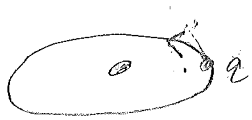
Atom in a magnetic field (Zeeman effect)






A circulating charge acts as a(n electro)magnet (dipole)

magnetic dipole moment $\mu = IA$

$A = \pi r^2$

$I = \frac{q}{T}$

period $T = \frac{2\pi r}{v}$

~~current~~ $I = \frac{qv}{2\pi r}$

~~magnetic dipole moment of a current loop is: IA~~

$$\mu = IA = \left(\frac{qv}{2\pi r}\right)(\pi r^2) = \frac{qvr}{2}$$

orbital angular momentum $L = mvr$

$$\Rightarrow \mu = \left(\frac{q}{2m}\right) L$$

Both $\vec{L}$ and $\vec{\mu}$ have direction

$$\vec{\mu} = \underbrace{\left(\frac{q}{2m}\right)}_{\equiv \text{ gyromagnetic ratio}} \vec{L}$$

For an electron in a Bohr orbit, $L = l\hbar$, $l \in \mathbb{Z}$, ...

$$\mu = \frac{e}{2m_e}(l\hbar) = \underbrace{\left(\frac{e\hbar}{2m_e}\right)}_{\text{smallest unit of dipole moment}} l$$

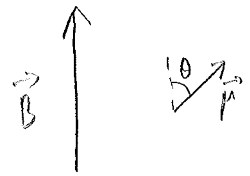
$\equiv$ Bohr magneton $= \mu_B$

$= 5.8 \times 10^{-5} \frac{eV}{Tesla}$

$F = qvB$ [Tesla $\equiv \frac{J \cdot s}{C \cdot m^2}$]

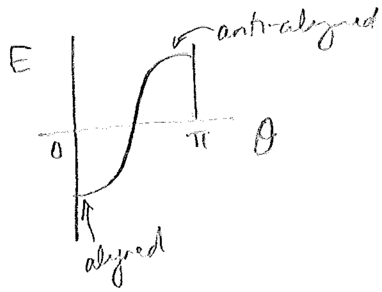
[gaussian $\Rightarrow \mu = \frac{e\hbar}{2mc}$]

[paramagnetic $\Rightarrow$ permanent dipole moment, e.g. Ag, H atoms]

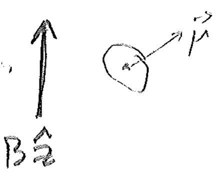



In an external $\vec{B}$ field, a magnetic dipole has energy

$$E = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\theta$$




Energy of a Bohr atom in an external $\vec{B}$ field:

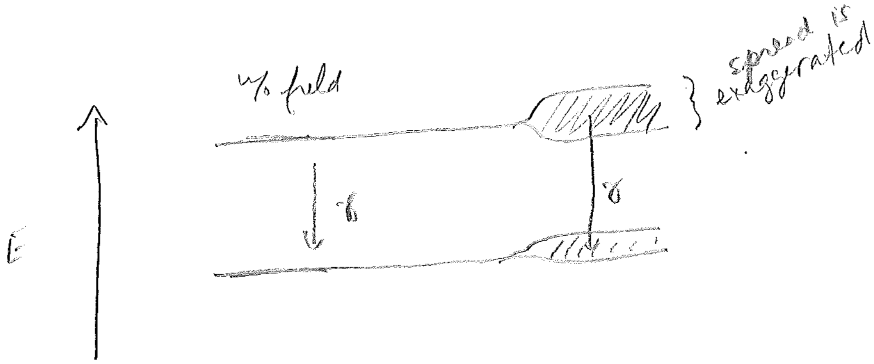



$$E = \underbrace{E_n}_{\text{w/o field}} - \vec{\mu}\cdot\vec{B}$$

$$= -\left(\frac{mk^2e^4}{2\hbar^2}\right)\frac{Z^2}{n^2} + \frac{e}{2m}L_z B, \qquad L_z = \hbar l\cos\theta$$

