Particle in a potential

$$E = T + V = \frac{p^2}{2m} + V(x)$$ non rel. [recall Bohr model]

$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V}(x)$$

[What is $\hat{V}$?]

Power series expan. of $V(x) = V_0 + V_1 x + V_2 x^2 + \ldots$

$$\Rightarrow \hat{V} = V_0 + V_1 \hat{X} + V_2 \hat{X}^2 + \ldots$$

$\hat{p}, \hat{x}$ are hermitian

$\hat{V}$ is hermitian if $V(x)$ is a real function, ie $V_0, V_1 \ldots$ are real

$$\hat{V}^\dagger = V_0^* + V_1^* \hat{X}^\dagger + V_2^* (\hat{X}^2)^\dagger + \ldots$$

But $(\hat{X}^n)^\dagger = \hat{X}^\dagger \hat{X}^\dagger \ldots \hat{X}^\dagger = \hat{X}^n$

$$\Rightarrow \hat{V}^\dagger = \hat{V}$$

$\Rightarrow \hat{H}$ is hermitian

Energy eigenvalue equation

$$\hat{H}|E\rangle = E|E\rangle$$

In position space

$$\underbrace{\langle x|\hat{H}|E\rangle}_{\equiv H_{pos}\langle x|E\rangle} = E\langle x|E\rangle$$

Define $\langle x|E\rangle = u_E(x)$ - energy eigenfunction in position space

$$H_{pos}\, u_E(x) = E\, u_E(x)$$

Particle in a potential

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

$$\Rightarrow H_{pos} = \frac{1}{2m}\left(\frac{\hbar}{i}\frac{d}{dx}\right)^2 + V(x)$$

$$-\frac{\hbar^2}{2m}\frac{d^2u_E}{dx^2} + V(x)\,u_E(x) = E\,u_E(x)$$

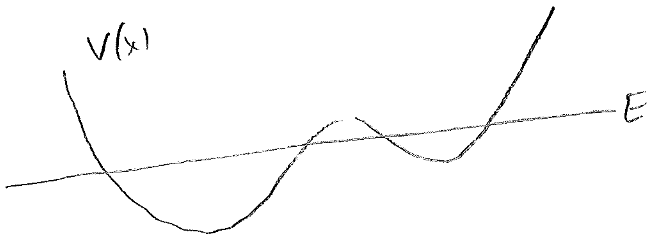
t.i.S.e. in position space [we solved this in 2140 for ptl in box, square well & harmonic oscillator]

please refer to notes

$$\Rightarrow \frac{1}{u_E}\frac{d^2u_E}{dx^2} = -\frac{2m}{\hbar^2}(E - V(x))$$

Qualitative solutions of t.i.s.e

t.i.s.e. $\Rightarrow$ $$\boxed{\frac{1}{u_E}\frac{d^2 u_E}{dx^2} = -\frac{2m}{\hbar^2}\left(E - V(x)\right)}$$

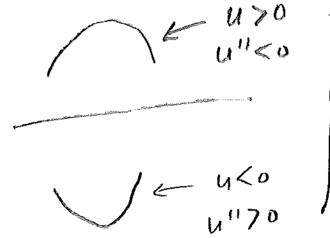


classically allowed regions $\Rightarrow E - V(x) > 0 \Rightarrow \frac{u''}{u} < 0 \Rightarrow$ concave toward x-axis

classically forbidden regions $\Rightarrow E - V(x) < 0 \Rightarrow \frac{u''}{u} > 0 \Rightarrow$ concave away from axis

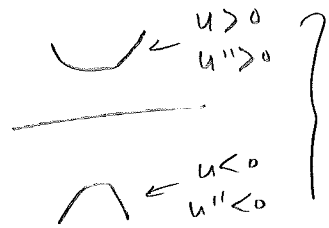
turning point $\Rightarrow E - V(x) = 0 \Rightarrow \frac{u''}{u} = 0 \Rightarrow$ point of inflection

