1925 Heisenberg + Born
Dirac
Schrödinger
} invent quantum mechanics

main features

Indeterminacy: there exist states that do not have definite values for all observables

Incompatibility: there exist pairs of observables such that if one observable has a definite value, the other cannot

e.g. position + momentum
different components of $\vec{L}$

[A new mathematical framework is required]

$\boxed{\text{State}}$

[The State of a qu. mech system (e.g. electron, atom, molecule, ...) is

described by a ket $|\psi\rangle$ $\in$ Hilbert space
(normed, complex vector space)
usually infinite dimensional

If $|\psi_1\rangle$ and $|\psi_2\rangle$ are possible states
so are all complex linear combinations
$\alpha_1|\psi_1\rangle + \alpha_2|\psi_2\rangle$ $\quad \alpha_i \in \mathbb{C}$

[very different from classical physics:
you can't add physical states

$|\psi\rangle$ and $e^{i\theta}|\psi\rangle$ describe the same state

$\Rightarrow$ $|\psi\rangle$ cannot be measured directly

Postulate: all information about the system is contained in $|\psi\rangle$

$\boxed{\text{observable}}$ A

Some measurable quantity of the system

eg S_z = z-component of spin of an electron

X = x-component of position of a particle

E = energy of the system

$\boxed{\text{eigenvalue}}$ of an observable (physical definition)

a possible result of a measurement of an observable (always a real number), a_n $(n=1,2,\ldots)$

- some observables have quantized eigenvalues
  eg. S_z : either $\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$
- some observables have continuous eigenvalues
  eg X : any real # from $-\infty$ to ∞.
- Some, eg: E, can have either, depending on the system

$\boxed{\text{eigenstate}}$ of an observable (physical def)

certain states, called eigenstates, are associated with the eigenvalues of an observable $\Rightarrow$ $|A=a_n\rangle$ or $|a_n\rangle$

They are states that, upon measurement of an observable, yield a definite (determinate) result.

eg. S_z has two eigenstates, $|S_z=\frac{\hbar}{2}\rangle = |+\rangle$ = "spin up"; $|S_z=-\frac{\hbar}{2}\rangle = |-\rangle$ = "spin down"

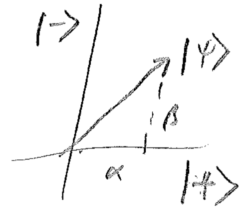
If measure S_z of an electron in state $|+\rangle$, (resp. $|-\rangle$) always get $\frac{\hbar}{2}$ (resp $-\frac{\hbar}{2}$)

$|+\rangle$ $|S_z$

Indeterminate States

[Even though eigenvalues are the only possible results of a measurement
eg $\pm\frac{\hbar}{2}$, eigenstates are not the only possible states.
Since Hilbert space is a vector space, can also have linear comb.]

$$|\psi\rangle = \alpha|+\rangle + \beta|-\rangle \qquad \alpha,\beta \in \mathbb{C}$$



[picture not accurate because $\alpha,\beta \in \mathbb{C}$]

[What is physical meaning of this state?
If measure S_z, what do you get?
Sometimes $\frac{\hbar}{2}$, sometimes $-\frac{\hbar}{2}$.]

Same state can give different results.

[non repeatability: Radical departure!]

QM is not deterministic.

QM can only predict probabilities.

[Einstein resisted]

"God does not play dice" [show 1st part]

Einstein/Podolsky/Rosen (EPR): QM is not complete

[there are additional (hidden) variables necessary to fully describe the state + ∴ give firm predictions]

Bell's theorem (1964): hidden variables ⇒ Bell inequalities
which are violated by QM, + by experiment

Bohr: "Stop telling God what to do"

"God does not play dice."

Albert Einstein

"Einstein, stop telling God what to do."

Niels Bohr

probabilities

[$|\psi\rangle = \alpha|+\rangle + \beta|-\rangle$]

Probability of measuring $\frac{\hbar}{2}$ is $|\alpha|^2 = \alpha^*\alpha$

Probability of measuring $-\frac{\hbar}{2}$ " $|\beta|^2 = \beta^*\beta$

(this assumes $|\psi\rangle$, $|+\rangle$, $|-\rangle$ are "normalized")
[explain later]

α, β = probability amplitudes

[check: $|\psi\rangle = |\pm\rangle \Rightarrow$ prob = 100%]

[Since only $\pm\frac{\hbar}{2}$ are possible]

$|\alpha|^2 + |\beta|^2 = 1$

eg $|\psi\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{i}{\sqrt{2}}|-\rangle$
[50% each one]

Do $|\psi\rangle$ and $e^{i\theta}|\psi\rangle$ yield same probabilities?