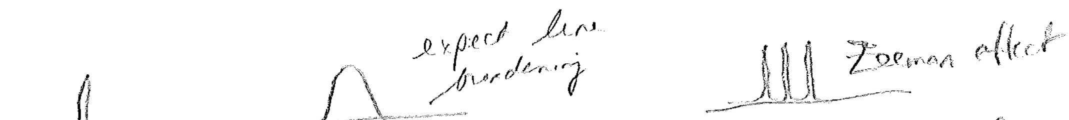
$$= (-13.6\,\text{eV})\frac{Z^2}{n^2} + \left(5.8\times10^{-5}\,\frac{\text{eV}}{\text{Tesla}}\right)\cdot B\,l\cos\theta$$

[eg of "perturbation theory"]



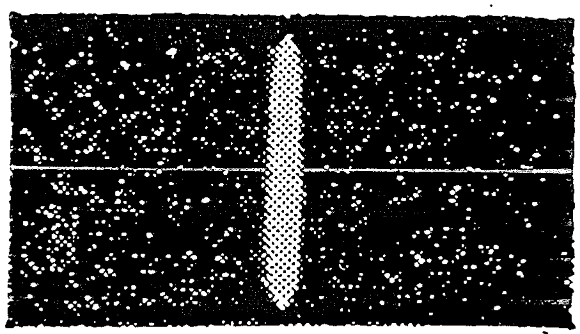

Instead [show transparency]

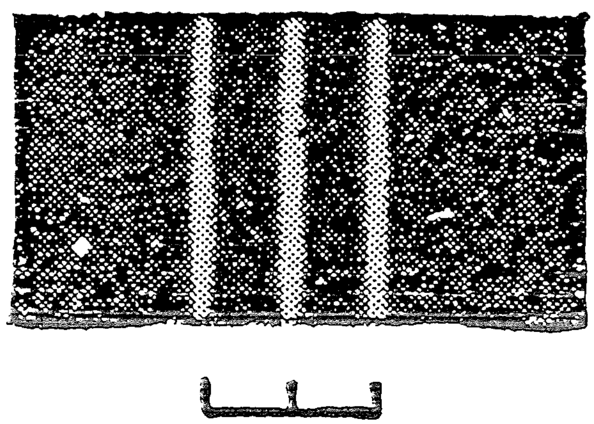
Spectra




Suggests discrete values for $L_z$

Zinc Singlet





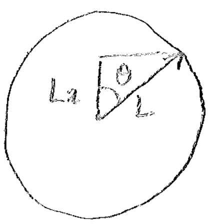
Normal Triplet

## Spatial quantization of angular momentum

Both the magnitude and the components of $\vec{L}$ are quantized

$L = l\hbar \qquad l = 0, 1, 2 \ldots$

$L_z = m\hbar \qquad \underbrace{m = -l, -l+1, \ldots, l-1, l}_{2l+1 \text{ possible values}} \qquad \text{since } |L_z| \le L$

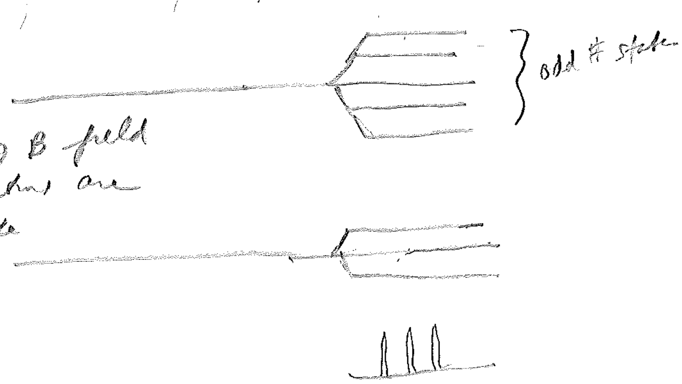



$\cos\theta = \frac{L_z}{L} = \frac{m}{l}$ so only certain angles are allowed "spatial quantization"

