Deficiencies of Bohr model

[doesn't predict spectra of elements other than hydrogen]

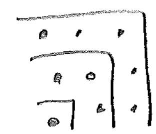
- predict that orbital angular momentum quantum no. l is equal to principle quantum no. n (which determines energy) but in fact $0 \leq l \leq n-1$

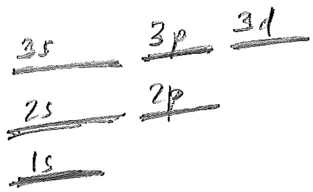
  [How can this be?]

- predicts that electrons travel in well-defined planar orbits but electrons are described by 3d probability distributions w/a spherically symmetric ground state (hence $l = 0$ for $n=1$)

- incorrect degeneracies for each energy level

n	l	m	degeneracy (due to Coulomb)
1	0	0	1
2	0, 1	0; −1, 0, 1	1 + 3 = 4
3	0, 1, 2	0; −1, 0, 1; −2, −1, 0, 1, 2	1 + 3 + 5 = 9
			$\sum_{l=0}^{n-1} (2l+1) = n^2$





- Pauli exclusion ⇒ one electron for each distinct set of quantum numbers
  "filled states" have 1, 5, 14, etc electrons
  but periodic table ⇒ twice these numbers.
- Pauli suggested 4th quantum #: "zweideutigkeit" twovaluedness

- anomalous Zeeman effect [show; pauli quote]

- Stern Gerlach doublets [show]

Goudsmit + Uhlenbeck suggest electron possesses <u>intrinsic</u> angular momentum (spin) $\vec{S}$

$S = s\hbar$

$S_z = m_s \hbar \qquad m_s = -s, \dots, s$

Multiplicity $= 2s+1$

Since $2s+1 = 2 \Rightarrow s = \frac{1}{2} \Rightarrow m_s = -\frac{1}{2}, \frac{1}{2}$ (doublet)

spin down ↓ ↑ spin up

3s 3p 3d   2+6+10   [3d gets shifted up due to electron interactions]

2s 2p   2+6

1s   2

[Lorentz didn't like it: electron equator rotates > c.]

[Pauli didn't like it + talked Kronig out of it.]

All particles have intrinsic spin either { half-integer → fermions; integer → bosons }

s		
0	scalar particle	Higgs, α
$\frac{1}{2}$	spinor	e^-, p, quarks
1	vector	γ, $W^\pm$, Z^0
2	tensor	graviton

Spin

Freshly encountering this situation, it was of obvious interest to physicists in the 1920s to specify a complete list of the quantum numbers associated with the electrons in atoms. A decisive contribution was made by a twenty-five-year-old Viennese named Wolfgang Pauli. Brilliant, acerbic, and fat, Pauli was already well on his way to becoming one of the great physicists of the century; even as a young man, his intellect and passionate style of argumentation intimidated many of his colleagues. He had a kind heart, an irascible temperament, and a streak of melancholia that he overcame by driving at physical questions as if his life depended on it, which perhaps it did. Like many theoreticians of the day, Pauli was concerned with understanding the spectral lines emitted by atoms. Bohr's original model worked for the relatively simple patterns emitted by hydrogen, but heavier, more complex elements were much harder to understand. For example, cesium, strontium, and barium—several of what chemists call the alkaline-earth metals—produce spectral lines that upon close examination are seen actually to be split in two; these lines, called doublets, are made of two almost identical frequencies. In December of 1924, Pauli suggested that a complete list of the quantum numbers of an orbiting electron would include its energy, angular momentum, and orientation in space; in addition, to explain the alkali doublets, he suggested that there had to be a fourth quantum number, which he called, rather unhelpfully, *Zweideutigkeit*—two-valuedness.[17]

In the summer of 1925, Samuel Goudsmit, a young Dutch physicist, was trying to explain Pauli's ideas to a young countryman, George Uhlenbeck, who had been out of Holland for a while and was trying to get back into physics. Uhlenbeck had been told by his teacher that he should be briefed by Goudsmit. During afternoon talks, Goudsmit explained Pauli's four quantum numbers to Uhlenbeck. "I was impressed," Uhlenbeck recalled later, "but since the whole argument was purely formal, it seemed like abracadabra to me. There was no picture that at least qualitatively connected Pauli's formula with the old Bohr atomic model."[18]

