Scanned 9-2019

$\boxed{\text{PHYS 3140}}$

Physicist's history of atomic physics

Atom = "uncuttable" but

1897 Thomson discovers e^- (cathode rays), first "subatomic" particle

[first subatomic particle, w/ $m \sim \frac{1}{2000}$ hydrogen
(rather $\frac{e}{m}$ ratio much bigger)]

Thomson model of atom
point-like electrons embedded in a "positive" jelly

(plum pudding model)

Atoms excited by heating them up or running a current through them emit electromagnetic radiation w/ certain discrete frequencies (characteristic of the type of atom). The set of frequencies (or wavelengths) is called its spectrum. [image: Balmer]

$\lambda = 365\text{ nm}\left(\frac{m^2}{m^2-4}\right)$

Thomson model seems to explain this
Hydrogen: single e^- in equilibrium at center of uniform density positive charge

if displace e^-, simple harmonic motion [problem]

classical EM theory: accelerating charge emits radiation w/ same freq.

But this predicts only one spectral line whereas hydrogen has many

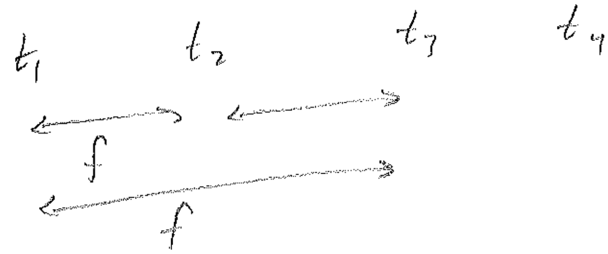
Using spectroscopic data, Balmer (1885) + Rydberg (1888) discovered the empirical formula for frequencies emitted by hydrogen

$$f = (3.29 \times 10^{15}\text{ Hz})\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

n, m are integers
$m > n$

$n=1$, $m=2,3,4,\ldots \Rightarrow$ Lyman series (ultraviolet) (1906–14)
$n=2$, $m=3,4,5 \rightarrow$ Balmer (visible) (1885)
$n=3$, $m=4,5,6 \Rightarrow$ Paschen (infrared) (1908)
$n=4$ Brackett " (1922)
$n=5$ Pfund " (1924)
$n=6$ Humphreys " (1953)

No analogous simple formula for atoms $> H$
but Rydberg-Ritz combination principle (1900/08)
revealed that frequencies could always be
expressed as the difference between two
of a set of characteristic "terms" t_n



$$f = t_n - t_m$$

1900 Planck
1905 Einstein } quantization of light

Light of frequency f consists of discrete photons of

$$E = hf$$

$$h = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$$

Hydrogen emits photons of energies

$$E = (13.6 \text{ eV})\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

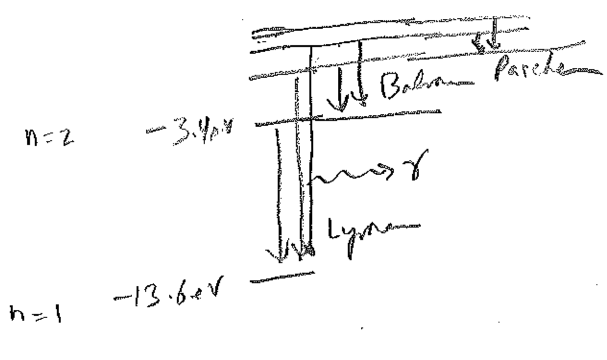
[other atoms emit photons whose energies are differences of terms]

Apply energy conservation to emission process

$$E_{photon} = E^{atom}_{init} - E^{atom}_{final}$$

This suggests hydrogen atoms have discrete energy levels

$$E_{hyd} = \frac{-13.6 \text{ eV}}{n^2}$$



But how can we understand these discrete energy levels?