Correspondence principle: $l \to \infty \Rightarrow$ all angles are allowed

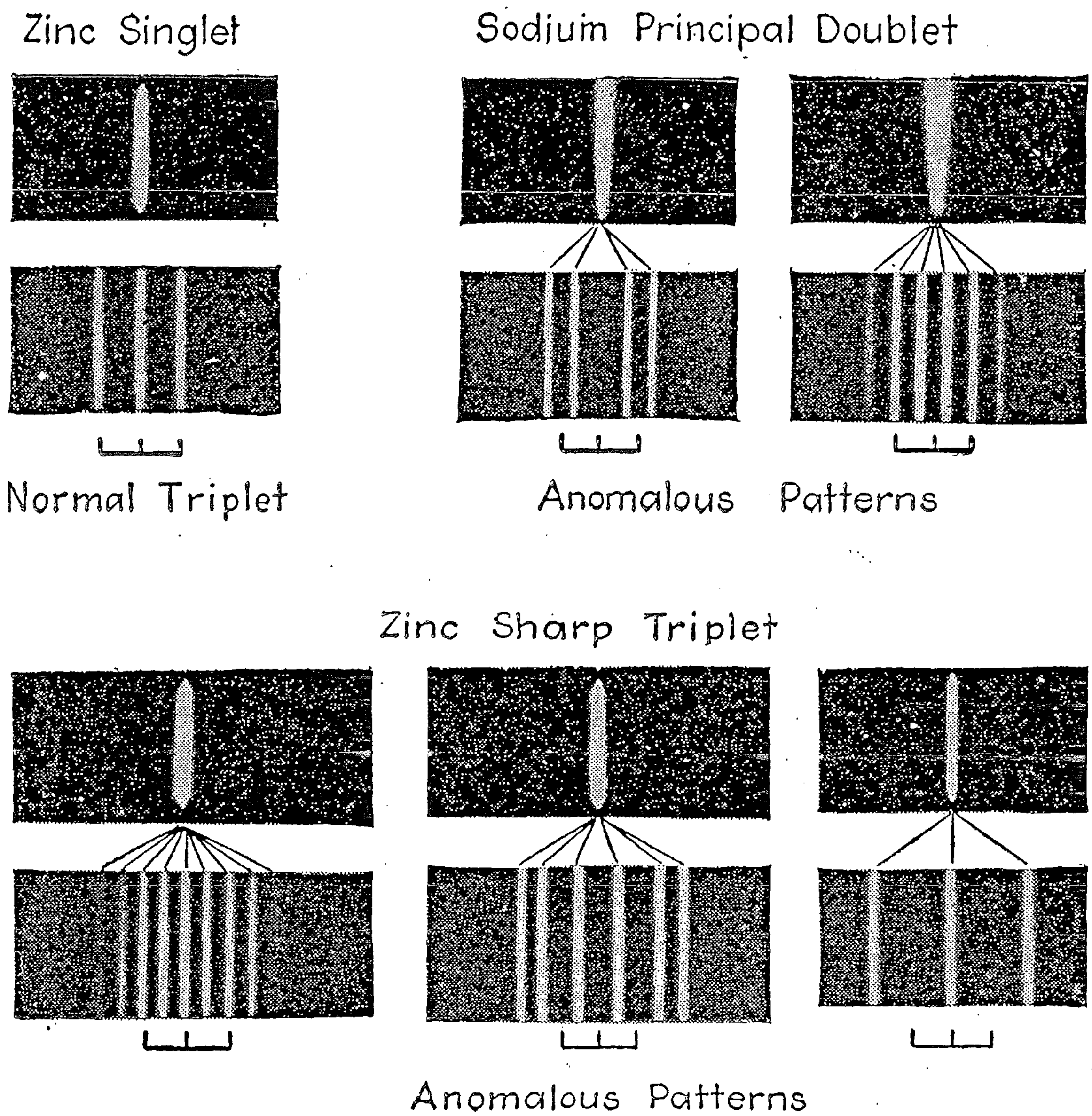
| $l$ | state | $m$ | multiplicity |
|---|---|---|---|
| 0 | s | 0 | singlet |
| 1 | p | $-1, 0, 1$ | triplet |
| 2 | d | $-2, -1, 0, 1, 2$ | quintet |
| 3 | f | $-3, -2, -1, 0, 1, 2, 3$ | septet |

$$E = (-13.6\,\text{eV})\frac{Z^2}{n^2} + \left(5\times10^{-5}\,\frac{\text{eV}}{\text{Tesla}}\right) B\, m \qquad |m| \le l$$




In absence of B field all the directions are degenerate

[only $|\Delta m| \le 1$ transitions]


Zinc Singlet
Sodium Principal Doublet
Normal Triplet
Anomalous Patterns
Zinc Sharp Triplet
Anomalous Patterns

"You look very unhappy."

