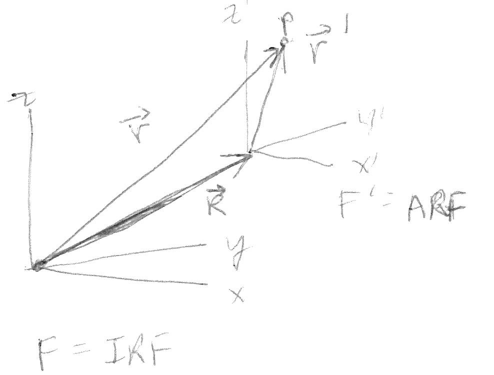
Non inertial (accelerating) reference frames



$$\vec{r}\,' = \vec{r} - \vec{R}$$

$$\vec{v}\,' = \vec{v} - \vec{v}_R$$

$$\vec{a}\,' = \vec{a} - \vec{a}_R$$

F : $\quad m\vec{a} = \vec{F}$

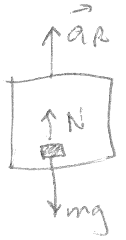
$$m(\vec{a}\,' + \vec{a}_R) = \vec{F}$$

$$m\vec{a}\,' = \vec{F} - m\vec{a}_R$$

$$= \vec{F} + \vec{F}_{pseudo}$$

$$\vec{F}_{pseudo} = -m\vec{a}_R$$

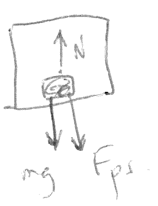
elevator



$$N - mg = ma_R$$

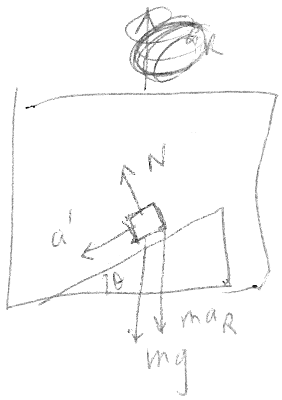
$$N = m(g + a_R)$$

ARF



$$N - mg - ma_R = 0$$

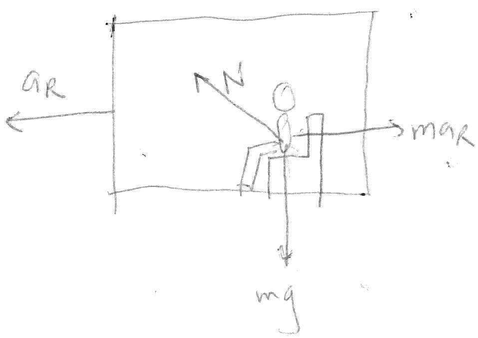
$$N = m(g + a_R)$$



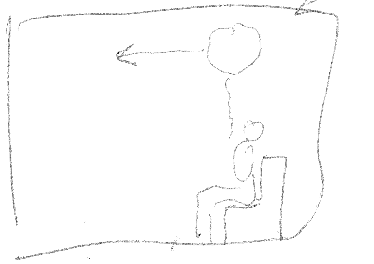
ARF

$$N = m(g + a_R)\cos\theta$$

$$m(g + a_R)\sin\theta = ma'$$

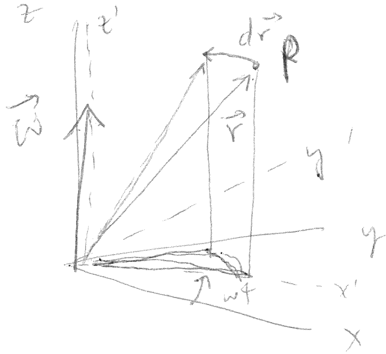


[which way does helium balloon go?]



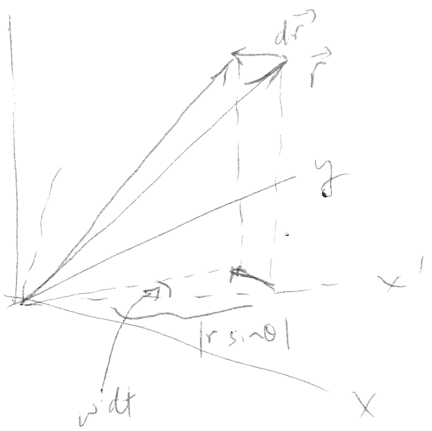
Rotating reference frame

F' rotating about $\hat{z}$ w/ ω wrt F



Let P be stationary in F'

