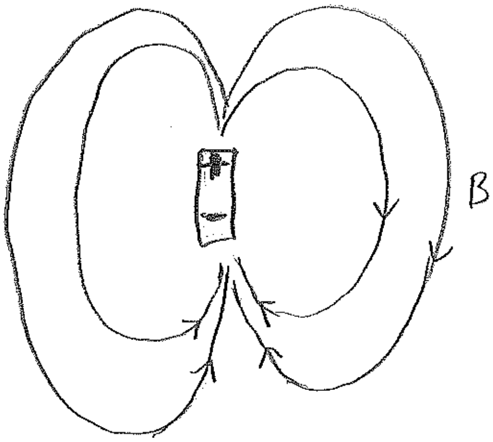
Non uniform (inhomogeneous) magnetic field



(common experience) → $\uparrow\vec{\mu}$ experiences an upward force, $\downarrow\vec{\mu}$ experiences a downward force } as we'll now show, this is due to gradient in magnetic field

Recall $E = -\vec{\mu}\cdot\vec{B}$

A homogeneous $\vec{B}$ field exerts a torque but not a net force on $\vec{\mu}$
An inhomogeneous $\vec{B}$ field exerts a force because
E depends on position not just orientation

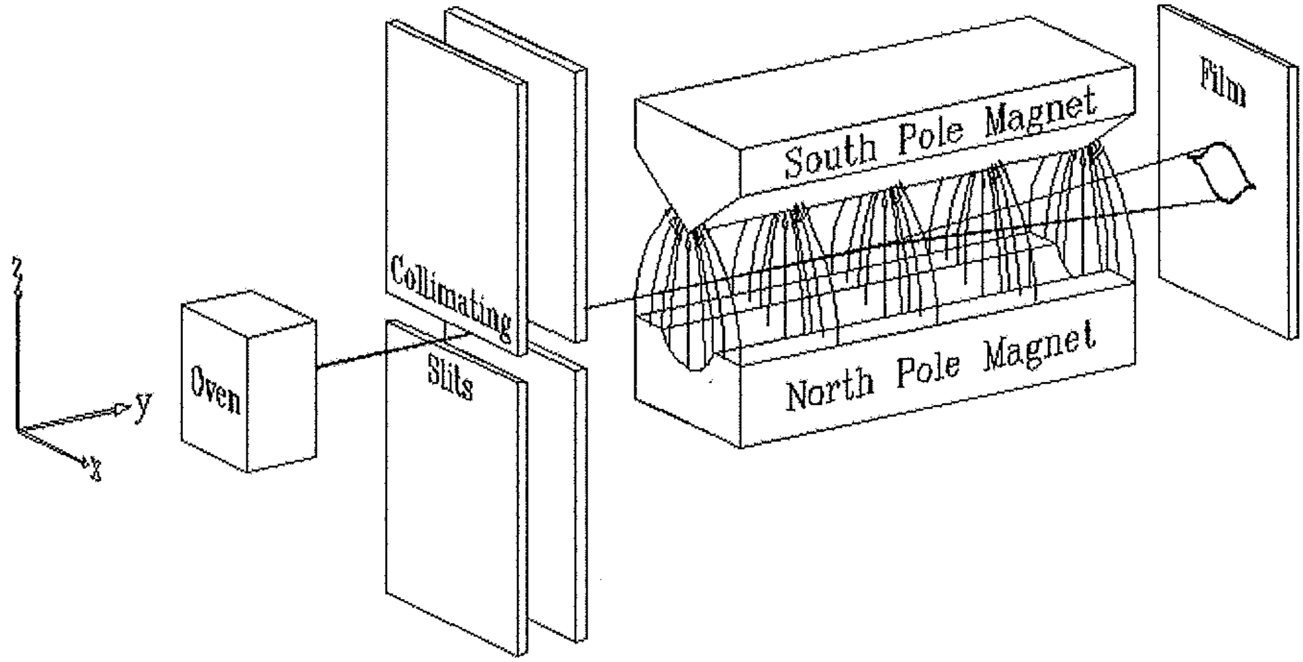
$$\vec{F} = -\vec{\nabla}E = \vec{\nabla}(\vec{\mu}\cdot\vec{B})$$

