PHYS 3120: Spring 2018

P.S 40%
exams 60%

A1

Newtonian mechanics

Newton's 2nd law (valid in any inertial reference frame)

$$\frac{d\vec{p}}{dt} = \vec{F} \qquad (*)$$

where the net force $\vec{F}$ is the vector sum $\sum_A \vec{F}^{(A)}$ of all forces acting on a point mass m.

Nonrelativistic momentum: $\vec{p} = m\vec{v}$

Assume m does not change in time: $\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt}$

$$\Rightarrow \vec{F} = m\vec{a}$$

[2nd most famous eqn in physics]

but (*) is more fundamental (e.g. relativity, or changing mass)

[Sommerfeld, vol I, § 1.4]

[This is just a] 2nd order o.d.e

$$\frac{d^2\vec{r}}{dt^2} = \frac{\vec{F}}{m}$$

If $\vec{F}$ is known as a function of t,

we can integrate (analytically or numerically)

once to obtain the velocity and

twice to obtain the position as a function of t.

The two constants of integration are the initial position and velocity.

Example: { gravitational force that is uniform (in space) + constant (in time) $\vec{F} = m\vec{g}$

$$\vec{a} = \vec{g}$$

$$\vec{v} = \vec{g}t + \vec{v}_0$$

$$\vec{r} = \frac{1}{2}\vec{g}t^2 + \vec{v}_0 t + \vec{r}_0$$

Work-energy theorem

Nonrelativistic kinetic energy $T = \frac{1}{2}mv^2 = \frac{1}{2}m\vec{v}\cdot\vec{v}$

$$\frac{dT}{dt} = m\vec{a}\cdot\vec{v} = \vec{F}\cdot\vec{v} = \vec{F}\cdot\frac{d\vec{r}}{dt}$$

$$dT = \vec{F}\cdot d\vec{r} = \sum_A \vec{F}^{(A)}\cdot d\vec{r}$$

Change in kinetic energy = work done on the point mass

= scalar sum of work done by each force acting on it

Analogously

$$\frac{d\vec{p}}{dt} = \vec{F}$$

$$d\vec{p} = \vec{F}\,dt = \sum \vec{F}^{(A)}\,dt$$

change in momentum = impulse = vector sum of impulses imparted to the point mass

Classification of forces

Let's classify forces as

1) constraint
2) conservative
3) neither of these

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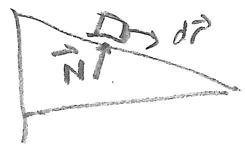
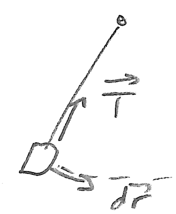
1) Constraint forces act <u>perpendicular</u> to the displacement to constrain the particle to move along a certain path or surface.

eg normal force of a stationary surface



or tension of a constant length string



<u>Constraint forces do no work:</u> $\vec{F}\cdot d\vec{r} = 0$

NB. floor of an elevator or a rope being wound up <u>can</u> do work.

2) Conservative forces (eg electrostatic, gravity)

are those for which

<u>work done is independent of path</u>

Therefore one can define a potential energy <u>function</u> U

$$U(\vec{r}) = U(\vec{r}_0) - \int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{r}$$

Since line integral is indep of path, $U(\vec{r})$ only depends on endpoint position $\vec{r}$.

For $\frac{1}{r^2}$ forces, define $U(\infty) = 0$ so $\vec{F} = \frac{KQq}{r^2}\hat{r} \Rightarrow U = \frac{KQq}{r}$

Infinitesimal displacement $dU = U(\vec{r}+d\vec{r}) - U(\vec{r}) = -\vec{F} \cdot d\vec{r}$

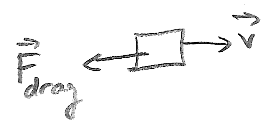
Taylor series $dU = \vec{\nabla} U \cdot d\vec{r}$

So

$$\vec{F} = -\vec{\nabla} U$$

3) Non constraint, non conservative force (eg. viscous or sliding friction)

Force depends on the direction of motion (usually opposing it)



$\therefore$ Work done around a closed path is non zero

$\oint \vec{F} \cdot d\vec{r} \neq 0$ ($\Rightarrow$ non conservative)

$$dT = \vec{F} \cdot d\vec{r} = \underbrace{\sum_{\text{constraint}} \vec{F} \cdot d\vec{r}}_{0} + \underbrace{\sum_{\text{conservative}} \vec{F} \cdot d\vec{r}}_{-dU} + \sum_{\substack{\text{non}\\ \text{cons.}}} \vec{F} \cdot d\vec{r}$$

$$dT + dU = \sum_{\substack{\text{non}\\ \text{cons}}} \vec{F} \cdot d\vec{r}$$

Define mechanical energy $E_{mech} = T + U$

$$dE_{mech} = \sum_{\substack{\text{non}\\ \text{cons}}} \vec{F} \cdot d\vec{r}$$

change in mechanical energy = work done by non conservative forces

(usually positive for driving forces, always negative for viscous forces)

$$\frac{dE_{mech}}{dt} = \sum_{\substack{\text{non}\\ \text{cons}}} \vec{F} \cdot \vec{v} = \text{power delivered by non cons. forces}$$

Corollary: if only constraint or conservative forces act on a point mass, its mechanical energy is conserved, $E_{mech} = \text{const}$

[problems]