$$\mathcal{L} = \overbrace{\underbrace{\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\mu^2\phi^2}_{\text{free massive scalar}}}^{\text{quadratic terms} \Rightarrow \text{linear e.o.m}} - \underbrace{\frac{1}{3!}g\phi^3 - \frac{1}{4!}\lambda\phi^4}_{\text{interactions}} + \underbrace{\cdots}_{\substack{\text{higher powers}\\ \text{nonrenormalizable}}}$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - \mu^2|\phi|^2$$

$$= -\frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu) - \mu^2\phi^*\phi + \underbrace{(\partial_\mu\phi + iqA_\mu\phi)(\partial^\mu\phi^* - iqA^\mu\phi^*)}$$

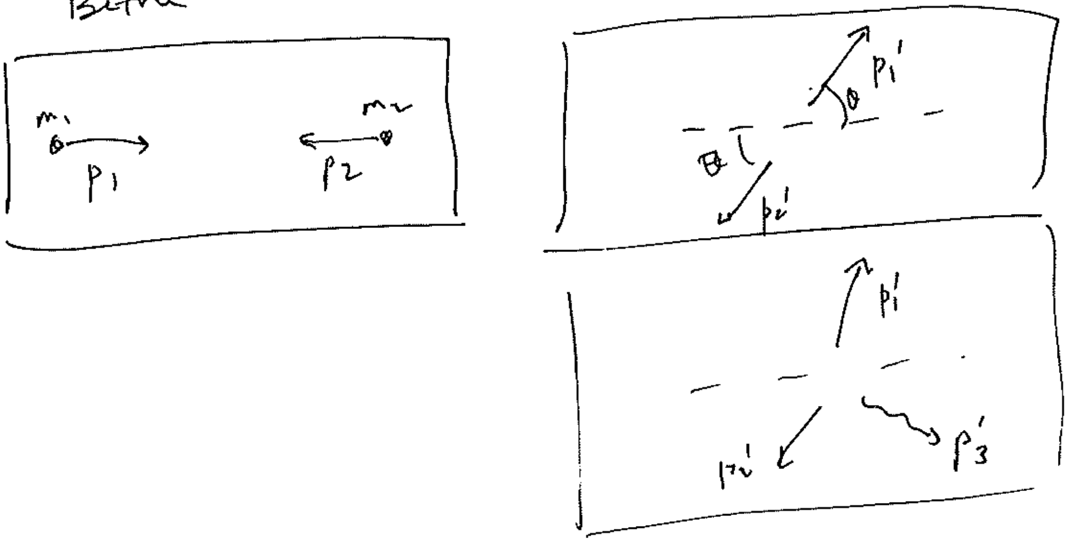
$$= \underbrace{\qquad\qquad\qquad \partial_\mu\phi\,\partial^\mu\phi^*}_{\text{free fields}} + \underbrace{iqA_\mu(\phi\,\partial^\mu\phi^* - \phi^*\partial^\mu\phi) + q^2A_\mu A^\mu\phi^*\phi}_{\text{interactions}}$$

seagull

Scattering processes

Before

After

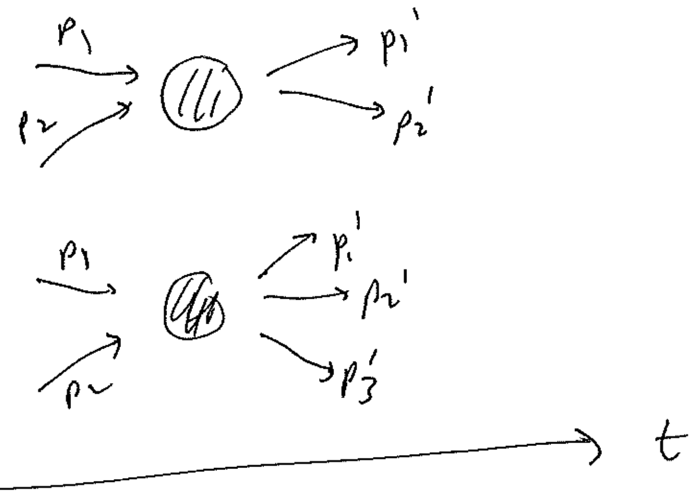


elastic

inelastic

elastic

inelastic



what is probability of each final state?

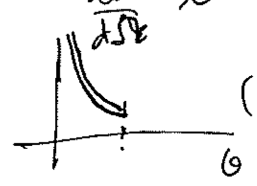
$$P(p_1' p_2' \leftarrow p_1 p_2) = ?$$

actually compute "differential cross-section"

$$\frac{d\sigma}{d\Omega}(p_1' p_2' \leftarrow p_1 p_2)$$

eg: Coulomb scattering

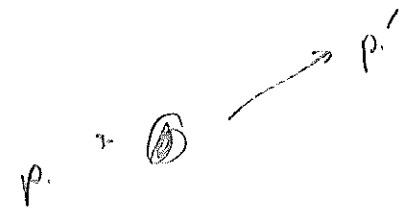
$$\frac{d\sigma}{d\Omega} \sim \frac{1}{\sin^4(\frac{\theta}{2})}$$