$\vec{B}$ mostly in z-direction

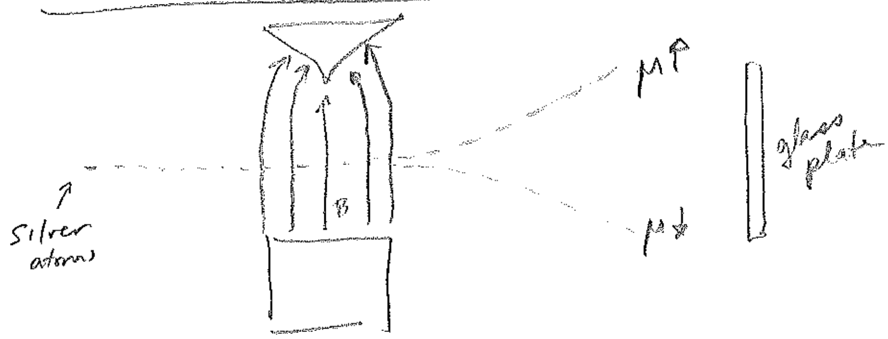
$$\vec{F} = \vec{\nabla}(\mu_z B_z) = \mu_z \underbrace{\vec{\nabla} B_z}$$

points in direction of increasing B_z (ie upward)

If $\begin{cases}\mu_z > 0 \\ \mu_z < 0\end{cases}$ then $\vec{F}$ is $\begin{cases}\text{up} \\ \text{down}\end{cases}$ [agrees w/ above]


z
y
x
Oven
Collimating
Slits
South Pole Magnet
North Pole Magnet
Film

Stern-Gerlach experiment (1922)



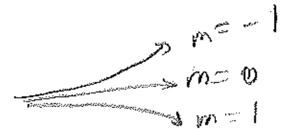
Atom passing thru inhomog mag field experiences

$$\vec{F} = \mu_z \vec{\nabla} B_z \qquad \mu_z = -\frac{e}{2m_e} L_z = -\frac{e\hbar}{2m_e} m$$

$$= \left(-\frac{e\hbar}{2m_e} \vec{\nabla} B_z \right) m$$

$\Rightarrow$ deflection proportional to m

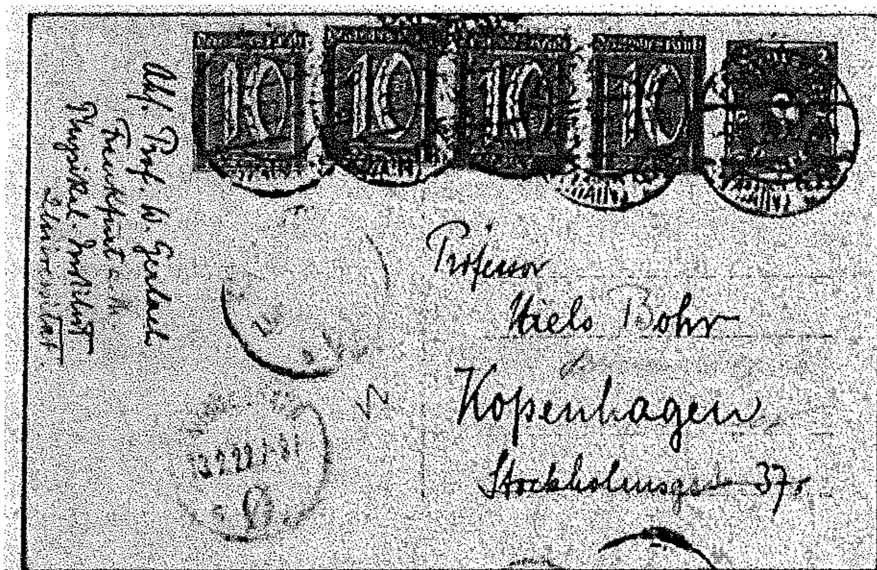
e.g. $l = 1 \Rightarrow$ expect



[more direct than Zeeman]

[Cohen-Tannoudji vol I, p 391
validity of classical trajectory]