"How can one look happy when he is thinking about the anomalous Zeeman effect?"

Wolfgang Pauli, 1923

[Zeeman is somewhat indirect]

Stern Gerlach effect : deflection of a beam of atoms in an inhomogeneous magnetic field

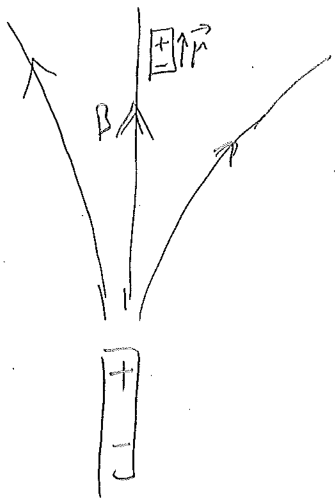
Energy of magnetic dipole in a magnetic field $E = -\vec{\mu}\cdot\vec{B}$

$\mu \cdot B$

In an inhomogeneous $\vec{B}$ field, E is position dependent

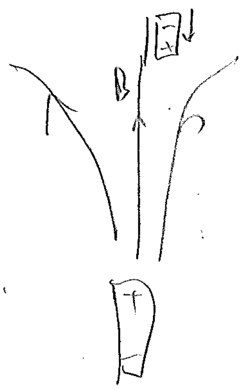
∴ ~~...~~ magnetic fd causes ~~...~~ a force (as well as a torque)

$$\vec{F} = -\vec{\nabla} U = +\vec{\nabla}(\vec{\mu}\cdot\vec{B})$$



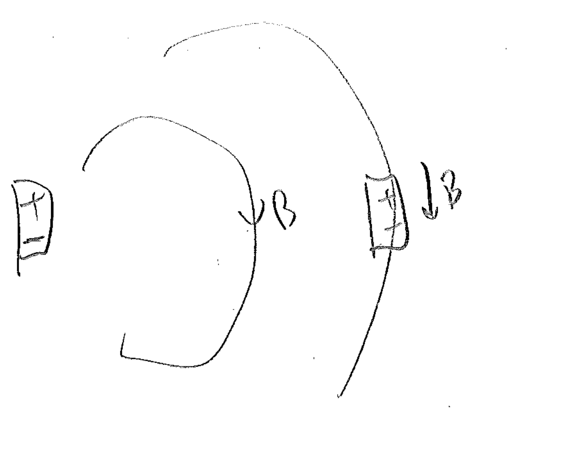
If $\vec{\mu}$ is aligned w/ $\vec{B}$, then $\vec{\mu}\cdot\vec{B} > 0$ and $\vec{\nabla}(\vec{\mu}\cdot\vec{B})$ points in the direction of increasing $\vec{B}$ i.e. down