$$|d\vec{r}| = (\omega\, dt)(r \sin\theta)$$



$$d\vec{r} = \vec{\omega}\, dt \times \vec{r}$$

$$\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$

Suppose P is moving in F'. Let $\left(\frac{d\vec{r}}{dt}\right)_{F'}^{0}$ be its velocity; then

$$\left(\frac{d\vec{r}}{dt}\right)_F = \left(\frac{d\vec{r}}{dt}\right)_{F'}^{0} + \vec{\omega} \times \vec{r} \qquad \vec{v} = \vec{v}' + \vec{\omega} \times \vec{r}$$

In general, any vector $\left(\frac{d\vec{A}}{dt}\right)_F = \left(\frac{d\vec{A}}{dt}\right)_{F'}^{0} + \vec{\omega} \times \vec{A}$

$\vec{A} = \vec{v}$

$$\left(\frac{d\vec{v}}{dt}\right)_F = \left(\frac{d\vec{v}}{dt}\right)_{F'} + \vec{\omega}\times\vec{v}$$

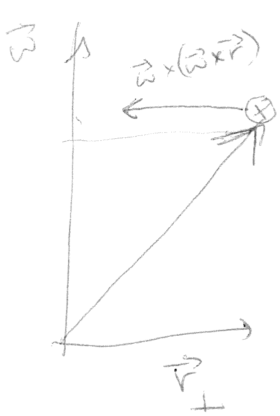
$$= \left(\frac{d}{dt}(\vec{v}' + \vec{\omega}\times\vec{r})\right)_{F'} + \vec{\omega}\times(\vec{v}' + \vec{\omega}\times\vec{r})$$

$$= \left(\frac{d\vec{v}'}{dt}\right)_{F'} + \vec{\omega} * \underbrace{\left(\frac{d\vec{r}}{dt}\right)_{F'}}_{\vec{v}'} + \vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$\vec{F} = m\vec{a} = m\vec{a}' + 2m(\vec{\omega}\times\vec{v}') + m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$m\vec{a}' = \vec{F} - \underbrace{\cancel{2}m\,\vec{\omega}\times(\vec{\omega}\times\vec{r})}_{\text{centrifugal}} - \underbrace{2m(\vec{\omega}\times\vec{v}')}_{\text{coriolis}}$$

$$\vec{\omega}\times(\vec{\omega}\times\vec{r}) = -\underbrace{\left(\omega^2\vec{r} - (\vec{\omega}\cdot\vec{r})\vec{\omega}\right)}_{\omega^2\vec{r}_\perp}$$