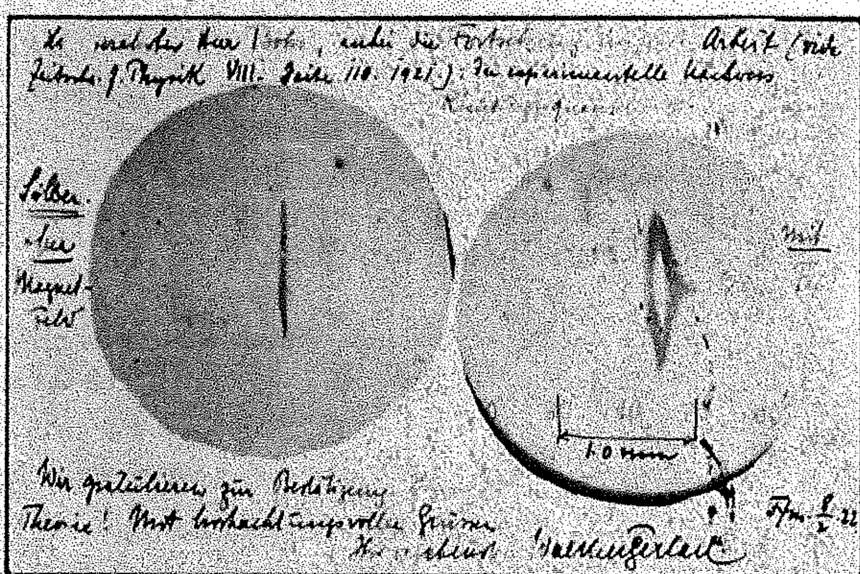
3-2b



Abs. Prof. W. Gerlach
Frankfurt a. M.
Physikal. Institut
Universität

Professor
Niels Bohr
Kopenhagen
Stockholmsgade 37

13.2.22



Hochverehrter Herr Bohr, anbei die Fortsetzung unserer Arbeit (vide Zeitschr. f. Physik VIII. Seite 110. 1921): Der experimentelle Nachweis der Richtungsquantelung.

