Spatial quantization

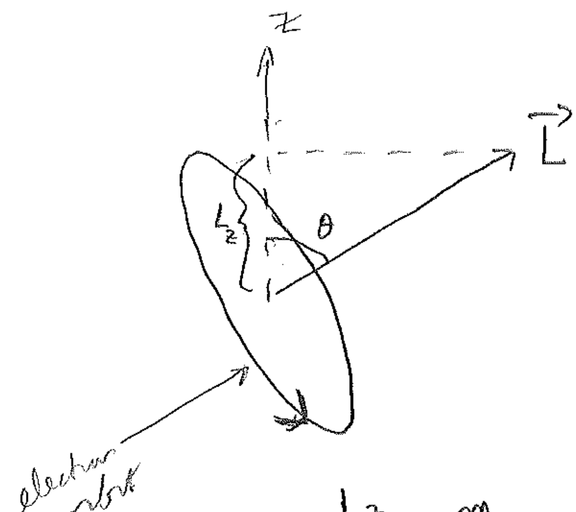
Angular momentum is a vector $\vec{L}$

Both the magnitude and the components of $\vec{L}$ are quantized

$L = l\hbar \qquad l = 0, 1, 2, \ldots$

$L_z = m\hbar \qquad m = \underbrace{-l,\ -l+1,\ \ldots,\ l}_{2l+1 \text{ possible values (multiplicity)}} \qquad \text{since } |L_z| \le L$

(do not confuse with mass)



(It might seem impossible for both magnitude & all components to be integer multiples of $\hbar$, but we'll see that only one component at a time can have a well-defined value)

let's pick L_z

$\cos\theta = \frac{L_z}{L} = \frac{m}{l}$ $\Rightarrow$ only certain directions of orbital plane allowed "spatial quantization"

[see AEC logos]

But directions w.r.t. what?? Also, ϕ is not determined.
Don't take this picture literally! but result is valid

Correspondence principle: as $l \to \infty$, all directions allowed
agrees w/ classical expectation

Nomenclature

name	l	m	multiplicity
s (sharp)	0	0	singlet
p (principle)	1	$-1, 0, 1$	triplet
d (diffuse)	2	$-2, -1, 0, 1, 2$	quintet
f (fundamental)	3	$-3, \ldots, 3$	septet

always odd

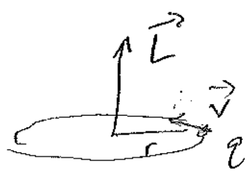
Empirical evidence for spatial quantization

1) Zeeman effect: splitting of spectra in a uniform magnetic field [image]

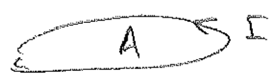
2) Stern-Gerlach experiment: deflection of trajectories of atoms in a nonuniform magnetic field

because atoms act like little magnets

Bohr model



A circulating charge q acts as an electromagnet of dipole moment $\mu = IA$



$$A = \pi r^2, \quad I = \frac{q}{T}, \quad T = \frac{2\pi r}{v}$$

classical result

$$\Rightarrow \mu = \frac{qvr}{2} = \frac{q}{2m_e} L$$

$\vec{\mu}$ & $\vec{L}$ are both vectors so $\vec{\mu} = \underbrace{\left(\frac{q}{2m_e}\right)}_{\text{called gyromagnetic ratio}} \vec{L}$

Since L is quantized in Bohr model, so is μ

$$\min L = \hbar \Rightarrow \min \mu = \underbrace{\frac{e\hbar}{2m_e}}$$

μ_B = Bohr magneton = smallest unit of dipole moment

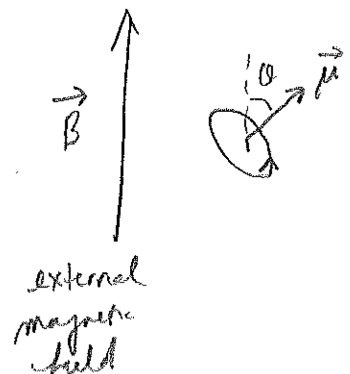
↳ typical magnetic moment of an atom (xc noble gases)

↳ cf French & Taylor p 441

[gaussian: $\mu = \frac{e\hbar}{2mc}$]

[paramagnetic as perm. dipole moment, eg Ag, H]

Uniform magnetic field



A magnetic dipole has least / most energy when it aligns / anti-aligns w/ an external magnetic field

[a compass needle aligns w/ $\vec{B}$ to lower its energy; an atom will instead precess around $\vec{B}$ axis like a gyroscope → we'll derive later]

$$E = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\theta$$

energy of Bohr atom in a magnetic field

$$E = E_n - \vec{\mu}\cdot\vec{B} \qquad \vec{\mu} = -\frac{e}{2m_e}L$$

(w/o field)

$$= E_n + \frac{e}{2m_e}\vec{B}\cdot\vec{L} \qquad \text{let } \vec{B} = B\hat{z}$$

$$= E_n + \frac{eB}{2m_e}L_z$$

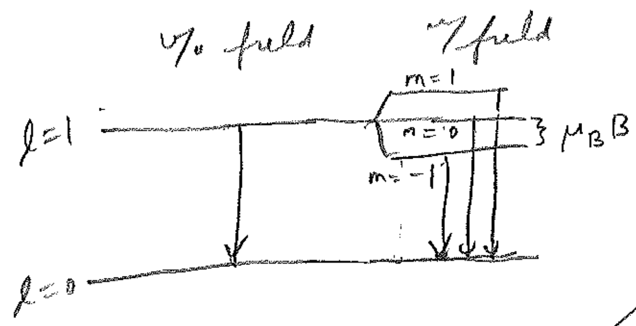
spatial quantization $L_z = m\hbar$

$$= E_n + \mu_B B m$$

$\mu_B = \frac{e\hbar}{2m_e} = 5.8\times10^{-5}\,\frac{eV}{Tesla}$

($\mu_B B$ is usually a small perturbation

[Tesla = $\frac{J\cdot s}{C\cdot m^2}$]