concave toward axis



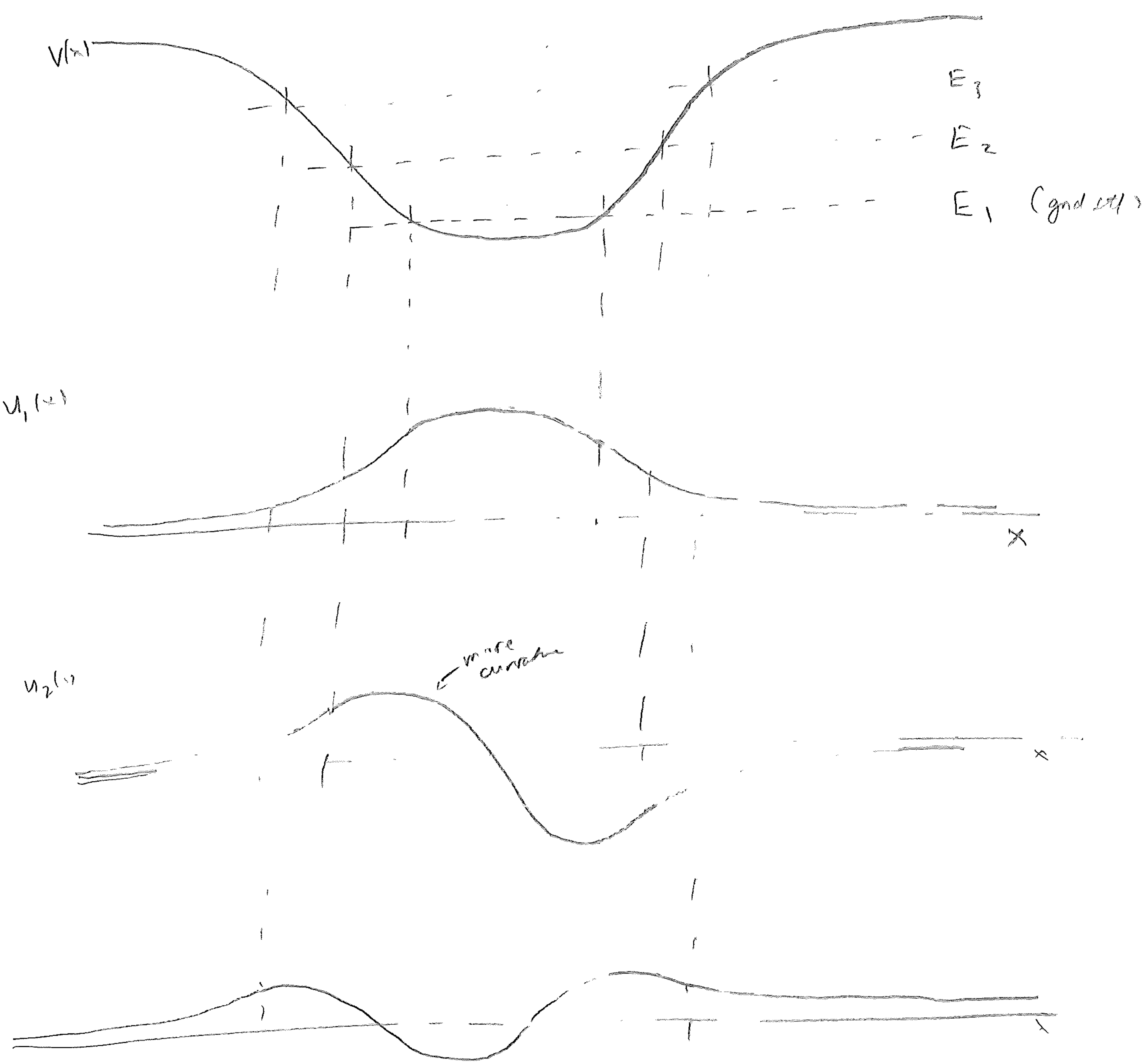
$\frac{u''}{u} < 0$

concave away from axis



$\frac{u''}{u} > 0$

Suppose the potential has the shape



For energies between the eigenvalues, $u(x) \not\to 0$ as $x \to \infty$

Suppose $E < V_{min} \Rightarrow$ all regions forbidden
$\Rightarrow$ always concave ... $\Rightarrow u(x) \not\to 0$
not normalizable

$\Rightarrow \nexists$ no energy eigenvalue $\forall E < V_{min}$