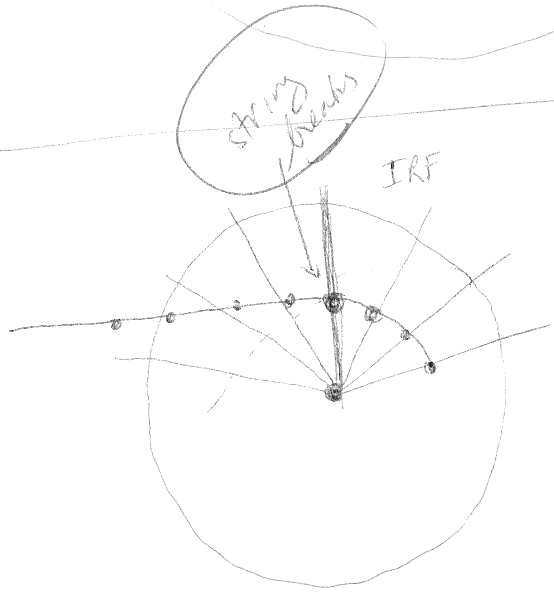


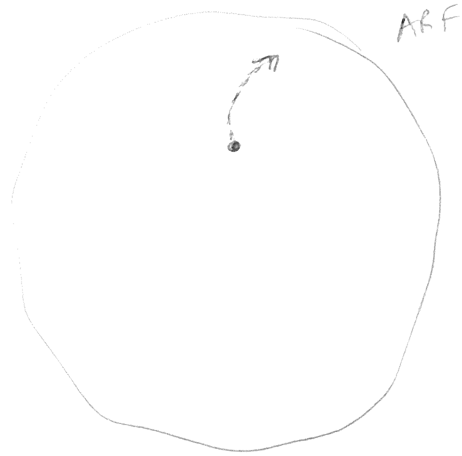
$F_{centrf} = +m\omega^2\vec{r}_\perp$

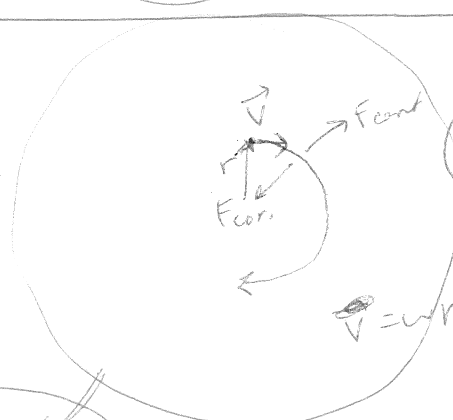
$F_{cor} = -2m(\vec{\omega}\times\vec{v}')$

m5

F_{cor} always at right angles; prop to velocity





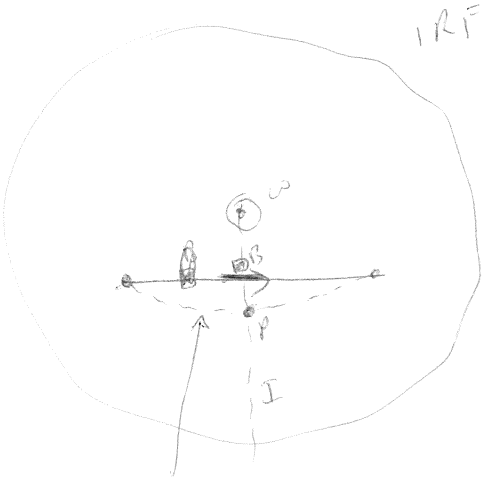


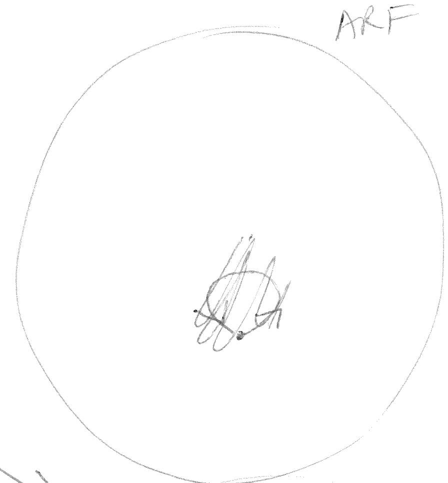
$$F' = ma'$$

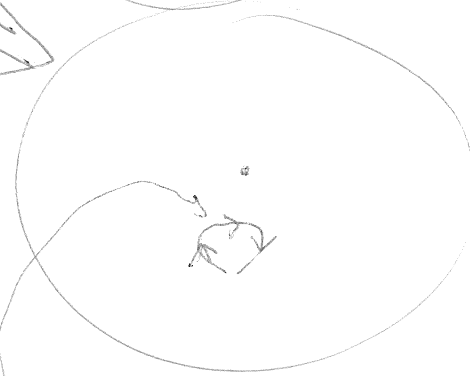
$$\underbrace{F_{cor}}_{-2m\omega^2 r} + \underbrace{F_{cent}}_{+m\omega^2 r} = \underline{-m\omega^2 r}$$

$F_{cent} = m\omega^2 r$ out

$F_{cor} = 2m\omega^2 r$ in







~~[ang speed on curved path is greater so at I, B appears to be moving to the left rel to the rotating frame~~

[At B is actually moving slower than ~~the~~ P, but its angular velocity is larger]

