$\boxed{\text{PHYS 3140}}$

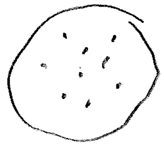
[Quantum mechanics began in]

Atomic physics

1897 Thomson disc. e^-

[first sub atomic particle
i.e. pcl w/ mass $\ll$ hydrogen
i.e. $\frac{e}{m}$ ratio $\gg$ hydrogen]

Thomson model of atom (plum pudding)



[hydrogen: single e^- at center of distrib. of pos. charge

if displaced, will return to center

uniform positive charge $\rightarrow$ SHM (problem)

classical electromagnetic theory:
will emit EM waves w/ same frequency

indeed, excited atoms do emit light
w/ characteristic discrete frequencies (1140 lab)]

1885 Balmer + Rydberg disc. formula for hydrogen atom freq.

$$f = (3.29 \times 10^{15}\,\text{Hz})\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) \qquad n, n' \in \mathbb{Z}, \quad n' > n$$

$n=1$ Lyman series (UV)
$n=2$ Balmer series (vis)
$n=3$ Paschen series (IR)

[no simple formula for other elements]

1908 Ritz combination principle

f = difference of "terms"

1911 Rutherford disc. nucleus [i.e. positive charge is not jelly]

Rutherford model of atom (planetary) $\rightarrow$ classically unstable due to emission of EM radiation

1900 Planck } disc. discrete nature of light
1905 Einstein }

$$E_{photon} = hf \qquad h = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}$$

[Ritz principle suggests E_{photon} = diff. of energies

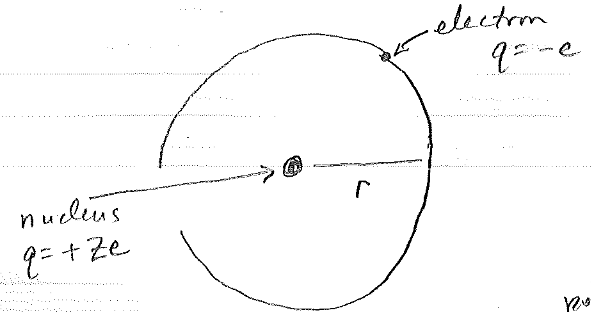
En. cons. suggests these are energy of atom before & after emission

Balmer/Rydberg suggests $E_{hyd} = \frac{-(13.6\,\mathrm{eV})}{n^2}$]

1913 Bohr model of atom (planetary model w/ quantized ang. mom.)
$\rightarrow$ solves stability problem
$\rightarrow$ predicts $E_{hyd} = -\frac{(13.6\,\mathrm{eV})}{n^2}$

1925-6 Heisenberg, Schrödinger invent quantum mechanics
$\rightarrow$ modern model of atom [electron described by probability distribution]

Bohr model: incorrect, but a useful mnemonic



Coulomb force: $|F| = \frac{KZe^2}{r^2}$, $K \equiv \frac{1}{4\pi\epsilon_0}$

potential energy $V = -\frac{KZe^2}{r}$

kinetic energy $T = \frac{1}{2}mv^2$ (assume non-relativistic)
[check later]

m = reduced mass $\Rightarrow \frac{1}{m} = \frac{1}{m_e} + \frac{1}{m_{Nuc}}$

$\Rightarrow m \approx m_e$

$E = T + V$

orbital angular mom $\vec{L} = \vec{r} \times \vec{p}$

You will show:

Assume circular orbit

1) $v \sim r^{-\frac{1}{2}}$

2) $E = -\frac{1}{2}mv^2 \sim -\frac{1}{r}$

[Virial thm: for $V \sim r^n$ $\Rightarrow V = \frac{2}{n}T$]

3) $L = mvr \sim r^{\frac{1}{2}}$

4) $f_{orbital} \sim r^{-3/2}$

[any r allowed]

[leave space in notes to derive these]

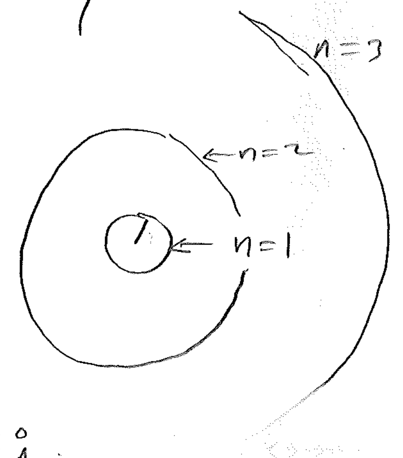
Orbit classically unstable since accelerating charge emits EM radiation

[HW: derive v, E, L, f for circular orbit]

Recall $\Delta x \Delta p \geq \frac{\hbar}{2}$. L and $\hbar$ have same units.