It occurred to Uhlenbeck that Pauli's *Zweideutigkeit* was not really another quantum number, but simply another property of an electron. He suggested that perhaps an electron spins on its axis like a toy top; unlike a top, however, the spin of the electron would be quantized, and it could only turn at certain speeds. Looking at Pauli's formulas, Uhlenbeck and Goudsmit realized that if electrons had a second angular momentum associated with a spin, this would perfectly account for both "two-valuedness" and the double spectrum lines from the alkaline earths. The amount of spin was one-half $\hbar$, where $\hbar$ is physics shorthand for $h/2\pi$, that is, Planck's constant divided by twice pi. Although both men were struggling in the sea of quantization, they "appreciated right away that if the [spin] angular momentum of the electron was $\hbar/2$, one had a picture of the alkali doublets as the two ways the electron could rotate with respect to its orbital motion."[19] The two ways of rotation would give the electrons two slightly different energy levels, and electrons with two slightly different energy levels would create two slightly different frequencies of light.

They took their idea to Uhlenbeck's teacher, Paul Ehrenfest, who headed the physics department in Leiden. Ehrenfest made some suggestions, had them write up a short paper about spin, and then told them to take it to Hendrik Lorentz, the grand old man of Dutch physics. In addition to inventing the Lorentz invariance on which special relativity is based, Lorentz was the first man to construct a theory of the electron. In 1925, Lorentz was seventy-two and ostensibly retired, but he still taught a class at Leiden every Monday morning from eleven to noon. After one class, Uhlenbeck and Goudsmit showed Lorentz their paper, which was only a few paragraphs long.

"Lorentz was not discouraging," Uhlenbeck once said. "He was a little bit reticent, [but] said that it was interesting and that he would think about it." *Thinking*, for Lorentz, was apparently an active occupation. "It was so typical of Lorentz that he immediately made very extensive calculations on the classical theory of rotating electrons. I think the next week, but maybe two weeks later, he gave me such a *stack* of papers with long calculations. Large white paper, I still remember. He tried to explain it to me, but it was so learned that I . . ."[20] Uhlenbeck's voice trailed off. Lorentz had explained several problems, one of the most grievous being that if the electron really had a spin angular momentum of $\hbar/2$, this implied that it rotated with a particular velocity. The old man had figured out the speed: ten times faster than the speed of light. Because nothing can go faster than the speed of light, this was, Uhlenbeck and Goudsmit decided, a devastating critique. They were most unhappy.

We visited Uhlenbeck at Rockefeller University, a set of isolated buildings of funereal modernity on the Upper East Side of Manhattan. The university is one of the more carefully guarded in the nation, and we were detained by an armed guard for some twenty minutes. It was one of those clear mornings when the gray, boxy magnificence of the cityscape is just slightly fuzzed over by smog. He was eighty-four, a tall man with thinning hair scattered across his head like so much straw. Impatient with his growing deafness, he asked us to sit close. "Talk in my left ear," he said cheerfully. "The other one is primarily for decoration." He kept a pair of glasses in his hand, twisting the frames around his fingers like worry beads. We asked him what he had done with Lorentz's calculations.

"I thought, well, therefore it is all wrong, what we have done. And I went back to Ehrenfest and said, 'You better not publish that paper, because Lorentz has shown that it is not correct.' He said, 'I sent it out right away. It will come out next week.' "

Ehrenfest had mailed it off without waiting to hear from the master?

Uhlenbeck roared with laughter.

"He *knew*!" he said. Ehrenfest had immediately seen the difficulties with spin. "But he said to us—he said it in German—'*Sie beiden sind jung genug sich eine Dummheit leisten zu können*!' Both of you are young enough to afford a stupidity!"[21]

Ehrenfest was not being entirely cavalier. It is important for theoreticians not to pay attention at all times to what is wrong with their ideas. They must not be too afraid to follow their intuition; every now and then, the best way to hit a target is to shoot from the hip. Spin, as an example, had been thought of earlier, by one Ralph de Laer Kronig, an American of Hungarian descent. Unfortunately, Kronig asked caustic Wolfgang Pauli for his reaction to the notion that *Zweideutigkeit* was due to the effects of a spinning electron. Pauli tore apart the idea and, to the later regret of both men, Kronig never published it.[22] Bohr, on the other hand, dismissed Lorentz's objections immediately, telling Uhlenbeck that the faster-than-the-speed-of-light problem would "disappear when the real quantum theory is found."[23]