1911 Rutherford disc. nucleus

Proposes alternative to Thomson model planetary model of atom: electrons orbiting a small heavy nucleus

hydrogen

Coulomb $|F| = \frac{Ke^2}{r^2}$ $K = \frac{1}{4\pi\epsilon_0}$

$[Ke^2 = 2.307E-28$ J·m$]$

Assume electron in a circular orbit w/ $a = \frac{v^2}{r}$

Then $F = m_e a \Rightarrow \frac{v^2}{r} \sim \frac{1}{r^2}$

① $v \sim r^{-1/2}$

Energy $E = T_e + V$ (ignore T_p because $m_p \gg m_e$

$V = -\frac{Ke^2}{r}$

$T_e = \frac{1}{2} m_e v^2$ (assume non relativistic: check later)

$\sim \frac{1}{r}$

② $E \sim -r^{-1}$

Any (negative) energy is allowed

Orbital angular momentum $\vec{L} = \vec{r} \times \vec{p}$

circular $\Rightarrow L = m_e v r$

③ $L \sim r^{1/2}$

Orbital frequency of electron $f \sim \frac{v}{r}$

④ $f \sim r^{-3/2}$

[HW: derive exact expressions]

Accelerating electron emits EM waves.

Problems: ① any frequency is allowed (not discrete)

② atom is unstable

1913 Bohr makes an amazing assumption:

orbital angular momentum must be quantized

$$L = n\hbar, \quad n = 1, 2, \ldots \quad \left(\hbar = \frac{h}{2\pi}\right) \quad [\hbar = 1.0546 \times 10^{-34}\ \mathrm{J \cdot s}]$$

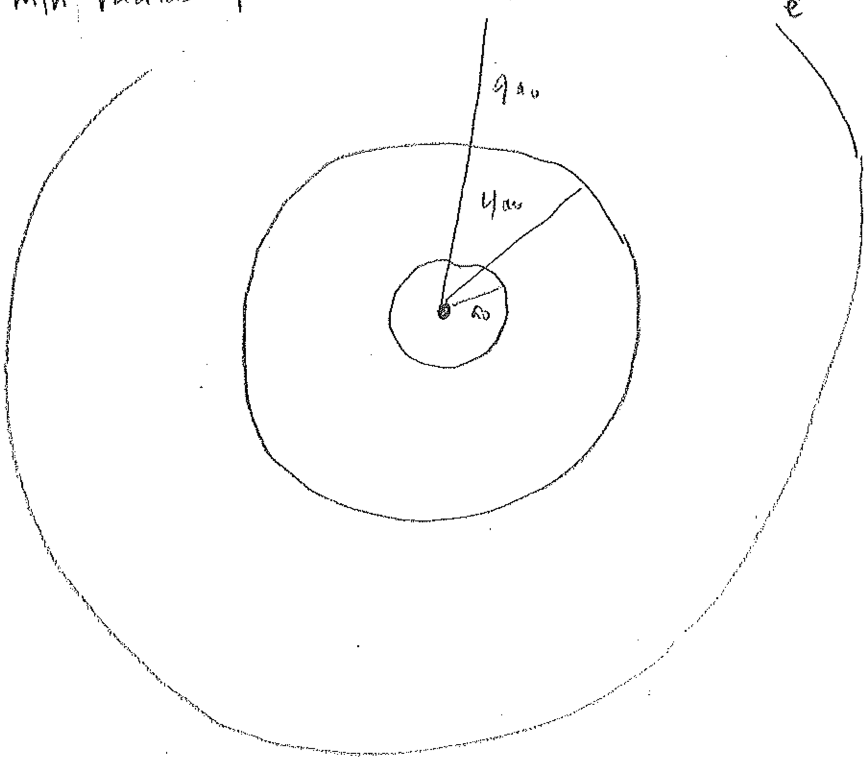
[This then fixes other variables, eg. radius]

Allowed radii

$$r_n = \left(\frac{\hbar^2}{m_e K e^2}\right) n^2 \qquad \text{[HW: show]}$$

min radius r_1 = Bohr radius $a_0 = \dfrac{\hbar^2}{m_e K e^2} \approx \frac{1}{2}\text{Å}$, $\quad \text{Å} = 10^{-10}\ \mathrm{m}$

$$r_n = n^2 a_0$$



NB size of hydrogen nucleus ~ 1 fm ~ 10^{-15} m

Allowed velocities

$$v_n = \left(\frac{Ke^2}{\hbar}\right)\frac{1}{n}$$ [HW: show]

