Spatial quantization

Angular momentum is a vector $\vec{L}$

Both the magnitude and the components of $\vec{L}$ are quantized

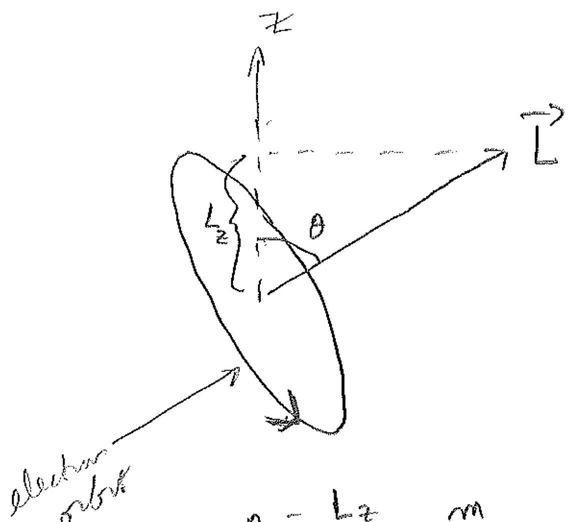
$L = l\hbar \qquad l = 0, 1, 2, \ldots$

$L_z = m\hbar \qquad m = \underbrace{-l,\ -l+1,\ \ldots,\ l}_{2l+1 \text{ possible values (multiplicity)}}$ since $|L_z| \leq L$

(do not confuse with mass)

(It might seem impossible for both magnitude & all components to be integer multiples of $\hbar$, but we'll see that only one component at a time can have a well-defined value)

let's pick L_z



$\cos\theta = \frac{L_z}{L} = \frac{m}{l}$ $\Rightarrow$ only certain directions of orbital plane allowed "spatial quantization"

[see AEC logo]

But directions w.r.t. what?? Also, ϕ is not determined.
Don't take this picture literally! but result is valid

Correspondence principle: as $l \to \infty$, all directions allowed
agrees w/ classical expectation

l	name	m	multiplicity
0	s (sharp)	0	singlet
1	p (principle)	$-1, 0, 1$	triplet
2	d (diffuse)	$-2, -1, 0, 1, 2$	quintet
3	f (fundamental)	$-3 \ldots 3$	septet

always odd

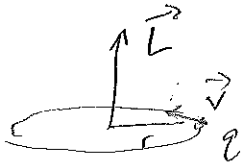
<u>Experimental Evidence for spatial quantization</u>

1) Zeeman effect: the splitting of spectra emitted by atoms in a <u>uniform</u> magnetic field

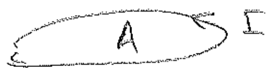
2) Stern-Gerlach experiment: deflection of the trajectories of atoms in a <u>nonuniform</u> magnetic field

[Atoms act like little magnets.]

Bohr model



A circulating charge q acts as an electromagnet of dipole moment $\mu = IA$



$A = \pi r^2, \quad I = \frac{q}{T}, \quad T = \frac{2\pi r}{v}$

classical result

$\Rightarrow \mu = \frac{qvr}{2} = \frac{q}{2m_e} L$

$\vec{\mu}$ + $\vec{L}$ are both vectors so $\vec{\mu} = \underbrace{\left(\frac{q}{2m_e}\right)}_{\text{called gyromagnetic ratio}} \vec{L}$

Since L is quantized in Bohr model, so is μ

$\mu = \underbrace{\left(\frac{e\hbar}{2m_e}\right)}_{\mu_B} \ell$

μ_B = Bohr magneton = smallest unit of dipole moment

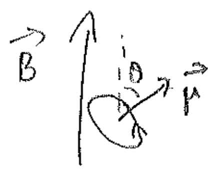
↳ typical magnetic moment of an atom (xc noble gases)

↳ cf French + Taylor p 441

[gaussian: $\mu = \frac{e\hbar}{2m_e c}$]

[paramagnetic ⇒ perm. dipole moment, eg Ag, H]

A magnetic dipole ~~wants~~ tries to align with an external magnetic field
$\Rightarrow \vec{B}$ exerts a torque on $\vec{\mu}$



$$E = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\theta$$ (min. energy when aligned)

[Unlike a compass needle, an atom will precess around $\vec{B}$ axis (like a gyroscope)] → will derive later

Energy of Bohr atom in an external magnetic field

$$E = E_n - \vec{\mu}\cdot\vec{B} \qquad \vec{\mu} = -\frac{e}{2m_e}\vec{L}$$

energy levels w/o B field

$$= E_n + \frac{e}{2m_e}\vec{B}\cdot\vec{L} \qquad \text{Let } \vec{B} = B\hat{z}$$

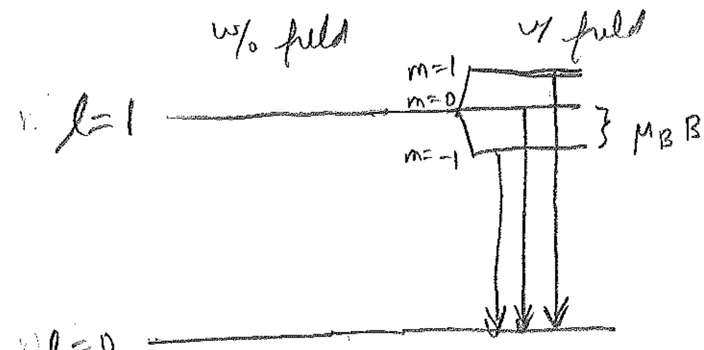
$$= E_n + \frac{eB}{2m_e}L_z$$

Spatial quantization $L_z = m\hbar$

$$E = E_n + (\mu_B B)\,m$$

$$\mu_B = \frac{e\hbar}{2m_e} = 5.8\times10^{-5}\ \frac{eV}{Tesla} \Rightarrow (\mu_B B)m \text{ is a small perturbation}$$

$[T = \frac{J\cdot s}{C\cdot m^2}]$

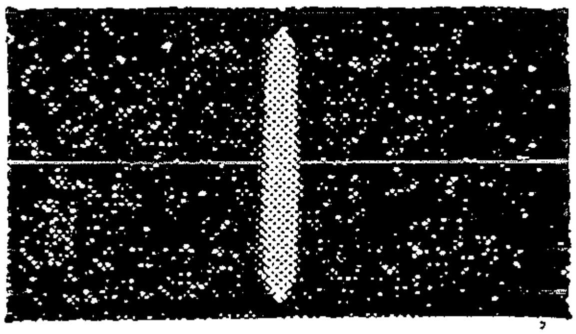


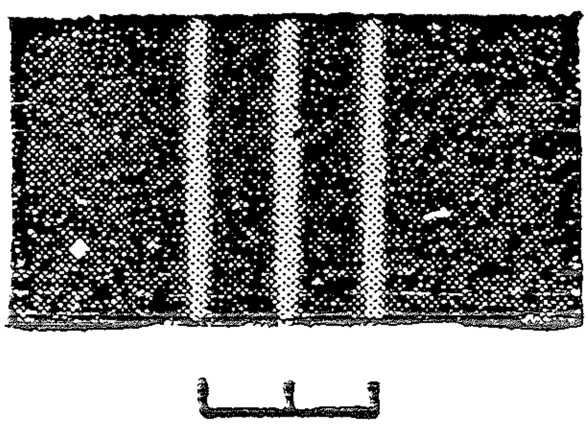
[If no spatial qu'zn then line broadening not splitting]

spectrum

Zeeman effect
[show image]

Zinc Singlet





Normal Triplet