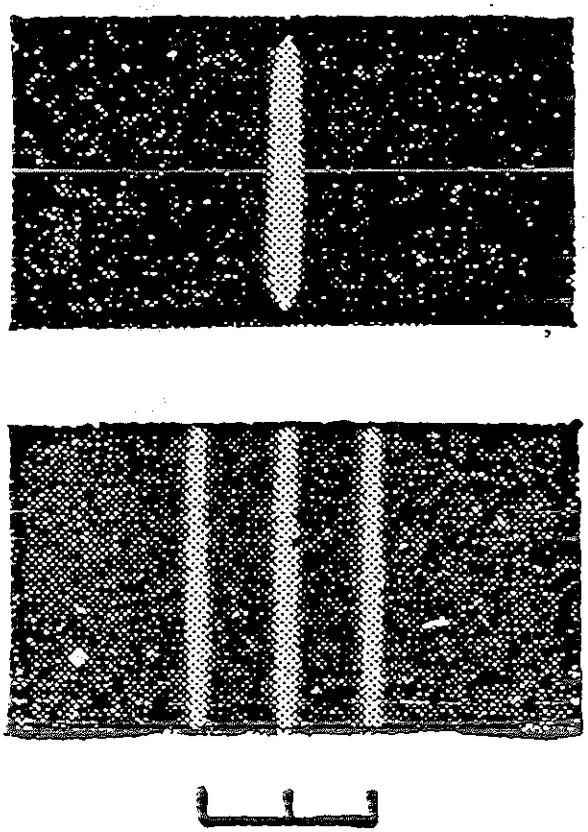


Zeeman effect [see image]
splitting, not broadening,
provides evidence for spatial quantization

spectra 

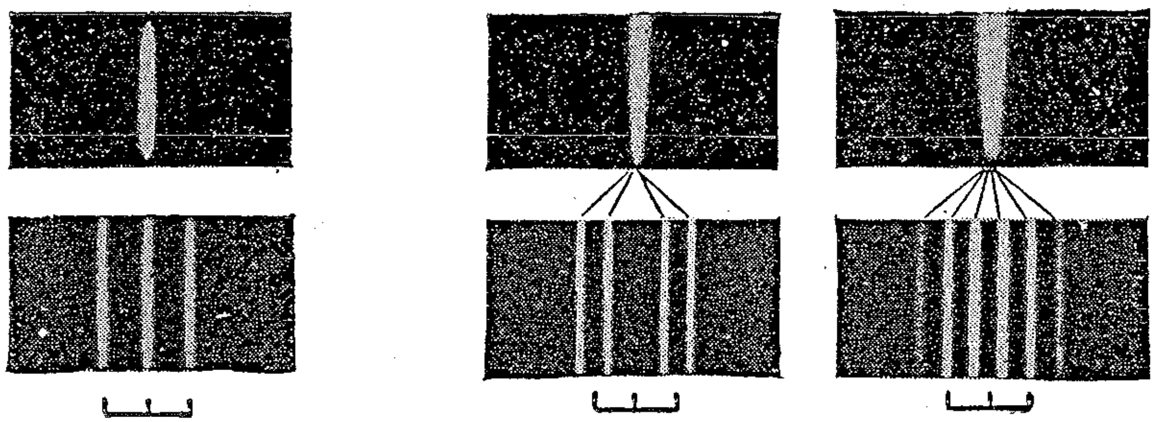
but many Zeeman spectra were anomalous (even multiplicity, etc.)

Zinc Singlet



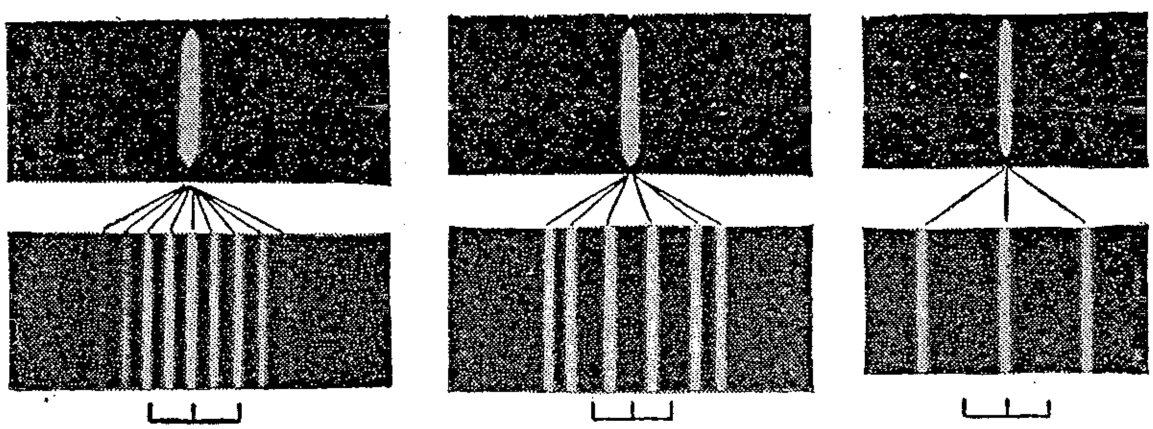
Normal Triplet

Zinc Singlet   Sodium Principal Doublet



Normal Triplet   Anomalous Patterns

Zinc Sharp Triplet



Anomalous Patterns

2-4c

"You look very unhappy."

"How can one look happy when he is thinking about the anomalous Zeeman effect?"

Wolfgang Pauli, 1923