Define fine structure constant $\underbrace{\alpha \equiv \frac{Ke^2}{\hbar c}}_{\text{memorize}} \approx \frac{1}{137}$

$$v_n = \frac{\alpha c}{n}$$

Non rel. assumption valid since $\frac{v}{c} = \frac{\alpha}{n} \ll 1$

[1% speed of light]

Allowed energies

$$E_n = -\underbrace{\left(\frac{m_e K^2 e^4}{2\hbar^2}\right)}_{13.6\,\mathrm{eV}}\frac{1}{n^2}$$ [HW: show]

Allowed frequencies of orbital motion

$$f_n = \left(\frac{m_e K^2 e^4}{2\pi \hbar^3}\right)\frac{1}{n^3}$$ [HW: show]

Acc. classical electromagnetic theory,
an electron orbiting with this frequency
would emit EM waves w/ the same frequency
losing energy and spiralling into the nucleus

[But classical EM theory is modified (→ quantum field theory)]

In Bohr model, an electron loses energy in discrete jumps from one allowed orbit to another, and is stable in the lowest allowed orbit. The frequency of the emitted photon is determined by energy conservation as

$$E_{photon} = E_{init} - E_{final} = \frac{m_e K^2 e^4}{2\hbar^2}\left(-\frac{1}{m^2} - \left[-\frac{1}{n^2}\right]\right)$$

$$f_{photon} = \frac{E_{ph}}{h} = \underbrace{\frac{m_e K^2 e^4}{4\pi\hbar^3}}_{3.29\times10^{15}\ Hz}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

Balmer ✓
Singly ionized He

Bohr's correspondence principle

Predictions of QM must agree w/ classical mechanics in the limit of large quantum numbers.

e.g. the frequency of a photon emitted in transition from state n to $n-1$ must approx. agree with the orbital frequency of the electron in state n when n is large.

[HW: show]

$$\left[\quad \frac{mK^2e^4}{4\pi\hbar^3}\left[\frac{1}{n^2} - \frac{1}{(n-1)^2}\right] \approx \frac{mK^2e^4}{4\pi\hbar^3}\,\frac{2}{n^3}\quad\right]$$

Despite the fact that the Bohr model, starting from the assumption that

$L = n\hbar, \quad n \in \mathbb{Z}$

gives the correct ~~non-relativistic~~ energy levels (in the non-relativistic approximation)

$E_n = -\frac{13.6\,eV}{n^2}$

for hydrogen, the model is fundamentally incorrect.

① Electrons do not travel in well-defined circular orbits (they are described by 3 dim'l probability distributions)

and ② ~~[illegible]~~ $L \neq n\hbar$.

Angular momentum is quantized

$L = l\hbar \quad w/\ l \in \mathbb{Z}$

~~Specifically l ranges from 0 to n-1.~~

but $l < n$.

The ground state ($n=1$) corresponds to a spherically symmetric probability distribution w/ $l = 0$ angular momentum

In general, l ranges from 0 to $n-1$.

We'll see this later when we solve the Schrodinger eqn for hydrogen

But Bohr model still useful for quick (back-of-envelope) estimates & for developing intuition

Mnemonics

Memorize $\alpha = \frac{Ke^2}{\hbar c} \sim \frac{1}{137}$

mnemonic $\alpha = \beta$ where $\beta = \frac{v}{c}$

$\Rightarrow \boxed{v_1 = \alpha c} \sim \frac{c}{137} = \frac{Ke^2}{\hbar}$

$E_1 = -\frac{1}{2} m v^2 = \boxed{-\frac{1}{2} mc^2 \alpha^2} \sim \frac{511{,}000}{2(137)^2} \sim 13.6$

$V = -2T$

$= -\frac{mKe^2}{2\hbar^2}$

virial
$V \sim r^n$
$n\langle V \rangle = 2T$
so $n = -1$
$\Rightarrow \langle V \rangle = -2T$

$L = mvr$ so $a_0 = \frac{\hbar}{mv} = \frac{\hbar}{\alpha mc} = \frac{\text{Compton}}{\alpha}$

$= \frac{\hbar^2}{mKe^2}$

Compton $\frac{\hbar}{mc} = \frac{\hbar c}{mc^2} = \frac{197 \text{ meV fm}}{0.511 \text{ fm}} \sim 400$ fm