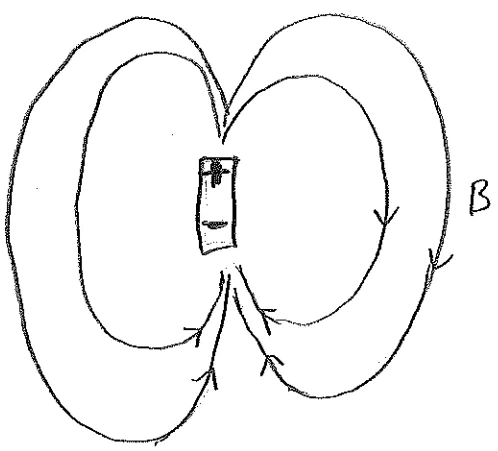
Non uniform (inhomogeneous) magnetic field



(common experience) → $\uparrow\vec{\mu}$ experiences an upward force
$\downarrow\vec{\mu}$ experiences a downward force } [as we'll now show, this is] due to gradient in magnetic field

Recall $E = -\vec{\mu}\cdot\vec{B}$

A homogeneous $\vec{B}$ field exerts a torque but not a net force on $\vec{\mu}$
An inhomogeneous $\vec{B}$ field exerts a force because then E depends on position not just orientation

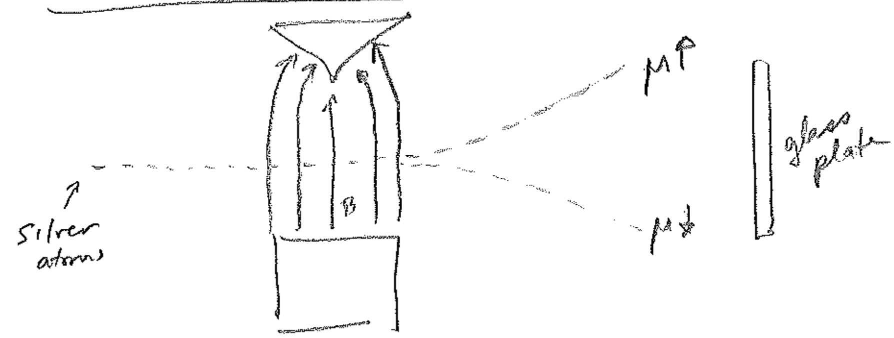
$$\vec{F} = -\vec{\nabla}E = \vec{\nabla}(\vec{\mu}\cdot\vec{B})$$

$\vec{B}$ primarily in z-direction

$$\vec{F} = \vec{\nabla}(\mu_z B_z) = \mu_z \underbrace{\vec{\nabla} B_z}_{\text{points in direction of increasing } B_z \text{ (i.e. upward)}}$$

If $\begin{cases}\mu_z > 0 \\ \mu_z < 0\end{cases}$ then $\vec{F}$ is $\begin{cases}\text{up}\\ \text{down}\end{cases}$ [agrees w/ above]

Stern-Gerlach experiment (1922)



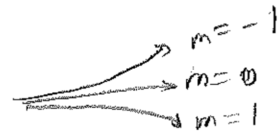
Atom passing thru inhomog mag field experiences

$$\vec{F} = \mu_z \vec{\nabla} B_z \qquad \mu_z = -\frac{e}{2m_e} L_z = -\frac{e\hbar}{2m_e} m$$

