Classical field theory: scalar fields

[We want our fields to be consistent w/ special relativity so choose them to have well defined transformations under Lorentz transformations: boosts + rotations. Simplest is a scalar field (Lorentz scalar)]

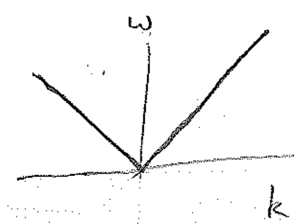
Wave travelling at speed of light obeys

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi = 0$$

Ansatz: $\phi = e^{i(\vec{k}\cdot\vec{x} - \omega t)}$

$$-\frac{\omega^2}{c^2}\phi + \vec{k}^2\phi = 0$$

Ansatz valid provided $\omega = c|\vec{k}|$



If $E = \hbar\omega$ (Einstein) & $\vec{p} = \hbar\vec{k}$ (de Broglie)

then $E = c|\vec{p}|$, holds for massless particles

Modify wave equation (maintaining linearity)

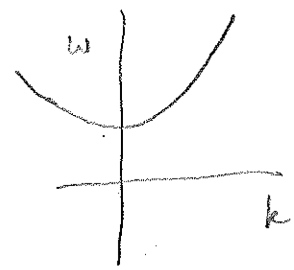
$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi + \mu^2 c^2 \phi = 0 \qquad \text{(Klein-Gordon eqn)}$$

[which as you might guess from the name was first written down by Schrödinger]
[Coleman, p. 43]

Travelling wave ansatz gives

$$\left(-\frac{\omega^2}{c^2} + k^2 + \mu^2 c^2\right)\phi = 0$$

valid provided $\omega = \sqrt{(ck)^2 + (\mu c^2)^2}$



"dispersion relation"

$$E = \sqrt{(cp)^2 + (\mu\hbar c^2)^2}$$

If we let $m = \mu\hbar$ then this describes a relativistic massive particle

$$E = \sqrt{(cp)^2 + (mc^2)^2}$$

Units: MKS = meters - kg - sec

length, mass, time

length + time related by c

mass, momentum, energy related by $c + c^2$

Set $c = 1$ $\Rightarrow$ [length] = [time]

[mass] = [momentum] = [energy]

$$E = \sqrt{p^2 + m^2}$$

$$\omega = \sqrt{k^2 + \mu^2}$$

ω, k, μ have units $\frac{1}{[\text{length}]}$

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length + momentum are related by $\hbar$

$$\Delta p \, \Delta x \sim \hbar$$

$$[\text{momentum}] = \frac{\hbar}{[\text{length}]}$$

If $\hbar = 1$, then only one unit left (length, mass, or time)

[Most QFT books set $\hbar = 1$ as well as $c = 1$
but we will try not to.]

but I may forget