Bohr postulated that orbital angular momentum is quantized

$\boxed{L = n\hbar}$ $n = 1, 2, 3, \ldots$



You will show $\Rightarrow r_n = n^2 \left(\frac{\hbar^2}{ZmKe^2} \right)$

[allowed orbits have specific radii]

hydrogen ground state $r_1 = \frac{\hbar^2}{mKe^2} \equiv a_0$ (Bohr radius) $\approx \frac{1}{2}$ Å

$= 0.529 \times 10^{-10}$ m (small!)

$\approx 50{,}000$ fm (large!)

You will show

$v_n = \frac{1}{n} \left(\frac{ZKe^2}{\hbar} \right)$ $\qquad r_n = \frac{a_0}{Z} n^2$

Is nonrel approx valid?

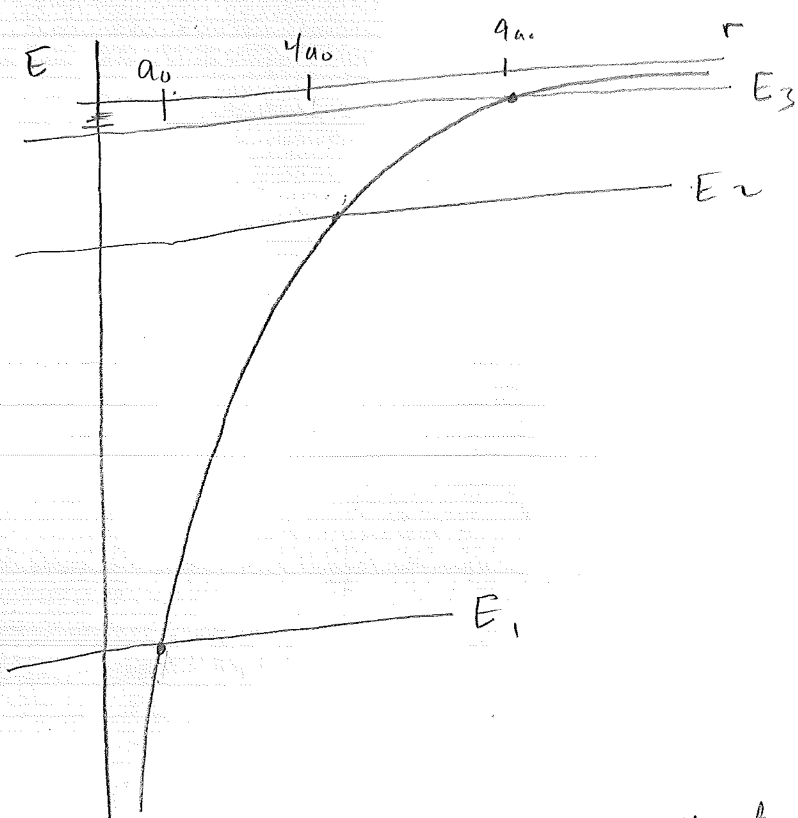
Define fine structure constant $\underbrace{\alpha \equiv \frac{Ke^2}{\hbar c} \approx \frac{1}{137}}_{\text{memorize}}$ ← dimensionless

Nonrel approx valid iff $v_1 \ll c$

ie if $\frac{v_1}{c} = Z\alpha \ll 1$ ie for small Z

Useful mnemonic: hydrogen gnd st: $\underline{\beta = \alpha}$

$$E_n = -\frac{1}{2} m v_n^2 = \quad -\frac{Z^2}{n^2}\underbrace{\left(\frac{mK^2e^4}{2\hbar^2}\right)}_{\text{Rydberg} \approx 13.6\,\text{eV}}$$



works well
for hydrogen
+ for innermost
levels of heavier
elements where
Ze is not screened;

also outer most e
of alkali elements
w/ Z→1

[Promise to derive this from Schrodinger eqn before end of course]

$\nearrow \alpha = \beta$

<u>useful mnemonics</u>: $v_1 = \beta c = \alpha c = \frac{Ke^2}{\hbar}$

$$\text{Rydberg} = \frac{1}{2}mv^2 = \frac{1}{2}mc^2\beta^2 = \frac{1}{2}\underbrace{mc^2}_{511{,}000\text{ eV}}\underset{\left(\frac{1}{137}\right)^2}{\alpha^2} = \frac{mK^2e^4}{2\hbar^2} \checkmark$$

no factors of c

$$\text{Bohr radius: } L = mva_0 = \hbar \Rightarrow a_0 = \frac{\hbar}{mv} = \frac{\hbar}{mc\beta} = \frac{\hbar}{mc\alpha} = \frac{\hbar^2}{mKe^2} \checkmark$$

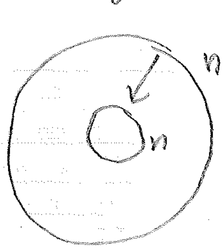
[Hwr: derive r_n, v_n, E_n, f_n using $L = n\hbar$]

You will show

orbital frequency $f_n = \frac{Z^2 m K^2 e^4}{2\pi \hbar^3 n^3}$

Classically, an orbiting charge would emit electromagnetic radiation with this frequency.