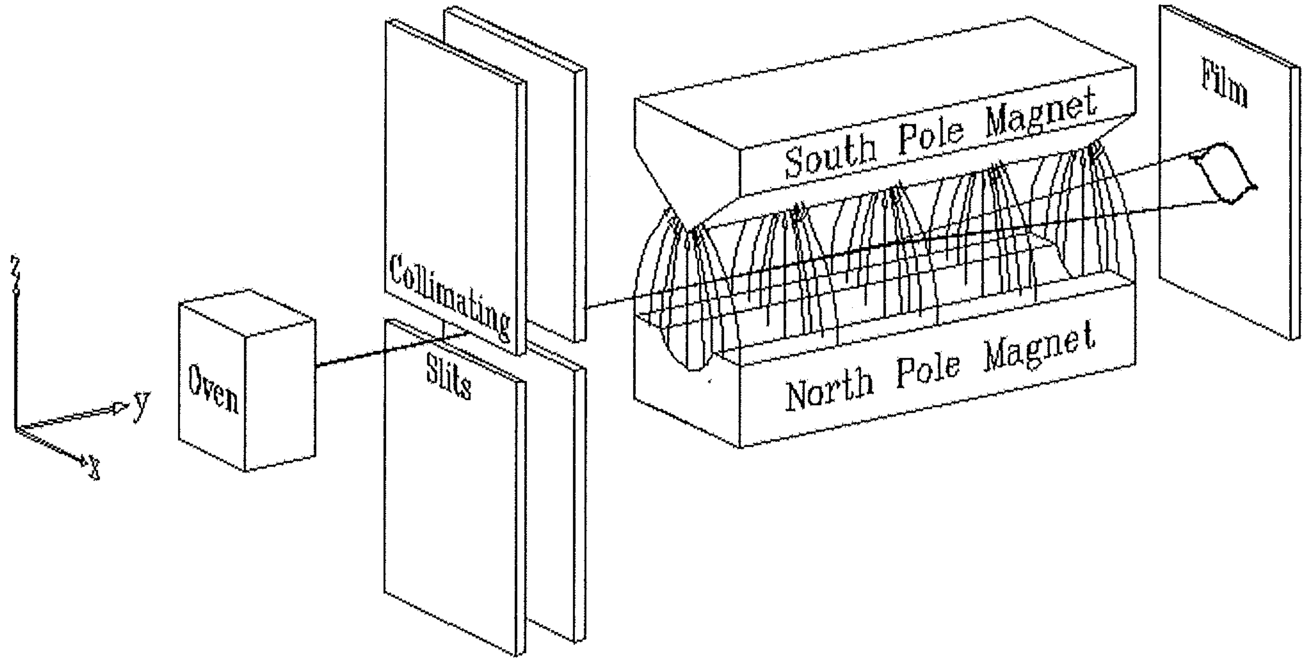
(magnets attract)



If $\vec{\mu}$ is anti aligned w/ $\vec{B}$ then $\vec{\mu}\cdot\vec{B} > 0$ & $\vec{\nabla}(\vec{\mu}\cdot\vec{B})$ points in direction of decreasing $\vec{B}$ i.e. up.

(magnets repel)





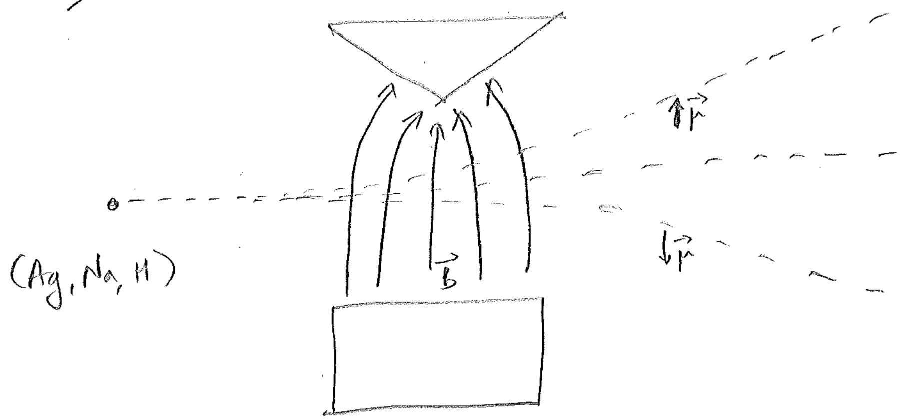
z
y
x
Oven
Collimating
Slits
South Pole Magnet
North Pole Magnet
Film

(Zeeman is somewhat indirect)

$2^{-5}$

Stern Gerlach ~~effect~~ experiment (1922) : ~~force on atoms in a nonuniform mag fld~~