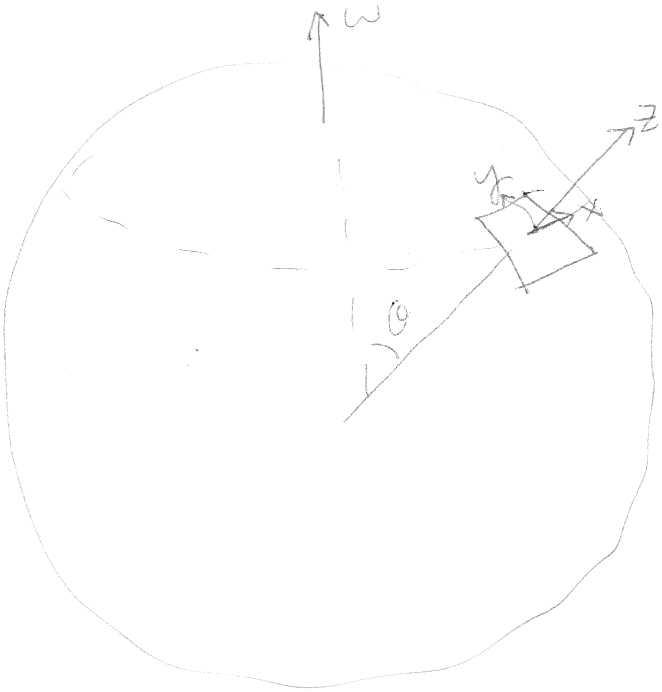
Falling object: fall east ...

(why not west: earth turns beneath it!)

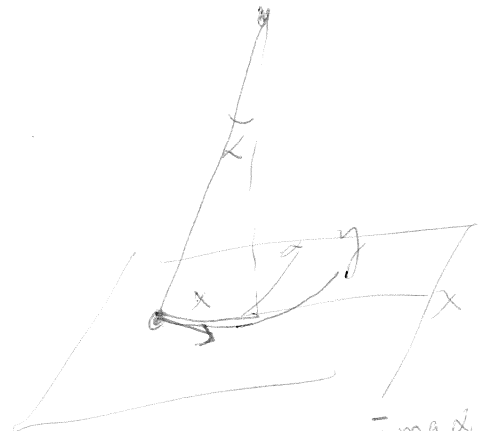
well $\ell = m\omega r^2 = $ const

$r\downarrow$, $\omega\uparrow$, speeds up rel to earth

Foucault pendulum



$\vec{\omega} = (0, \omega\sin\theta, \omega\cos\theta)$



$F = mg\sin\alpha \approx mg\alpha \approx -\frac{mg\,x}{l}$

$\vec{F}_{cor} = -2m(\vec{\omega}\times\vec{v})$

$$= -2m \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \omega\sin\theta & \omega\cos\theta \\ \dot{x} & \dot{y} & 0 \end{vmatrix} = -2m(-\dot{y}\,\omega\cos\theta,\ \dot{x}\,\omega\cos\theta,\ -\omega\dot{x}\sin\theta)$$

$$m\ddot{x} = -\frac{mg}{l}x + 2m\omega\cos\theta\,\dot{y}$$

$$m\ddot{y} = -\frac{mg}{l}y - 2m\omega\cos\theta\,\dot{x}$$

$$\ddot{x} + \alpha^2 x = 2\omega_z \dot{y}$$

$$\ddot{y} + \alpha^2 y = -2\omega_z \dot{x}$$

Trick:

$$\ddot{x} + i\ddot{y} + \alpha^2(x+iy) = 2\omega_z \underbrace{(\dot{y} - i\dot{x})}_{-i\dot{q}}$$

$$q = x + iy$$

$$\ddot{q} + 2i\omega_z \dot{q} + \alpha^2 q = 0$$

$$q = Ae^{i\omega t}$$

$$-\omega^2 - 2\omega_z\omega + \alpha^2 = 0$$

$$\omega = -\omega_z \pm \sqrt{\omega_z^2 + \alpha^2} \approx \pm\alpha - \omega_z, \qquad \omega_z \ll \alpha$$

$$q = Ae^{i(\alpha-\omega_z)t} + Be^{-i\alpha t - i\omega_z t}$$

$$= e^{-i\omega_z t}(Ae^{i\alpha t} + Be^{-i\alpha t})$$

If $\omega_z = 0$, $q = Ae^{i\alpha t} + Be^{-i\alpha t} = \tilde{x} + i\tilde{y}$

$$x + iy = \underbrace{e^{-i\omega_z t}}_{(\cos\omega_z t - i\sin\omega_z t)}(\tilde{x} + i\tilde{y})$$

$$= \underbrace{\tilde{x}\cos\omega t + \tilde{y}\sin\omega t}_{x} + i(\underbrace{\tilde{y}\cos\omega t - \tilde{x}\sin\omega t}_{y})$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{pmatrix}\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}$$ $\Rightarrow$ rotation of motion w/ freq $\omega_z = \omega\cos\theta$

at equator.