(Rutherford)

Prob amplitude $\;_{out}\langle \underset{k_1}{p_1'}\, \underset{k_2}{p_2'} \,|\, \underset{k_1}{p_1}\, \underset{k_2}{p_2} \rangle_{in}$

How about 1 particle



$_{out}\langle k' | k \rangle_{in}$

no interaction here

$|k\rangle = a^\dagger(k)|0\rangle$

$\langle k| = \langle 0|\, a(k)$

$\langle k' | k\rangle = \langle 0|\, a(k')\, a^\dagger(k)\,|0\rangle$

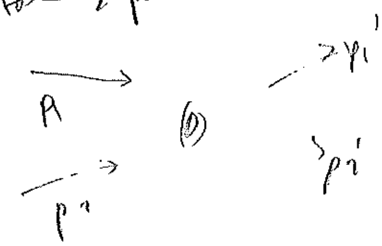
$= \langle 0|\, [a(k'), a^\dagger(k)]\,|0\rangle + a^\dagger(k)\, a(k)\,|0\rangle$

$\hbar(2\pi)^3\, 2\omega_k\, \delta(k-k')$

$= \hbar(2\pi)^3 (2\omega_k)\, \delta(k-k')$

$\neq 0$ iff $k = k'$

~~identical particles~~ 2 particles



If no interact

$\langle p_1'\, p_2' | p_1\, p_2\rangle$

$= \langle 0|\, a_{p_1'}\, a_{p_2'}\, a^\dagger_{p_1}\, a^\dagger_{p_2}\,|0\rangle$

$= 0$ unless $p_1 = p_1'$ and $p_2 = p_2'$

or $p_1' = p_2$ & $p_2' = p_1$

identical

$\delta(p_1 - p_1')\,\delta(p_2 - p_2') + \delta(p_1 - p_2')\,\delta(p_2 - p_1')$

If there are interactions between fields,

we use Dyson's formula (time-dependent pert. th see Sakurai)

$$ {}_{out}\langle p_1' p_2' | p_1 p_2 \rangle_{in} = \langle p_1' p_2' | \; T \; e^{-\frac{i}{\hbar}\int_{-\infty}^{\infty} dt\, H_{int}} \; | p_1 p_2 \rangle $$

time-ordered exponential of interaction Hamiltonian in the interaction picture.

$$ A_H = e^{\frac{iHt}{\hbar}} \hat{A} e^{-\frac{iHt}{\hbar}} $$

$$ A_I = e^{\frac{iH_0 t}{\hbar}} \hat{A} e^{-\frac{iH_0 t}{\hbar}} $$

(Recall time evolution operator $= e^{-\frac{iHt}{\hbar}}$)

If $\mathcal{L}(\phi) = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$

then $H_{int} = \int d^3x : \frac{\lambda}{4!}\phi^4 :$

Expand exponential:

$O(\lambda)$: $\langle k_1' k_2' | \; -\frac{i\lambda}{4!\hbar}\int d^4x \; \phi^4(x) \; | k_1 k_2 \rangle$

but $\phi = \int \tilde{dk} \left[a(k) e^{-ikx} + a^\dagger(k) e^{ikx} \right]$

$\phi(x) | k_1 \rangle = \int \tilde{dk}\; a(k) e^{-ikx} | k_1 \rangle = e^{-ik_1 x} | 0 \rangle$

$\langle p_1' | \phi(x) = \langle p_1' | e^{ik_1' x}$

$$ \langle p_1' p_2' | \left(\frac{-i\lambda}{4!\hbar}\right) a(k_1') a(k_2') a^\dagger(k_1) a(k_2) \underbrace{\int d^4x \; e^{i(k_1+k_2-k_1'-k_2')x}}_{(2\pi)^4 \delta^{(4)}(k_1+k_2-k_1'-k_2')} | p_1 p_2 \rangle $$

momentum conservation!

amplitude $\approx -\frac{i\lambda}{\hbar} \delta^{(4)}(p_1+p_2-p_1'-p_2')(2\pi)^4$

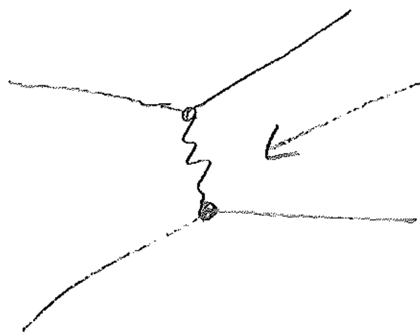
$\longrightarrow$ Isotropic

~~2nd order~~

$$H_{int} \sim \phi \partial_\mu \phi A^\mu$$

$$\langle p_1' p_2' | \, T\left(H_{int}(t_1) \, H_{int}(t_2) \right) | p_1 p_2 \rangle$$

$$= \langle p_1' p_2' | \, a^\dagger a^\dagger \, T(A^\mu(x_1) A^\mu(x_2)) \, a a \, | p_1 p_2 \rangle$$



propagator

"exchange of photon"

gives Coulomb cross-section