~~beam of atoms, are deflected by an inhomogeneous magnetic field~~




$$\vec{F} = \vec{\nabla}(\vec{\mu}\cdot\vec{B}) =$$

$$= \nabla\left(-\frac{e}{2m_e}B_z L_z\right)$$

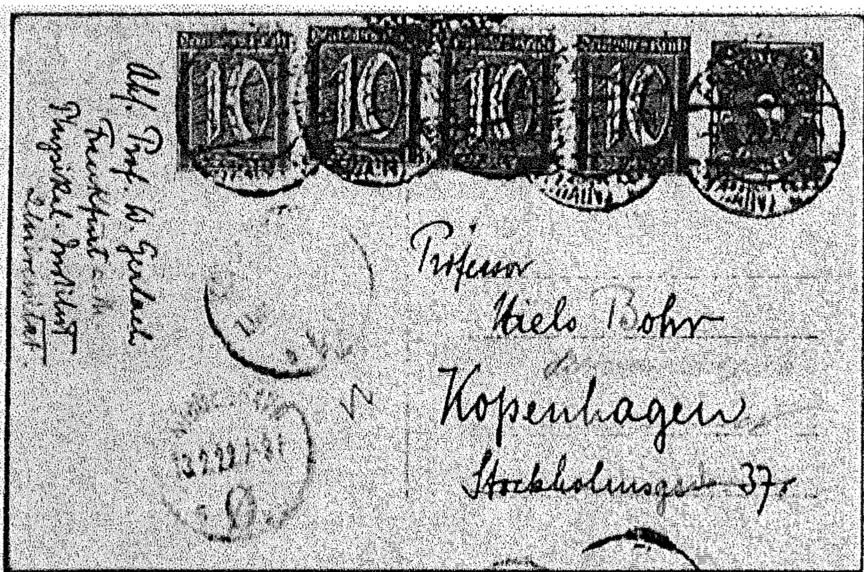
$$= \left(-\frac{e\hbar}{2me}\nabla B_z\right) m$$

deflection proportional to $m$

[ discrete spots on screen mean
[ evidence of spatial qu'zn
[ depending ~~only~~ directly on energy
[ levels, not on differences of ]

[cf Cohen Tannoudji]
vol I, p. 391
$\Rightarrow$ validity of treating
classical trajectories

~~Drawbacks: (1) Ag or H in gnd st. shd have $l=0$~~
~~$\Rightarrow$ no degeneracy, shd not deflect~~
~~(2) # of spots is even!~~



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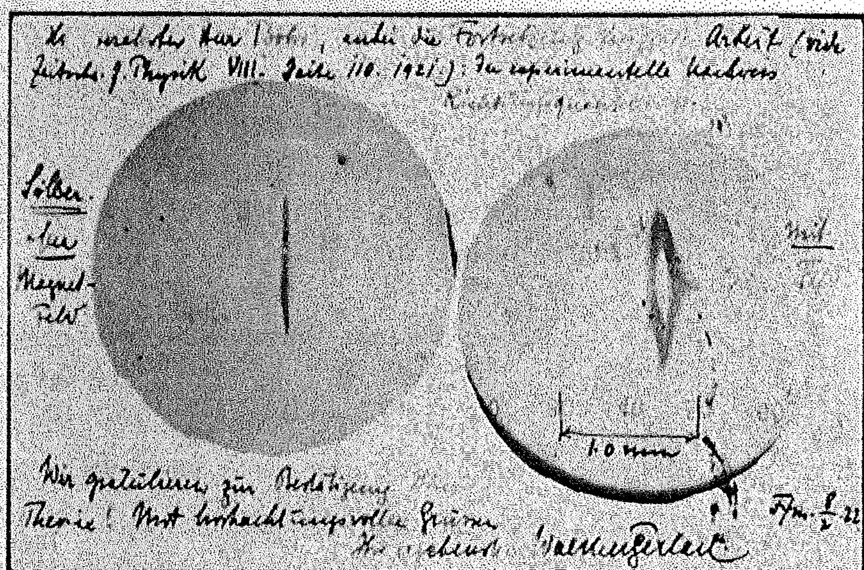

Hier, verehrter Herr Bohr, anbei die Fortsetzung unserer Arbeit (vgl. Zeitschr. f. Physik VIII. Seite 110. 1921): Der experimentelle Nachweis der Richtungsquantelung.



Wir gratulieren zur Bestätigung Ihrer Theorie! Mit hochachtungsvollen Grüssen
Ihr ergebener Walther Gerlach

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