3 dimensions

position operators $\hat{X}, \hat{Y}, \hat{Z}$ or $\hat{X}_i$ $(i = 1, 2, 3)$

momentum operators $\hat{P}_x, \hat{P}_y, \hat{P}_z$ or $\hat{P}_i$ $(i = 1, 2, 3)$

$[\hat{X}_i, \hat{X}_j] = 0$

$[\hat{P}_i, \hat{P}_j] = 0$ } necessary to do this in order to have them compute $[\hat{L}_i, \hat{L}_j]$ later

$[\hat{X}_i, \hat{P}_j] = i\hbar \delta_{ij}$

To prove these, use position space operators

$\hat{X}_i \rightarrow x_i$ $\quad \Rightarrow [x_i, x_j]\psi = 0$

$\hat{P}_i \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial x_i}$ $\quad \Rightarrow \left[\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j}\right]\psi = 0$

$$[\hat{X}_i, \hat{P}_j]|\psi\rangle \rightarrow x_i \frac{\hbar}{i}\frac{\partial}{\partial x_j}\psi - \frac{\hbar}{i}\frac{\partial}{\partial x_j}(x_i \psi)$$

$$= i\hbar \frac{\partial x_i}{\partial x_j}\psi = i\hbar \delta_{ij}\psi$$

use this to derive uncertainty relations $\quad \Delta x \, \Delta p_x \geq \frac{\hbar}{2}$

$$\Delta y \, \Delta p_y \geq \frac{\hbar}{2}$$

$$\Delta z \, \Delta p_z \geq \frac{\hbar}{2}$$

but no constraints on $\Delta x \, \Delta p_y$ for example

(Also $\underset{\sim}{\Delta t}\, \Delta E \geq \frac{\hbar}{2}$)

τ = decay time of a state

Let $|\vec{x}\rangle = |x_1, x_2, x_3\rangle$ be a mutual eigenstate 51-2

$X_i |x_i\rangle = x_i |x_i\rangle$

$$|x_1 x_2 x_3\rangle = |\vec{x}\rangle = \text{particle "localized" at a pt. } (x_1, x_2, x_3)$$

orthonormality $\underbrace{\langle x_1', x_2', x_3' | x_1, x_2, x_3\rangle}_{\langle \vec{x}' | \vec{x}\rangle} = \underbrace{\delta(x_1 - x_1')\,\delta(x_2 - x_2')\,\delta(x_3 - x_3')}_{\text{call this } \delta^{(3)}(\vec{x} - \vec{x}')}$

completeness $\int dx_1 dx_2 dx_3 \, |x_1 x_2 x_3\rangle\langle x_1 x_2 x_3|$

$$= \int d^3x \, |\vec{x}\rangle\langle\vec{x}| = \mathbb{1}$$

arbitrary state $|\psi\rangle = \int d^3x \, |\vec{x}\rangle \underbrace{\langle\vec{x}|\psi\rangle}_{}$

$\psi(x_1, x_2, x_3) = \psi(\vec{x})$ = wavefunction in 3d position space

normalization

$$\langle\psi|\psi\rangle = \int d^3x \, \underbrace{\langle\psi|\vec{x}\rangle}_{\psi^*(\vec{x})} \underbrace{\langle\vec{x}|\psi\rangle}_{\psi(\vec{x})} = \int d^3x \, |\psi(\vec{x})|^2$$

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momentum space wavefunction

$$|\psi\rangle = \int d^3p \, |\vec{p}\rangle \underbrace{\langle\vec{p}|\psi\rangle}_{\phi(\vec{p})}$$

$$\langle\vec{x}|\vec{p}\rangle = \frac{1}{(2\pi\hbar)^{3/2}} e^{\frac{i\vec{p}\cdot\vec{x}}{\hbar}}$$

$$\left.\begin{aligned} \psi(\vec{x}) &= \frac{1}{(2\pi\hbar)^{3/2}} \int d\vec{p} \, e^{\frac{i\vec{p}\cdot\vec{x}}{\hbar}} \phi(\vec{p}) \\ \phi(\vec{p}) &= \frac{1}{(2\pi\hbar)^{3/2}} \int d\vec{x} \, e^{-\frac{i\vec{p}\cdot\vec{x}}{\hbar}} \psi(\vec{x}) \end{aligned}\right\} \text{Fourier transform in 3d}$$