<u>Zeeman effect</u>

In a magnetic field $\delta E = -\vec{\mu}\cdot\vec{B}$

orbital motion of an electron $\Rightarrow \vec{\mu} = -\frac{e}{2m_e}\vec{L}$

Spin also contributes to magnetic moment $\vec{\mu} = -g\frac{e}{2m_e}\vec{S}$

Dirac equation for electron predicts $g=2$

(QED corrections change this slightly: $g = 2.002319\ldots$)

$$\therefore \quad \delta E = \frac{e}{2m_e}(\vec{L} + g\vec{S})\cdot\vec{B}$$

Suppose $\vec{B} = B\hat{z} \Rightarrow \delta E = \frac{eB}{2m_e}(L_z + gS_z)$

Acting on L_z, S_z eigenstates this gives $\delta E = \frac{eB\hbar}{2m_e}(m_\ell + g m_s)$

This accurately predicts Zeeman effect in a strong mag field, where Zeeman splitting $\gg$ hyperfine splitting

In a weak magnetic field,
Zeeman splitting << hyperfine splitting
so must use J^2, J_z eigenstates rather than L_z S_z eigenstates

Then a perturbation theory calculation gives [Wigner-Eckart]

$$\langle j, m | -\vec{\mu}\cdot\vec{B} | j, m \rangle = \delta E = \frac{eB\hbar}{2m_e}\left(g_j \, m_j \right)$$

where g_j = Landé g-factor

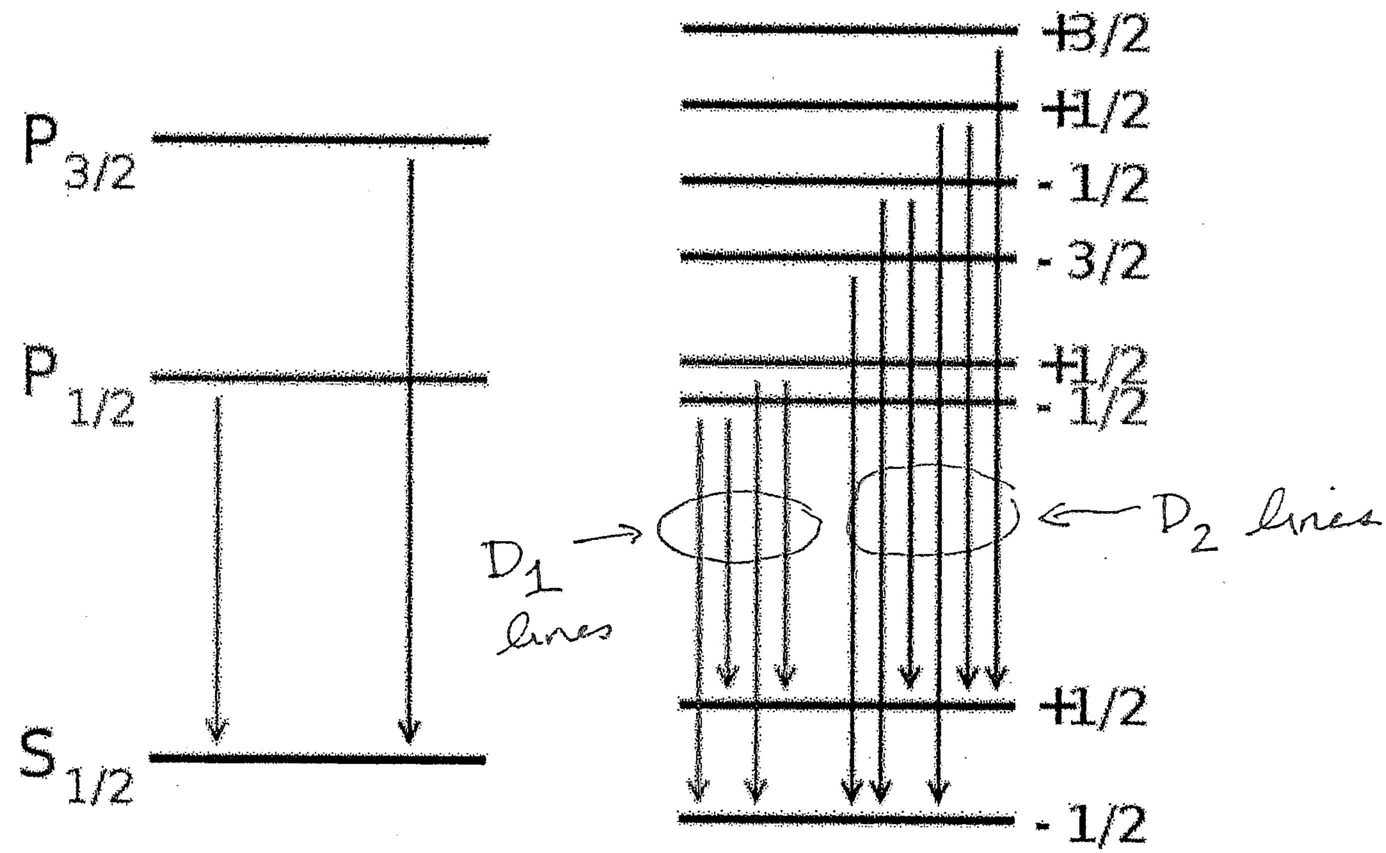
$$g_j = 1 - (g-1)\left[\frac{j(j+1) + \frac{3}{4} - l(l+1)}{2j(j+1)} \right]$$

