

Particle in a 3d potential

$$H = \frac{p^2}{2m} + V(x_1, x_2, x_3)$$

$$p^2 = p_1^2 + p_2^2 + p_3^2 \rightarrow -\hbar^2 \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \right) = -\hbar^2 \underbrace{\nabla^2}_{\text{Laplacian}}$$

In position space

$$H \mapsto -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r})$$

$$|E\rangle \mapsto u_E(\vec{r}) = \langle \vec{r} | E \rangle$$

$$H|E\rangle = E|E\rangle \rightarrow \left(-\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}) \right) u_E = E u_E$$

3d time-indep. Schrödinger eqn

First

examine 3d potentials of specific types

$$V(x_1, x_2, x_3) = \tilde{V}(x_1) + \tilde{V}(x_2) + \tilde{V}(x_3)$$

for some function $\tilde{V}(x)$

(Symmetric under permutation of x_1, x_2, x_3)

Consider a 3d potential of the special form

$$V(x_1, x_2, x_3) = \tilde{V}(x_1) + \tilde{V}(x_2) + \tilde{V}(x_3)$$

→ Symmetric
~~under~~
~~exchange~~
 $x_1 \leftrightarrow x_2$
 or
 any perm
 of x_1, x_2, x_3

- eg
- 1) particle in a cube
 - 2) isotropic harmonic oscillator
- $$V(\vec{r}) = \frac{1}{2} k \vec{r} \cdot \vec{r}$$

Then

$$\sum_{i=1}^3 \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_i^2} + \tilde{V}(x_i) \right] u(\vec{r}) = E u(\vec{r})$$

Try a separable ansatz: $u(\vec{r}) = u^{(1)}(x_1) u^{(2)}(x_2) u^{(3)}(x_3)$

$$\sum \underbrace{\frac{1}{u^{(i)}} \left[-\frac{\hbar^2}{2m} \frac{d^2}{dx_i^2} + \tilde{V}(x_i) \right] u^{(i)}(x_i)}_{E^{(i)}} = E$$

$$E = E^{(1)} + E^{(2)} + E^{(3)}$$

$$\left[-\frac{\hbar^2}{2m} \frac{d^2 u^{(i)}}{dx^2} + \tilde{V}(x) \right] u^{(i)} = E^{(i)} u^{(i)} \quad \leftarrow \text{1d eqn}$$

same eqn for all i

Let $u_n(x)$ be solutions of this eqn w/ eigenenergies E_n

For each i , choose one of these solutions

$$u(x_1, x_2, x_3) = u_{n_1}(x_1) u_{n_2}(x_2) u_{n_3}(x_3)$$

$$E = E_{n_1} + E_{n_2} + E_{n_3}$$

		$\frac{E}{3E_1}$	<u>degeneracy</u>
gnd state	$u_1(x_1) u_1(x_2) u_1(x_3)$		1
1 st excited	$u_2(x_1) u_1(x_2) u_1(x_3)$	$2E_1 + E_2$	3
	$u_1(x_1) u_2(x_2) u_1(x_3)$		
	$u_1(x_1) u_1(x_2) u_2(x_3)$		
not all in this order	$E_1 + 2E_2$		3
	$2E_1 + E_3$		3
	$E_1 + E_2 + E_3$		6

Bond states in 1d. are always non degenerate

In >1d, there are often degeneracies
especially when potential has symmetry

