]
  int x = start; // index into run #1
  int end1 = Math.min(start+inc, in.length); // boundary for run #1
  int end2 = Math.min(start+2*inc, in.length); // boundary for run #2
  int y = start+inc; // index into run #2 (could be beyond array boundary)
  int z = start; // index into the out array
  while ((x < end1) && (y < end2))
    if (c.compare(in[x],in[y]) <= 0) out[z++] = in[x++];
    else out[z++] = in[y++];
  if (x < end1) // first run didn't finish
    System.arraycopy(in, x, out, z, end1 - x);
  else if (y < end2) // second run didn't finish
    System.arraycopy(in, y, out, z, end2 - y);
}
```

Visualizations

Sorting Algorithms

Efficiency?

- Can't just count loop iterations!
- How many levels of recursion?
- How much non-recursive work done at each level?
- Need to solve a "recurrence equation"