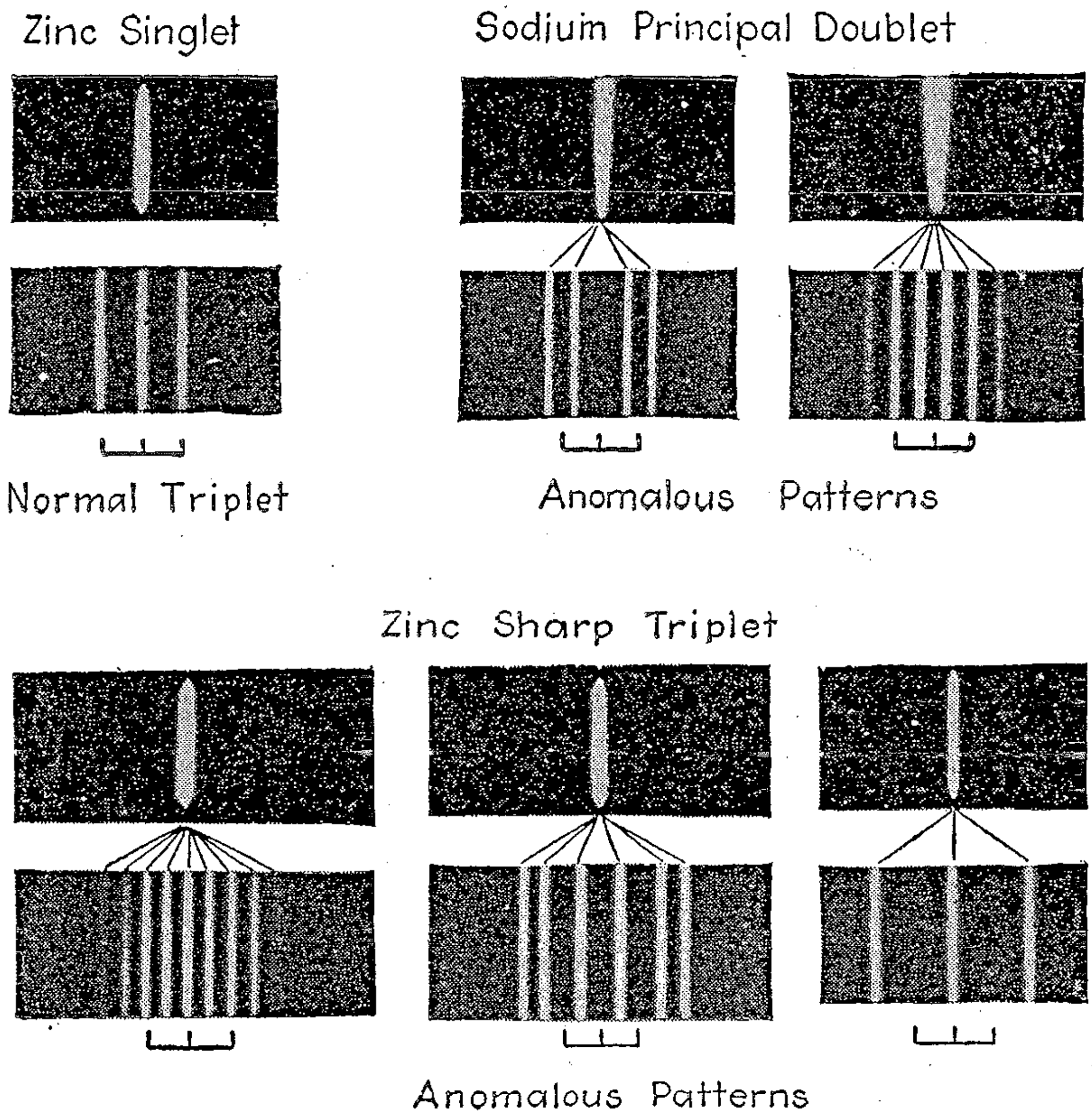
Silber.
ohne
Magnetfeld

mit

1,0 mm

Wir gratulieren zur Bestätigung Ihrer Theorie! Mit hochachtungsvollen Grüßen
Ihr ergebenster Walther Gerlach

Ffm. 8/2.22


Zinc Singlet
Sodium Principal Doublet
Normal Triplet
Anomalous Patterns
Zinc Sharp Triplet
Anomalous Patterns

3-2d

"You look very unhappy."

"How can one look happy when he is thinking about the anomalous Zeeman effect?"

Wolfgang Pauli, 1923

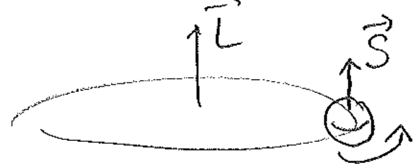
Recall: multiplicity of different directions
for an orbit of integer angular momentum $L = \ell\hbar$
is $2\ell+1$, an odd number.

But: Stern Gerlach expt of Ag shows a multiplicity of $\underline{2}$

Also: certain Zeeman spectra (called "anomalous") have even multiplicity

[show: anom Zeeman]
[show: Pauli quote]

1925 Goudsmit + Uhlenbeck proposed electron possesses
<u>intrinsic angular momentum</u>, or <u>spin</u>, $\vec{S}$



[earth sun analogy]

Do not take picture literally! Electron is a point particle

Spin is also quantized $\quad S = s\hbar \quad$ but $s = \frac{1}{2}$

$$S_z = m_s\hbar \qquad m_s = \underbrace{-\tfrac{1}{2}, \tfrac{1}{2}}_{\text{doublet}}$$

Spatial quant: $\begin{cases} \uparrow & \text{spin up} \\ \downarrow & \text{spin down} \end{cases}$

All particles have integer or half-integer spin

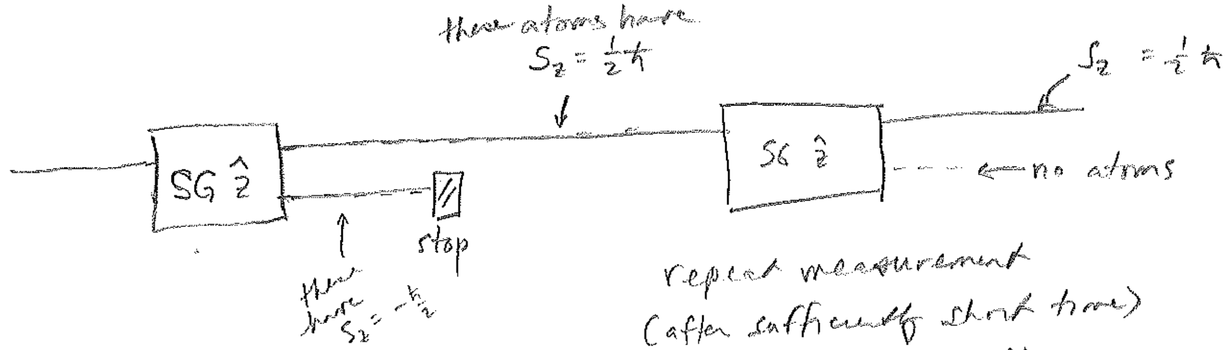
s	
0	α particles, Higgs
$\frac{1}{2}$	e^-, p, n, quarks
1	γ, $W^\pm$, Z^0
$\frac{3}{2}$	Δ^{++} (composite of 3 quarks)
2	gravitons