Let $g = 2$

$l = 0, \quad j = \frac{1}{2} \Rightarrow g_j = 2$

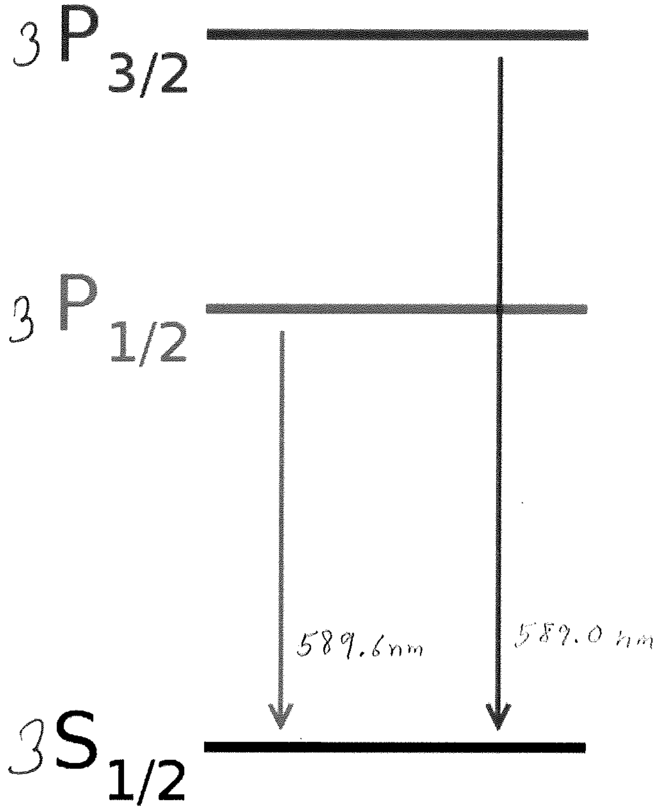
$l = 1, \quad j = \frac{1}{2} \Rightarrow g_j = \frac{2}{3}$

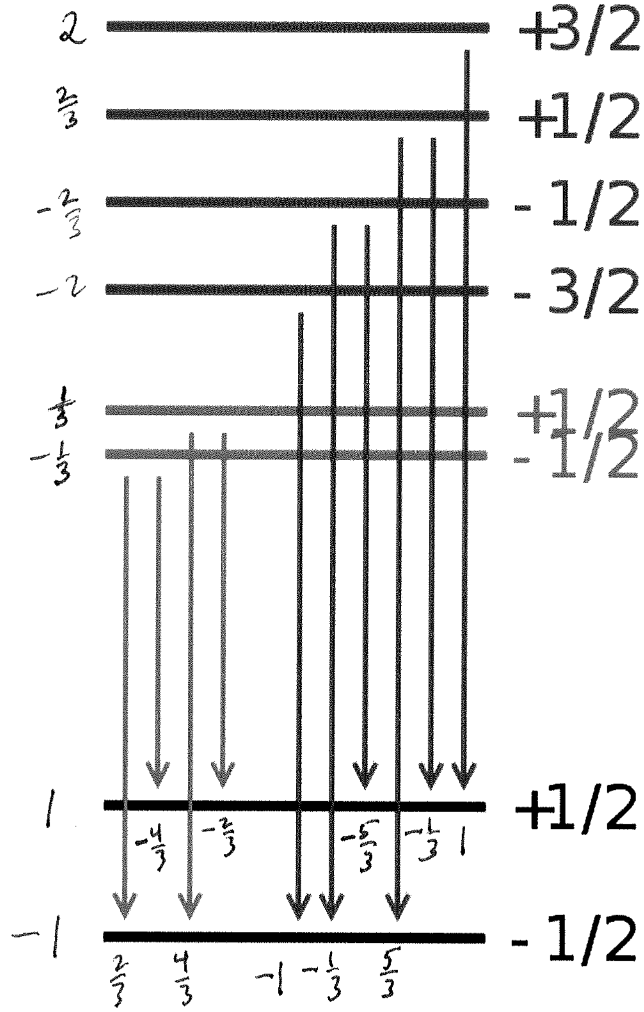
$\qquad\quad j = \frac{3}{2} \Rightarrow g_j = \frac{4}{3}$

