2014

Zeeman effect

rewrite, to skip!

orbital motion of an electron $\Rightarrow \vec{\mu} = -\frac{e}{2m_e}\vec{L}$

Spin also contributes to magnetic moment $\vec{\mu} = -g\frac{e}{2m_e}\vec{S}$ [already did this]

Dirac equation for electron predicts $g = 2$

(QED corrections change this slightly: $g = 2.002319\ldots$)

$g = 2 + \frac{\alpha}{\pi} + O(\alpha^2)$

In a magnetic field

$$\delta E = -\vec{\mu}\cdot\vec{B} = \frac{e}{2m_e}(\vec{L} + g\vec{S})\cdot\vec{B}$$

Suppose $\vec{B} = B\hat{z} \Rightarrow \delta E = \frac{eB}{2m_e}(L_z + gS_z)$

So 2 corrections to atomic energy levels: ⓐ fine structure: $\delta E \sim$ rel + $\vec{L}\cdot\vec{S}$ (spin orbit)

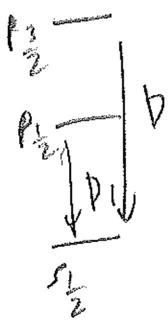
ⓑ Zeeman $\delta E \sim B(L_z + gS_z)$

but these don't commute

2 regimes

① Strong magnetic field: Zeeman >> fine str, ; use product state $|m_\ell; m_s\rangle$

then $\delta E = \frac{eB\hbar}{2m_e}(\underbrace{m_\ell + \overset{\approx 2}{g} m_s}_{\text{integer}})$ [normal Zeeman]



~~In~~ (2) weak magnetic field, ?

Zeeman splitting << hyperfine splitting

so must use [$\vec L\cdot\vec S$ eigenstates] ($\vec J^2$, J_z eigenstates) rather than L_z, S_z eigenstates

$|j,m\rangle$

Then a perturbation theory calculation gives [Wigner-Eckart] → see next

$$\delta E = \langle j,m| -\vec\mu\cdot\vec B\,|j,m\rangle = \mu_B B\,(g_j\, m)$$

where g_j = Landé g-factor, μ_B = Bohr mag = $\frac{e\hbar}{2m_e} = 5.79E{-5}\,\frac{eV}{T}$

$$g_j = 1 + (g-1)\left[\frac{j(j+1) + \frac{3}{4} - l(l+1)}{2j(j+1)}\right]$$

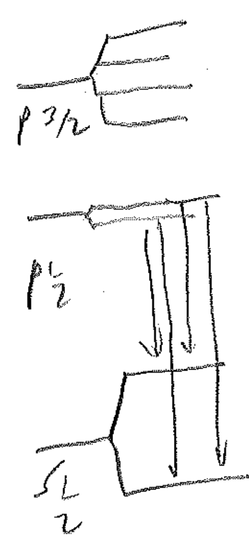
$\hookrightarrow j = l \pm \frac{1}{2} \Rightarrow$

Let $g = 2$

$l = 0,\quad j = \frac{1}{2} \Rightarrow g_j = 2$

$l = 1,\quad j = \frac{1}{2} \Rightarrow g_j = \frac{2}{3}$

$\qquad\quad j = \frac{3}{2} \Rightarrow g_j = \frac{4}{3}$



rework on chap?

[HW. compute wavelength shifts $\Delta\lambda$ & discover selection rules]

Instead of ~~[illegible]~~ invoking a mysterious part-K, Wigner-Eckhart calculation we could do this (following Sakurai) as follows

$$\vec{\mu} = \gamma(\vec{L} + g\vec{S})$$
$$= \gamma(\vec{J} + [g-1]\vec{S}).$$

Then evaluate $\vec{\mu}\cdot\vec{B}$ between $\langle j, m|$ and $|j, m\rangle$

where $j = l \pm \frac{1}{2}$. Since

$$|l \pm \tfrac{1}{2}, m\rangle = \pm\sqrt{\frac{l \pm m + \frac{1}{2}}{2l+1}}\,|m-\tfrac{1}{2}; \tfrac{1}{2}\rangle + \sqrt{\frac{l \mp m + \frac{1}{2}}{2l+1}}\,|m+\tfrac{1}{2}; -\tfrac{1}{2}\rangle$$

so we have

$$\langle l \pm \tfrac{1}{2}, m|\, S_z\, |l \pm \tfrac{1}{2}, m\rangle = \left(\frac{l \pm m + \frac{1}{2}}{2l+1}\right)\left(\frac{\hbar}{2}\right) + \left(\frac{l \mp m + \frac{1}{2}}{(2l+1)}\right)\left(-\frac{\hbar}{2}\right)$$

$$= \frac{\pm m\hbar}{2l+1}$$

Then $\langle l \pm \frac{1}{2}, m|-\vec{\mu}\cdot\vec{B}\,|l \pm \frac{1}{2}, m\rangle$