However, a Bohr electron loses energy in discrete jumps from one allowed orbit to another (quantum leap)



$$\Delta E = E_{n'} - E_n = \frac{Z^2 m K^2 e^4}{2\hbar^2}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$$

A photon with energy ΔE has frequency

$$f = \frac{\Delta E}{h} = Z^2 \underbrace{\left(\frac{m K^2 e^4}{4\pi \hbar^3}\right)}_{3.29\times 10^{+15}\ s^{-1}} \left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$$

Balmer/Rydberg formula!

Note: also explains Ritz principle

Bohr's correspondence principle

The predictions of QM must agree w/ those of classical physics in the limit of large quantum numbers

(eg large n)

eg. transition between n and n−1 emits a photon whose frequency is approx equal, when n is large, to the orbital frequency of the electron in state n

Show this!

[problem]

$$\left[\; f = \frac{mK^2e^4}{4\pi\hbar^3}\left[\frac{1}{(n+1)^2} - \frac{1}{n^2}\right] = \frac{mK^2e^4}{4\pi\hbar^3}\left[\frac{2}{n^3}\right] \;\right]$$

Fundamental constants $c \cong 3 \times 10^{8} \frac{m}{s}$

$$\hbar \cong 1. \times 10^{-27} \text{ erg sec}$$

$$\boxed{\hbar c \cong 200 \text{ MeV fm}}$$

$$\text{fm} = 10^{-15} \text{ m}$$

Units: $[c] = \frac{cm}{s}$

$$[\hbar] = \frac{g\,cm^2}{s}$$

$$\left[\frac{\hbar}{c}\right] = g\ cm$$

$$\left[\frac{\hbar}{c^2}\right] = g\ s$$

Use $\hbar, c$ to convert between units.

$$\begin{tikzcd} \text{LENGTH} \arrow[r, leftrightarrow, "c"'] \arrow[rr, bend right, leftrightarrow, "\frac{\hbar}{c}"'] & \text{TIME} \arrow[r, leftrightarrow, "\frac{\hbar}{c^2}"'] & \text{MASS} \end{tikzcd}$$

e.g. $t = 1$ year $\Rightarrow$ $x = ct = 1$ light year

mass m $\Rightarrow$ associated length $\boxed{\lambda = \frac{\hbar}{mc}}$ = Compton wavelength

e.g. electron: $mc^2 =$ 511,000 eV $\approx \frac{1}{2}$ MeV

$$\lambda = \frac{\hbar}{mc} = \frac{\hbar c}{mc^2} \cong 400 \text{ fm} = 4 \times 10^{-13} \text{ m}$$

~~de Broglie wavelength~~ $\cancel{\lambda = \frac{\hbar}{p}}$

Momentum p → de Broglie wavelength $\lambda = \frac{\hbar}{p} = \frac{\hbar}{mv} = \frac{\hbar}{mc\beta}$

Hyd atom $\lambda = \frac{\hbar}{mc\alpha} = \frac{\hbar}{mc}\left(\frac{\hbar c}{e^2}\right) = \frac{\hbar^2}{me^2}$ = Bohr radius a_0

↑ (~~destr~~ constructive interference of matter wave)

$$a_0 = \frac{\hbar}{mc\alpha} \approx 137\left(4\times10^{-13}\ \text{m}\right) \approx 5\times10^{-11}\ \text{m} \approx 0.5\ \text{Å}$$

$$\frac{e^2}{mc^2} = \alpha\frac{\hbar}{mc} \xleftarrow{\frac{1}{137}} \frac{\hbar}{mc} \xrightarrow{137} \frac{1}{\alpha}\frac{\hbar}{mc} = a_0$$

classical radius of electron (~3 fm) | Compton (400 fm) | Bohr (50000 fm)

~~make ... potential energy of ball of charge ... then $mc^2 = \frac{e^2}{r} \Rightarrow r = \frac{e^2}{mc^2}$~~

If mass due to potential energy of ball of charge of radius r

then $mc^2 = \frac{e^2}{r} \Rightarrow r = \frac{e^2}{mc^2}$