$$= \left(-\frac{e\hbar}{2m_e} \vec{\nabla} B_z \right) m$$

force, and $\therefore$ deflection, proportional to m

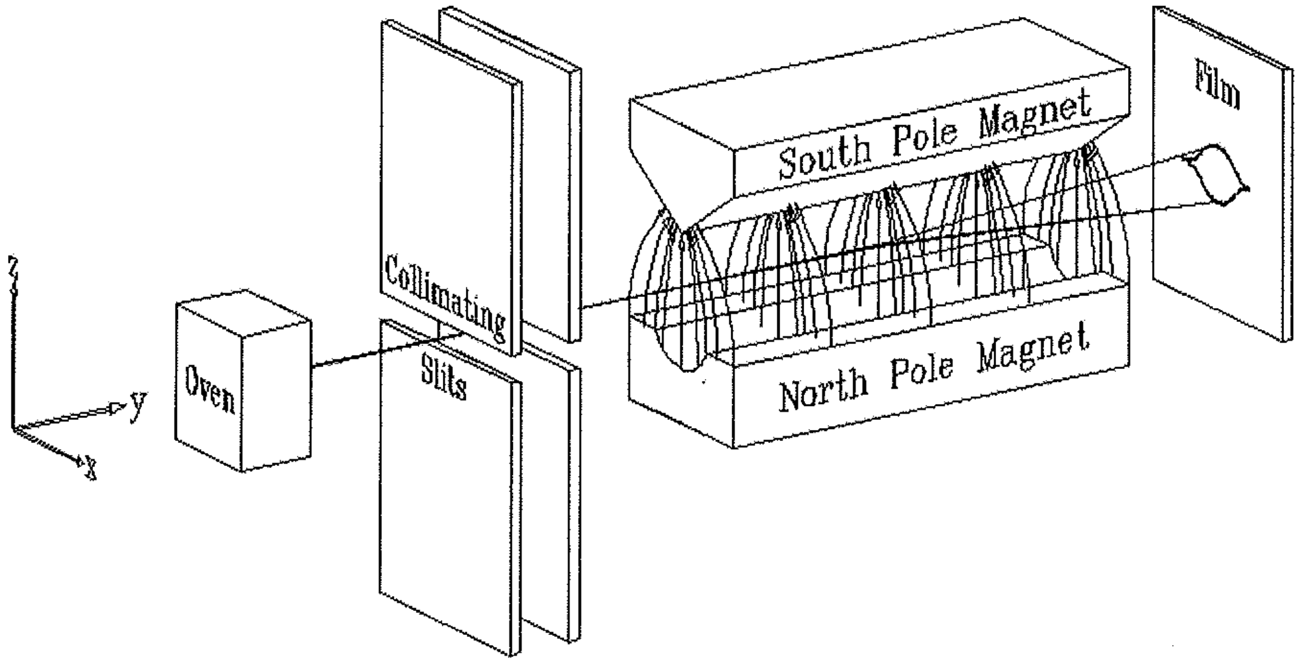
for $l=1 \Rightarrow$ expect



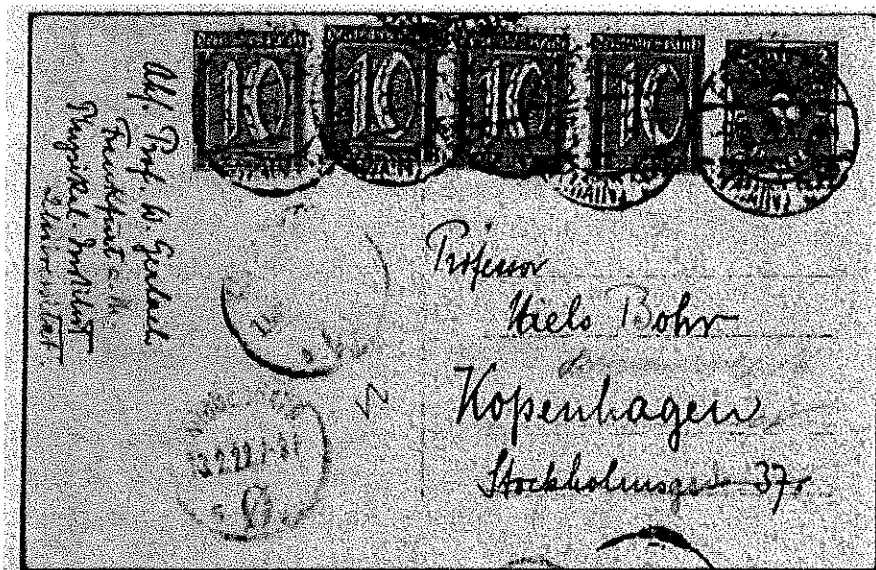
[more direct than Zeeman]