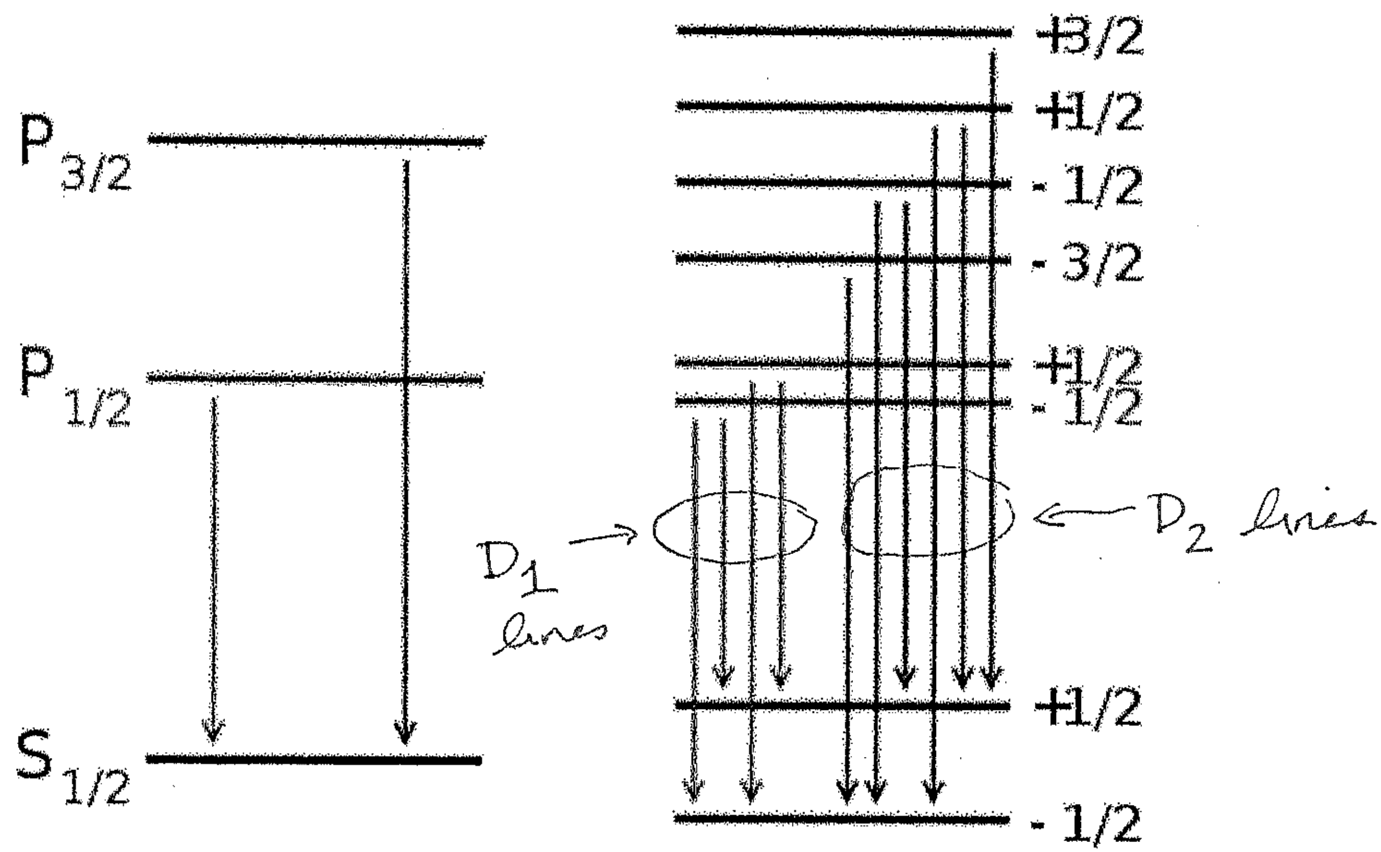
$$= -\gamma B\left[\underbrace{\langle l \pm \tfrac{1}{2}, m|\; J_z + (g-1)S_z\; |l \pm \tfrac{1}{2}, m\rangle}_{m\hbar + (g-1)\frac{\pm m\hbar}{2l+1}}\right]$$

$$= \mu_B B\, g_j m \quad \text{where} \quad \boxed{g_j = 1 + (g-1)\left[\frac{\pm 1}{2l+1}\right]}$$