[Sakurai p. 5]

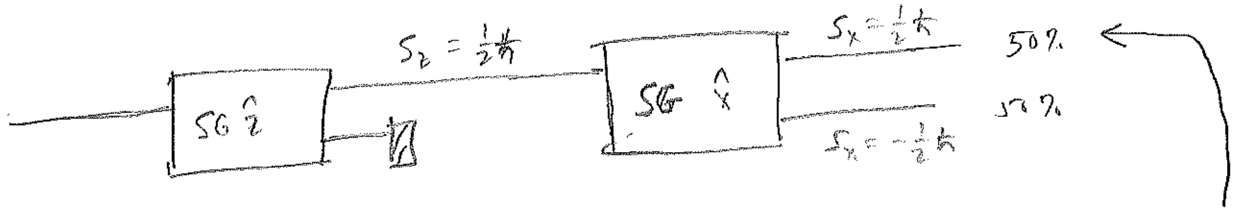
Stern Gerlach: Ag atoms have 1 unpaired electron whose spin determines which path it takes

possible values of $S_z = m_s \hbar = \pm \frac{1}{2}\hbar$ Also $S_x = \pm \frac{1}{2}\hbar$ and $S_y = \pm \frac{\hbar}{2}$ (but not at the same time!)

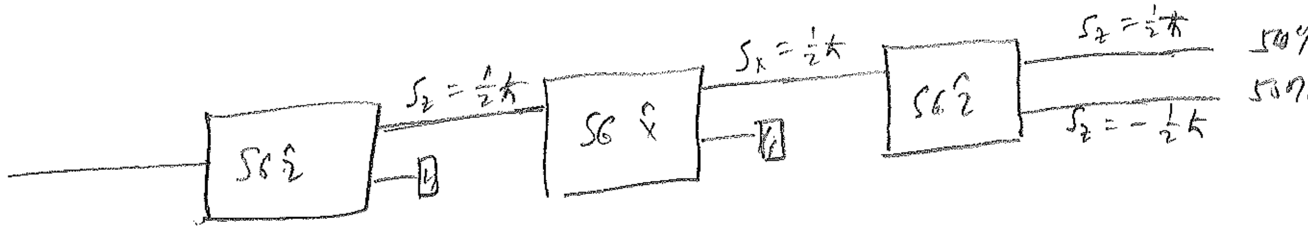
By orienting the S-G apparatus in different directions can determine component of spin along different axes



repeat measurement (after sufficiently short time) yields same result



Can we say that these atoms have $S_z = \frac{1}{2}\hbar$ <u>and</u> $S_x = \frac{1}{2}\hbar$?



Measurement of S_x has disturbed the value of S_z

Both S_x & S_z are quantized, but an atom cannot have well-defined values for both ⇒ <u>incompatible</u> observables

[Consider the $S_z = \frac{1}{2}\hbar$ state.
Can we say that 50% have $S_x = \frac{1}{2}\hbar$ & 50% have $S_x = -\frac{1}{2}\hbar$
and SG $\hat{x}$ has just separated them?
But why then do we get $S_z = -\frac{1}{2}\hbar$?
Rather $S_z = \frac{1}{2}\hbar$ atoms are indeterminate w.r.t. S_x
and only a measurement of S_x can cause the atoms
to assume a determinate value.
That measurement then makes the atom indeterminate
w.r.t. S_z]

features of QM

indeterminacy: there exist states that do not have
definite (or determinate) values for
all observables

incompatibility: there exist pairs of observables
($e.g.$ S_x & S_z, or x & p)
such that, if a state has a
definite value of one, it cannot have
" " the other.

1925 Heisenberg
Born
Dirac
1926 Schrödinger] invented QM,
a framework that
encompasses these features