but S-G experiment on Ag showed $\longrightarrow$ multiplicity of two

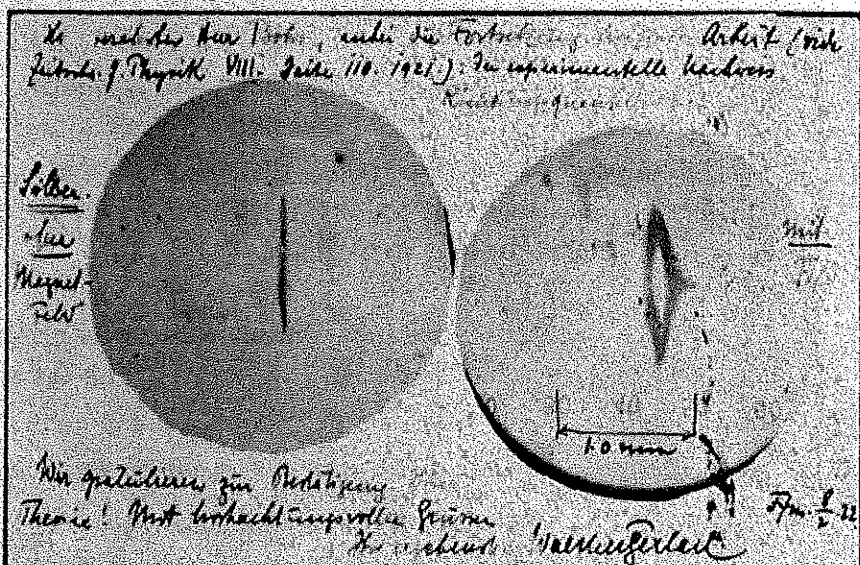
[Cohen-Tannoudji vol I, p 391
validity of classical trajectory]


z
y
x
Oven
Collimating
Slits
South Pole Magnet
North Pole Magnet
Film

3-2b


Abs. Prof. W. Gerlach
Frankfurt a. M.
Physikal. Institut
Universität

Professor
Niels Bohr
Kopenhagen
Stockholmsgade 37


Sehr verehrter Herr Bohr, anbei die Fortsetzung unserer Arbeit (vgl. Zeitschr. f. Physik VIII. Seite 110. 1921): der experimentelle Nachweis der Richtungsquantelung.

Silber ohne Magnetfeld

Mit Magnetfeld

1.0 mm

Wir gratulieren zur Bestätigung Ihrer Theorie! Mit hochachtungsvollen Grüssen
Ihr ergebenster Walther Gerlach

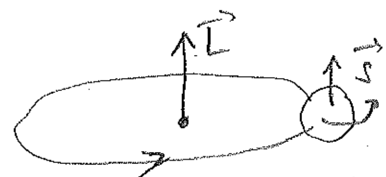
Ffm. 8/2 22

[Dutch graduate students]

Goudsmit & Uhlenbeck (1925) propose that electron possesses <u>intrinsic</u> angular <u>momentum</u> or <u>spin</u> $\vec{S}$

[intrinsic means not associated to orbital motion]

[planetary analogy]



do not take picture literally: electron is point particle

electron spin is quantized $|\vec{S}| = s\hbar$ w/ $s = \frac{1}{2}$ (<u>not</u> integer)

$S_z = m_s \hbar$ w/ $m_s = -\frac{1}{2}, \frac{1}{2}$ (doublet)

spin down ↓ ... spin up ↑

Stern-Gerlach: Ag atoms have one unpaired electron whose spin (up or down) determines which path it follows

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QFT ⇒ all particles have <u>integer</u> or <u>half-integer</u> spin

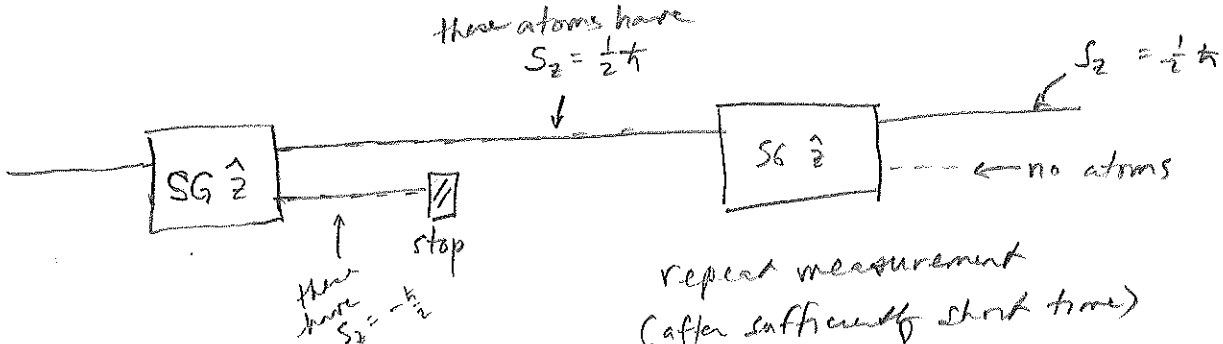
integer → bosons; half-integer → fermions