$j = \frac{1}{2}, l = 0 \Rightarrow 1 + (g-1)1 = 2$

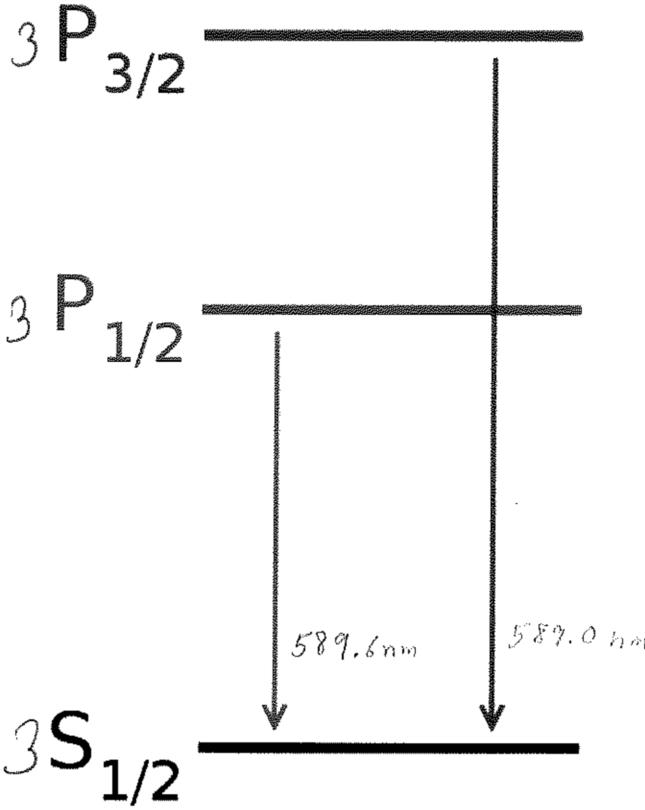
$j = \frac{1}{2}, l = 1 \quad 1 + (g-1)(-\frac{1}{3}) = \frac{2}{3}$

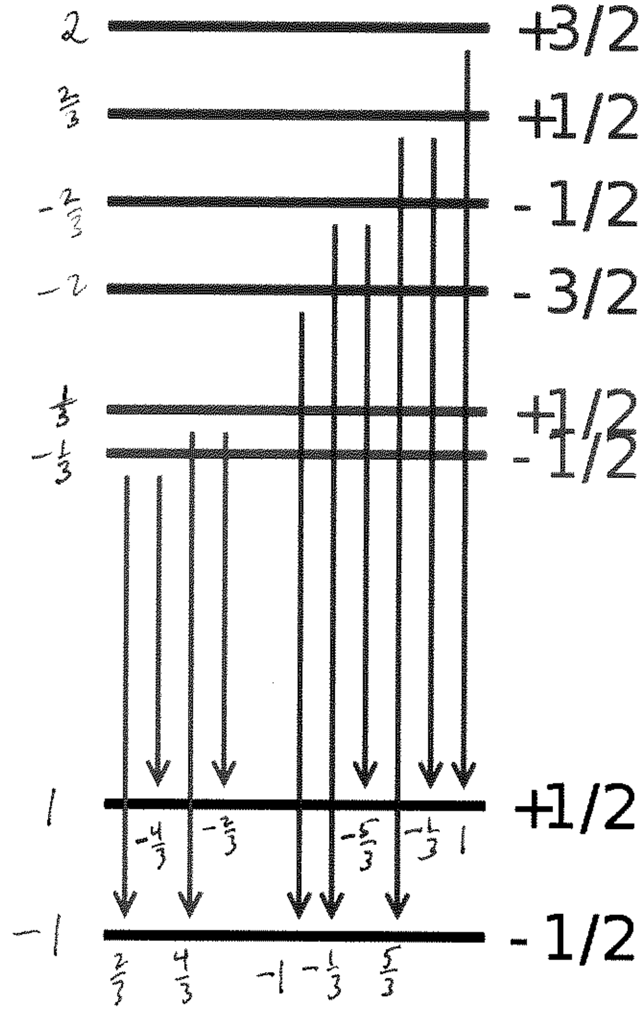
$j = \frac{3}{2}, l = 1, \quad 1 + (g-1)(\frac{1}{3}) = \frac{4}{3}$

