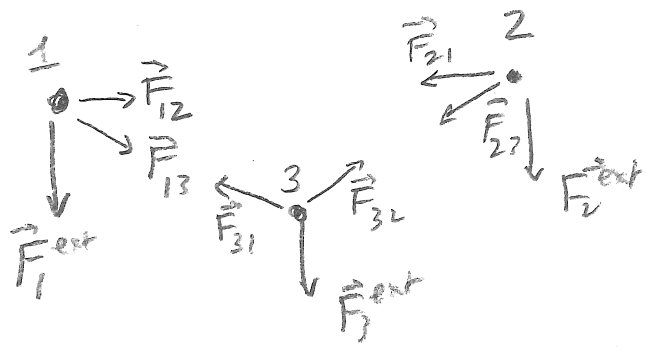
System of particles

All forces are due to interactions between objects (even external forces are caused by some unknown objects)

Consider a system of N point particles interacting in pairs and possibly also by external forces



$$m_i \ddot{\vec{r}}_i = \vec{F}_i^{ext}(\vec{r}_i) + \sum_{j \neq i} \vec{F}_{ij}(\vec{r}_{ij})$$

where $\vec{r}_{ij} \equiv \vec{r}_i - \vec{r}_j$

means sum j over all values except i

Momentum of the system

$$\vec{p}^{\,sys} = \sum_{i=1}^{N} m_i \vec{v}_i$$

$$\frac{d\vec{p}^{\,sys}}{dt} = \sum_i m_i \ddot{\vec{r}}_i = \sum_i \left(\vec{F}_i^{ext} + \sum_{j \neq i} \vec{F}_{ij} \right)$$

$$= \underbrace{\sum_i \vec{F}_i^{ext}}_{\vec{F}^{ext}} + \underbrace{\sum_i \sum_{j \neq i} \vec{F}_{ij}}_{\sum_{\substack{\text{pairs } i,j \\ (i<j)}} (\vec{F}_{ij} + \vec{F}_{ji})}$$

But $\vec{F}_{ij} + \vec{F}_{ji} = 0$ by Newton's 3rd law

$$\frac{d\vec{p}^{\,sys}}{dt} = \vec{F}^{ext}$$

change in total momentum = sum of <u>external</u> force
(internal interactions cancel out)
regardless of whether conservative or not

[consider a piece of chalk; only external forces affect its tot mom]

Corollary: total momentum of an isolated sys is conserved

If certain components of $\vec{F}^{ext}$ vanish, the corresponding components of $\vec{p}^{\,sys}$ are conserved

Center of mass

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M}$$

$$M = \sum_{i=1}^{N} m_i$$

Continuous distribution of matter

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\, dm = \frac{1}{M}\int \rho \vec{r}\, d^3r$$

$$M = \int dm = \int \rho\, d^3r$$

velocity of cm

$$\vec{V}_{cm} = \frac{\sum m_i \vec{v}_i}{M}$$

$$M\vec{V}_{cm} = \sum m_i \vec{v}_i = \vec{P}^{sys}$$

The total momentum of a system equal that of a single particle of mass M moving w/ velocity of cm

$$M\ddot{\vec{r}}_{cm} = \frac{d\vec{P}^{sys}}{dt} = \vec{F}^{ext}$$

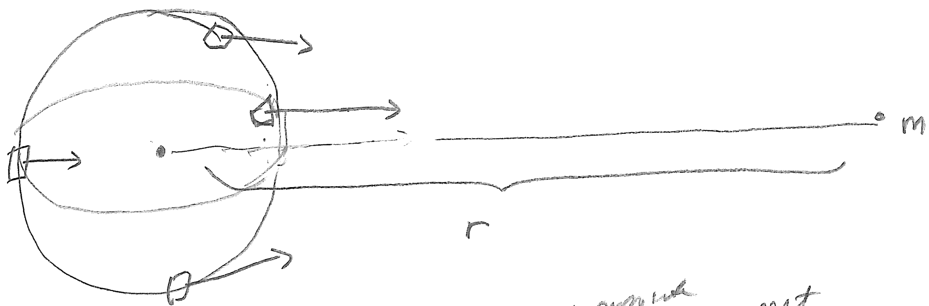
Center of mass moves as if it were a single particle of mass M subject to $\vec{F}^{ext}$