[spin-statistics theorem]

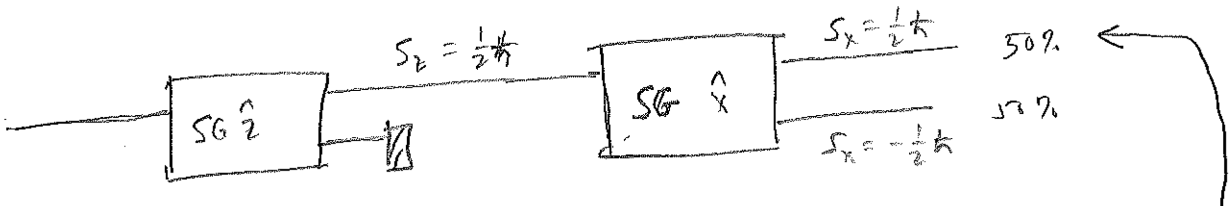
s	
0	α particles, Higgs boson
$\frac{1}{2}$	e^-, p, n, quarks
1	$\underbrace{\gamma}_{\text{EM}}$; $\underbrace{W^\pm\, Z^0}_{\text{weak}}$; $\underbrace{\text{gluons}}_{\text{strong}}$
$\frac{3}{2}$	Δ^{++} (composed of 3 quarks)
2	gravitons

[Sakurai p. 5]

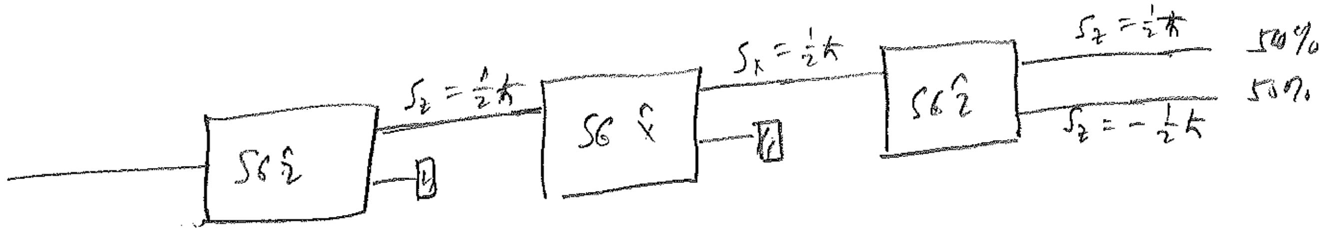
By orienting the S-G apparatus in different directions can determine component of spin along different axes



repeat measurement (after sufficiently short time) yields same result



Can we say that these atoms have $S_z = \frac{1}{2}\hbar$ and $S_x = \frac{1}{2}\hbar$?
No!



measurement of S_x has disturbed the value of S_z

Both S_x & S_z are quantized, but an atom cannot have well-defined values for both ⇒ <u>incompatible</u> observables

[Consider the $S_z = \frac{1}{2}\hbar$ state.
Can we say that 50% have $S_x = \frac{1}{2}\hbar$ & 50% have $S_x = -\frac{1}{2}\hbar$?
and SG$\hat{x}$ has just separated them?
But why then do we get $S_z = -\frac{1}{2}\hbar$?
Rather $S_z = \frac{1}{2}\hbar$ atoms are indeterminate w.r.t. S_x
and only a measurement of S_x can cause the atoms
to assume a determinate value.
That measurement then makes the atom indeterminate
w.r.t. S_z]

2 essential features of QM
indeterminacy: there exist states that do not have
well defined (or determinate) values for
all observables

incompatibility: there exist pairs of observables
(e.g. S_x & S_z, or x & p)
such that, if a state has a
definite value of one, it cannot have
" " " the other.

1925-6 Heisenberg/Born Munich
Dirac Cambridge
Schrödinger Zurich] invented QM,
a framework that
encompasses these features