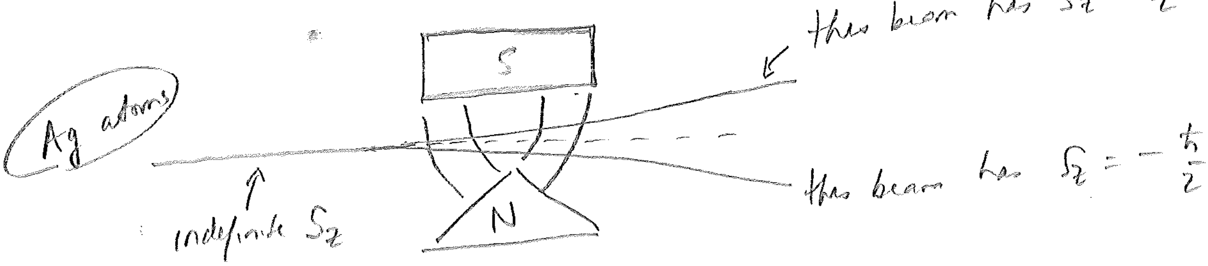
$$e^{i\theta}|\psi\rangle = \underbrace{e^{i\theta}\alpha}_{\alpha'}|+\rangle + \underbrace{e^{i\theta}\beta}_{\beta'}|-\rangle$$

$|\alpha'|^2 = |\alpha|^2$
$|\beta'|^2 = |\beta|^2$ $\Rightarrow$ yes

$|\psi\rangle = \frac{i}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$
is same state as above

measurement

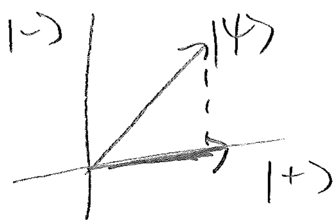
measure S_z using Stern Gerlach



After measurement, atoms do have definite value of S_z
because a repeat measurement yields same result w/ certainty

$\Rightarrow$ the act of measurement changes the state

$$|\psi\rangle \xrightarrow[\text{if measure } S_z = \frac{\hbar}{2}]{} |+\rangle$$ "reduction of the state vector"



"projection onto an eigenstate"

"collapse of the wavefunction"

$\boxed{\text{completeness}}$

Claim: the eigenstates of an observable A form a complete basis for the space of states

ie any state can be written as a linear combination

$$|\psi\rangle = \sum_{n=1}^{d} \psi_n |a_n\rangle$$

d = Dimension of state space = # of independent eigenstates (eg 2 for S_z)

ψ_n = coefficients of $|\psi\rangle$ in the A-basis

= probability amplitude for measuring a_n

Probability of measuring a_n is $p_n = |\psi_n|^2$

(provided $|\psi\rangle$ and $|a_n\rangle$ are normalized)

$$\sum_{n=1}^{d} p_n = 1$$

$\langle A \rangle$ = expectation value of A

mean value (weighted by probabilities) of a measurement of A on state $|\psi\rangle$

$$\langle A \rangle = \sum p_n a_n = \sum |\psi_n|^2 a_n$$

well omit writing the index

e.g. $\langle S_z \rangle = |\alpha|^2 \frac{\hbar}{2} + |\beta|^2 \left(-\frac{\hbar}{2}\right)$

$= \left(|\alpha|^2 - |\beta|^2\right) \frac{\hbar}{2}$

e.g. $\langle S_z \rangle = 0$ for $|\psi\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{i}{\sqrt{2}}|-\rangle$

[so not one of the possible results]

"expected value" indeed!

Uncertainty

ΔA = uncertainty of A for state $|\psi\rangle$

= r.m.s. deviation from $\langle A\rangle$

Let $\Delta = A - \langle A\rangle$ deviation from the mean

The possible results of Δ are $a_n - \langle A\rangle$.

$$\text{mean square deviation} = \sum p_n (a_n - \langle A\rangle)^2$$

$$= \sum p_n a_n^2 - 2\langle A\rangle \sum p_n a_n + \langle A\rangle^2 \sum p_n$$

$$= \langle A^2\rangle - 2\langle A\rangle\langle A\rangle + \langle A\rangle^2 \cdot 1$$

$$= \langle A^2\rangle - \langle A\rangle^2$$

$$\therefore \quad \Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}$$

For eigenstates, $\Delta A = 0$ because $\langle A\rangle = a_n$, $\langle A^2\rangle = a_n^2$