[fireworks]

Isolated system $\Rightarrow$ c-m moves at const velocity

Gravitational force on a sphere

Consider the force exerted on a spherical shell of uniformly distributed mass M by a point mass m outside the shell and located a distance r from the center of the shell



By 3rd law, this force is equal + opposite to the net force exerted by the shell on the point mass

In Phys 3000, we calculated this force to be exactly $|\vec{F}| = \frac{GMm}{r^2}$

Therefore $|\vec{F}^{ext}| = \frac{GMm}{r^2}$

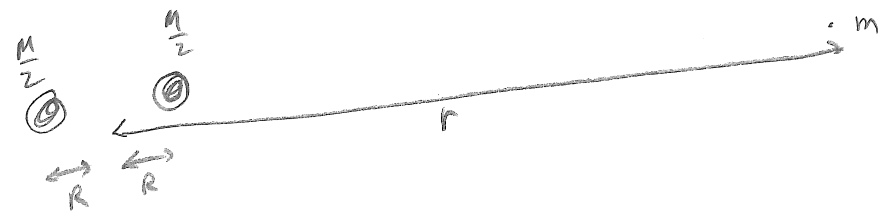
on shell

Since $\vec{F}^{ext} = M\ddot{\vec{r}}_{cm}$, the cm acts exactly as the shell were a point mass at $\vec{r}_{cm}$.

(Now replace point mass by a shell, a sphere of mass)

RJ?

We could not replace the distributed mass M w/a point mass if the shell were not spherical



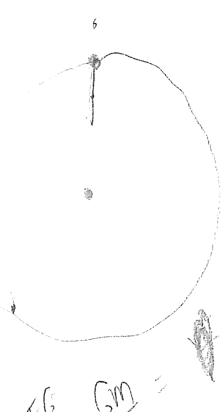
$$|\vec{F}^{ext}| = \frac{G\left(\frac{M}{2}\right)m}{(r+R)^2} + \frac{G\left(\frac{M}{2}\right)m}{(r-R)^2}$$

$$= \frac{GMm}{2}\frac{\left[(r-R)^2 + (r+R)^2\right]}{(r^2-R^2)^2}$$

$$= \frac{GMm}{2}\frac{\left[2r^2+2R^2\right]}{\left[r^2-R^2\right]^2}$$

$$= \frac{GMm}{r^2}\frac{\left[1+\frac{R^2}{r^2}\right]}{\left[1-\frac{R^2}{r^2}\right]^2} \approx \frac{GMm}{r^2}\left[1 + \frac{3R^2}{r^2} + \dots\right] \neq \frac{GMm}{r^2}$$

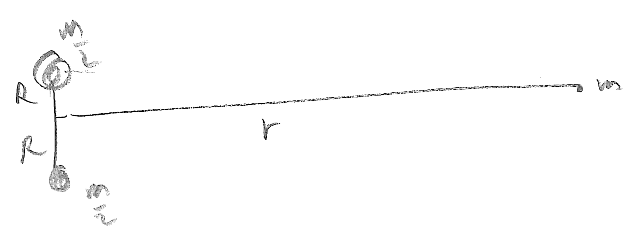
or just this?



F_G $\frac{GM}{r^2} = \omega^2 R$

v/ω^2

$\omega^2 R$ $\omega^2 R$



$$\frac{2GMm}{2(r^2+R^2)}\frac{r}{\sqrt{r^2+R^2}} = \frac{GMm}{r^2}\left[1+\frac{R^2}{r^2}\right]^{-\frac{3}{2}}$$

$$= \frac{GMm}{r^2}\left[1 - \frac{3R^2}{2r^2}\right]$$

$$\frac{GM\cancel{m}}{r^2}\left(1+\alpha\frac{R^2}{r^2}\right) = (\cancel{m})\,\omega^2 r$$

$\omega^2 =$