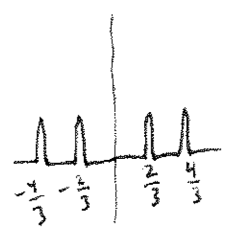


Sodium Zeeman effect (weak field)

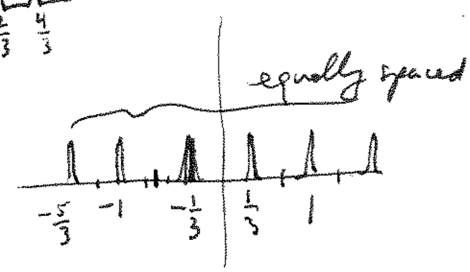
Sodium Zeeman







D_1



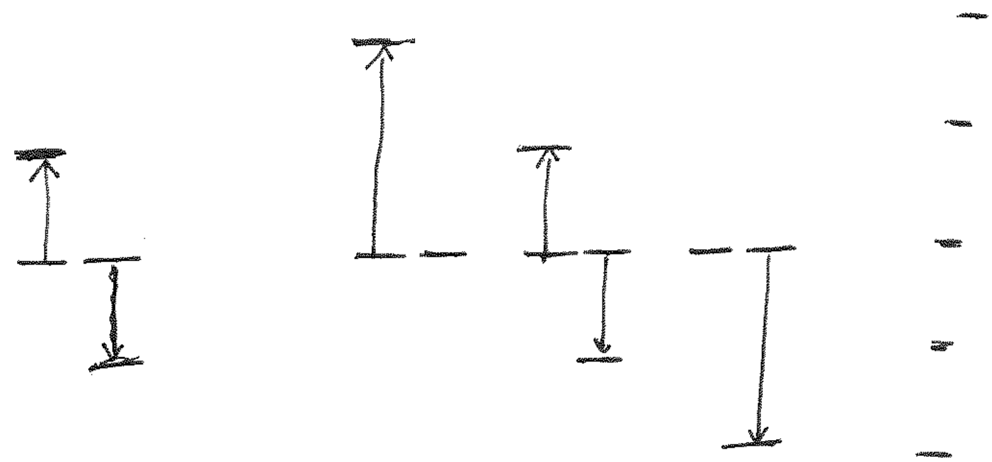
D_2

This is just for me

2s 2p

L_z, S_z eigenstates

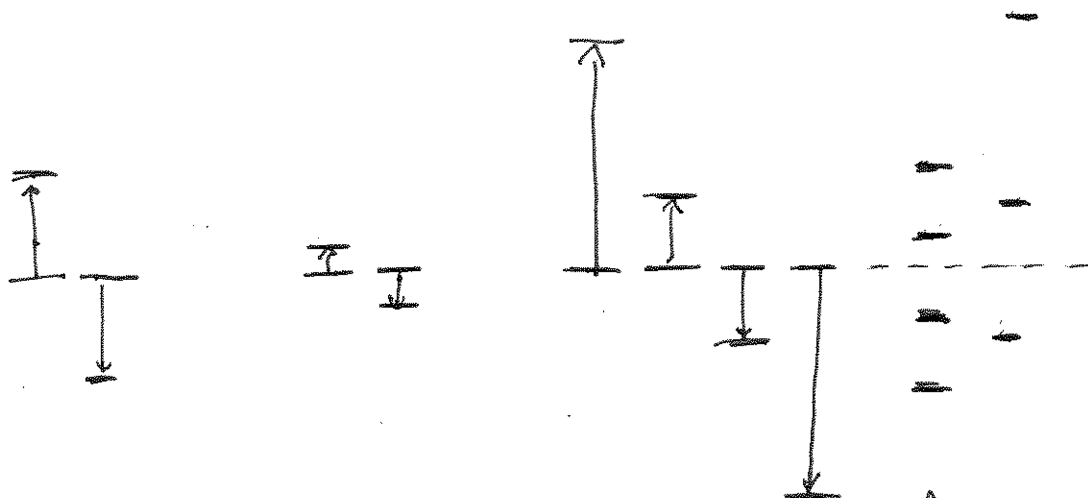
$\Delta E = \frac{e\hbar B}{2m}(m_\ell + 2m_s)$