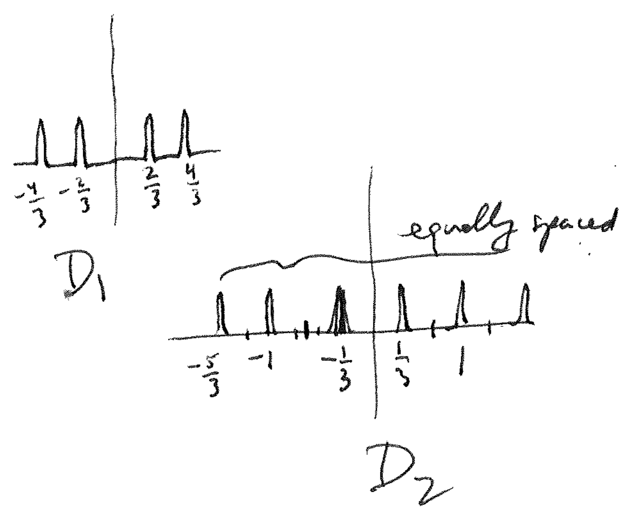


Sodium Zeeman effect (weak field)

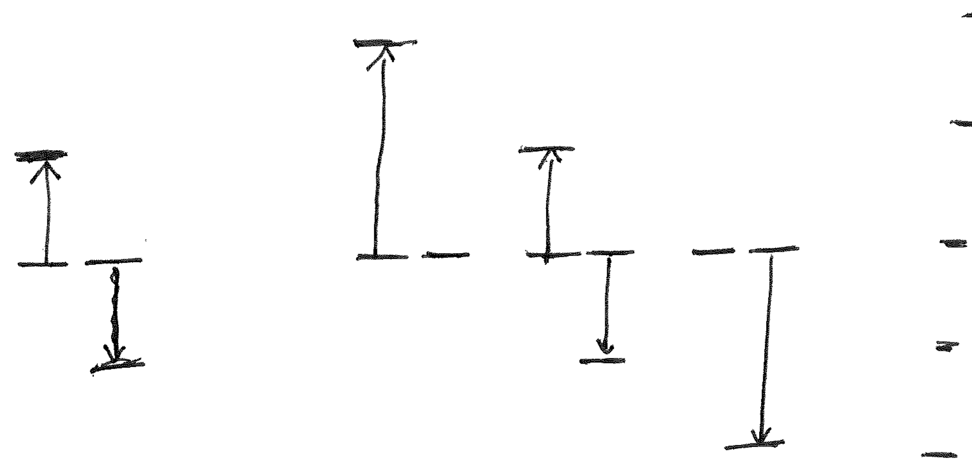
Sodium Zeeman







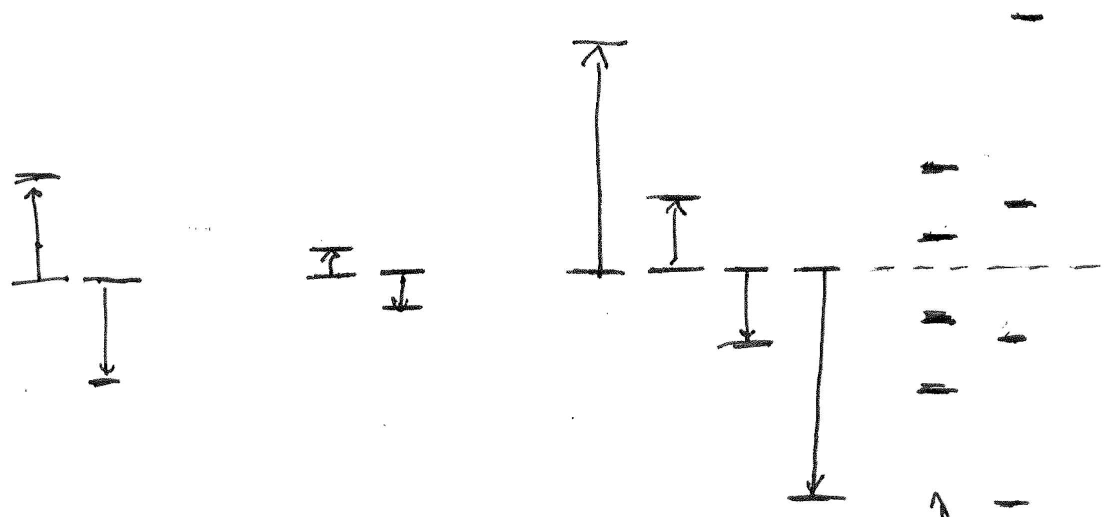
2s 2p



L_z, S_z eigenstates

$$\Delta E = \frac{e\hbar B}{2m}(m_\ell + 2m_s)$$

$m_\ell + 2m_s$ 1 −1 2 0 1 −1 0 2



J^2, J_z eigenstate

$$\Delta E = \frac{e\hbar B}{2m}\begin{cases} 2m_j & \text{for } s_{1/2} \\ \frac{2}{3}m_j & \text{for } p_{1/2} \\ \frac{4}{3}m_j & \text{for } p_{3/2} \end{cases}$$

1 −1 $\frac{1}{3}$ $-\frac{1}{3}$ 2 $\frac{2}{3}$ $-\frac{2}{3}$ −2

$S_{\frac{1}{2}} + P_{\frac{1}{2}}$ $P_{3/2}$

these are degenerate for hydrogen w/o any field