

3 dimensions:

position operators $\hat{X}_1, \hat{Y}_1, \hat{Z}_1 \propto \hat{X}_i \quad (i=1, 2, 3)$

momentum operators $\hat{P}_1, \hat{P}_2, \hat{P}_3 \propto \hat{P}_i \quad (i=1, 2, 3)$

$$[\hat{X}_i, \hat{X}_j] = 0$$

$$[\hat{P}_i, \hat{P}_j] = 0$$

$$[\hat{X}_i, \hat{P}_j] = i\hbar \delta_{ij}$$

To prove these, use position space operators

$$\hat{X}_i \rightarrow x_i$$

$$\Rightarrow [\hat{X}_i, \hat{X}_j] \psi = 0$$

$$\hat{P}_i \rightarrow \frac{\hbar}{i} \frac{\partial}{\partial x_i}$$

$$\Rightarrow \left[\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j} \right] \psi = 0$$

$$[\hat{X}_i, \hat{P}_j] \psi \rightarrow x_i \cdot \frac{\hbar}{i} \frac{\partial \psi}{\partial x_j} - \frac{\hbar}{i} \frac{\partial}{\partial x_j} (x_i \psi)$$

$$= i\hbar \frac{\partial x_i}{\partial x_j} \psi = i\hbar \delta_{ij} \psi$$

use this to derive
uncertainty relations

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

$$\Delta y \Delta p_y \geq \frac{\hbar}{2}$$

$$\Delta z \Delta p_z \geq \frac{\hbar}{2}$$

but no constraints on $\Delta x \Delta p_y$ for example

$$\left(\text{Also } \underbrace{\Delta t}_{\tau} \Delta E \geq \frac{\hbar}{2} \right)$$

$\tau = \text{decay time of a state}$

Let $|\vec{x}\rangle = |x_1, x_2, x_3\rangle$ be a mutual eigenstate 5/1-2

$$x_i |\vec{x}\rangle = x_i |\vec{x}\rangle$$

~~same as $|\vec{x}\rangle$ we can find mutual eigenstates~~

~~all these are~~ $|\vec{x}\rangle = |x_1, x_2, x_3\rangle = |\vec{x}\rangle$ = particle "localized" at a pt. (x_1, x_2, x_3)

orthonormality $\underbrace{\langle x'_1, x'_2, x'_3 |}_{\langle \vec{x}' | \vec{x} \rangle} |x_1, x_2, x_3\rangle = \underbrace{\delta(x_1 - x'_1) \delta(x_2 - x'_2) \delta(x_3 - x'_3)}_{\text{call this } \delta^{(3)}(\vec{x} - \vec{x}')}$

completeness $\int dx_1 dx_2 dx_3 |x_1, x_2, x_3\rangle \langle x_1, x_2, x_3|$
 $= \int d^3x |\vec{x}\rangle \langle \vec{x}| = 1$

arbitrary state $|\psi\rangle = \int d^3x |\vec{x}\rangle \langle \vec{x} | \psi \rangle$

$$\psi(x_1, x_2, x_3) = \psi(\vec{x}) = \text{wavefunction in 3d position space}$$

normalize

$$\langle \psi | \psi \rangle = \int d^3x \underbrace{\langle \psi | \vec{x} \rangle}_{\psi^*(\vec{x})} \underbrace{\langle \vec{x} | \psi \rangle}_{\psi(\vec{x})} = \int d^3x |\psi(\vec{x})|^2$$

momentum space wavefunction

$$|\psi\rangle = \int d^3p |\vec{p}\rangle \underbrace{\langle \vec{p} | \psi \rangle}_{\phi(\vec{p})}$$

$$\langle \vec{x} | \vec{p} \rangle = \frac{1}{(2\pi\hbar)^{3/2}} e^{i\frac{\vec{p} \cdot \vec{x}}{\hbar}}$$

$$\psi(\vec{x}) = \frac{1}{(2\pi\hbar)^{3/2}} \int d^3p e^{i\frac{\vec{p} \cdot \vec{x}}{\hbar}} \phi(\vec{p})$$

$$\phi(\vec{p}) = \frac{1}{(2\pi\hbar)^{3/2}} \int d^3x e^{-i\frac{\vec{p} \cdot \vec{x}}{\hbar}} \psi(\vec{x})$$

} Fourier transform in 3d