$m_\ell + 2m_s$ 1 −1 2 0 1 −1 0 2

J^2, J_z eigenstate

$$\Delta E = \frac{e\hbar B}{2m}\begin{cases} 2m_j & \text{for } s_{1/2} \\ \frac{2}{3}m_j & \text{for } p_{1/2} \\ \frac{4}{3}m_j & \text{for } p_{3/2} \end{cases}$$



1 −1 $\frac{1}{3}$ $-\frac{1}{3}$ 2 $\frac{2}{3}$ $-\frac{2}{3}$ −2

$\underbrace{S_{\frac{1}{2}} + P_{\frac{1}{2}}}$ $P_{3/2}$

these are degenerate for hydrogen w/o any field spin-orbit effect and decouples **L** and **S** so that they precess about **B** nearly independently; thus, the projections of **L** behave as if $\mathbf{S} \approx 0$, and the effect reduces to three lines, each of which is a closely spaced doublet.

EXAMPLE 7-5 Magnetic Field of the Sun The magnetic field of the Sun and stars can be determined by measuring the Zeeman-effect splitting of spectral lines. Suppose that the sodium D_1 line emitted in a particular region of the solar disk is observed to be split into the four-component Zeeman effect (see Figure 7-30). What is the strength of the solar magnetic field B in that region if the wavelength difference $\Delta\lambda$ between the shortest and the longest wavelengths is 0.022 nm? (The wavelength of the D_1 line is 589.8 nm.)

SOLUTION

The D_1 line is emitted in the $3^2P_{1/2} \rightarrow 3^2S_{1/2}$. From Equation 7-72 we compute the Landé g factors to use in computing the ΔE values from Equation 7-71 as follows:

For the $3^2P_{1/2}$ level:

$$g = 1 + \frac{1/2(1/2+1) + 1/2(1/2+1) - 1(1+1)}{(2)(1/2)(1/2+1)} = 2/3$$

For the $3^2S_{1/2}$ level:

$$g = 1 + \frac{1/2(1/2+1) + 1/2(1/2+1) - 0}{(2)(1/2)(1/2+1)} = 2$$

and from Equation 7-71,

For the $3^2P_{1/2}$ level:

$$\Delta E = (2/3)(\pm 1/2)(5.79 \times 10^{-9}\ \text{eV/gauss})B$$

For the $3^2S_{1/2}$ level:

$$\Delta E = (2)(\pm 1/2)(5.79 \times 10^{-9}\ \text{eV/gauss})B$$

The longest-wavelength line ($m_j = -\frac{1}{2} \rightarrow m_j = +\frac{1}{2}$) will have undergone a net energy shift of

$$-1.93 \times 10^{-9}\,B - 5.79 \times 10^{-9}\,B = -7.72 \times 10^{-9}\,B\ \text{eV}$$

The shortest-wavelength line ($m_j = +\frac{1}{2} \rightarrow m_j = -\frac{1}{2}$) will have undergone a net energy shift of

$$1.93 \times 10^{-9}\,B + 5.79 \times 10^{-9}\,B = 7.72 \times 10^{-9}\,B\ \text{eV}$$

The total energy difference between these two photons is

$$\Delta E = -1.54 \times 10^{-8}\,B\ \text{eV}$$

Since $\lambda = c/f = hc/E$, then $\Delta\lambda = -(hc/E^2)\Delta E = 0.022$ nm. We then have that

$$\Delta E = -0.022\ \text{nm}(E^2/hc) = -1.54 \times 10^{-8}\,B$$

where $E = hc/\lambda = hc/(589.9\ \text{nm})$. Finally, we have

$$B = \frac{(0.022 \times 10^{-9}\ \text{nm})hc}{(589.8 \times 10^{-9}\ \text{nm})^2(1.54 \times 10^{-8}\ \text{eV/T})(1.60 \times 10^{-19}\ \text{J/eV})}$$

$$B = 0.51\ \text{T} = 5100\ \text{gauss}$$

For comparison, the Earth's magnetic field averages about 0.5 gauss.