- course roster
- office hours
- no text

PHYS 3000 : "the physics may be theorized, but the fun is real" – Sheldon Cooper

- covers several areas of physics:

  mechanics, electrostatics, gravity, thermal flow, fluids

~~formulate~~

- use physical principles to formulate diff eq.

- present a variety of mathematical tools for solving
  - complex variables, normal mode analysis
    Fourier analysis, vector calculus, special fns

- heart of the course is the problems
  6 per week (hand out)
  significant time investment ~~([illegible])~~

  start early; sometimes needs to percolate

  → acknowledge collaborators

- expect you'll work together; eg Gedankenlab, Anak Alcove
  (I expect you will not consult students from previous years.)
- not just solns: full explanation of your assumptions
  + process. ~~Won't need~~ Want to see explicit steps + reasoning
  Don't need mathematica or Wolfram Alpha. All the integrals you can do by hand.
  - Can use mathematica to generate plots
    but must include the commands you used
  - Hand plot on graph paper is acceptable

→ checks on problems: you'll ~~loses~~ more points if error would be caught by check.

- you don't need to rederive results we've derived in class (altho you can if you wish)
- oral presentations?
- late problems?
- ~~office hours~~
- grades ~~([illegible])~~

## Functions

$f(x)$
independent variable
dependent variable

## Power series expansion (a.k.a. Taylor series)

Any function that is sufficiently-well behaved at $x=0$ can be expressed as

$$f(x) = b_0 + b_1 x + b_2 x^2 + \dots = \sum_{n=0}^{\infty} b_n x^n$$

where $b_n = \frac{1}{n!} \left.\frac{d^n f}{dx^n}\right|_{x=0}$

analytic: $f$ and all its derivatives must be finite at $x=0$.

<u>examples</u>

① $f(x) = e^x$

$$\frac{d^n f}{dx^n} = e^x \quad \Rightarrow b_n = \frac{1}{n!}$$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

useful for approximations near $x = 0$:

$$e^{0.1} = 1 + 0.1 + \dots = 1.1$$

can use next term in series to estimate error

$\frac{1}{2}(0.1)^2 = .005$

(exact: 1.10517 .....)

② $f(x) = \cos x$

$$\Rightarrow b_0 = 1, \quad b_1 = 0, \quad b_2 = -\frac{1}{2!}, \quad b_3 = 0$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \dots$$

N.B. $x$ in <u>radians</u>

$1 \text{ rad} = \frac{360^\circ}{2\pi} \approx 57.3^\circ$

③ $f(x) = \sin x$

$$\Rightarrow b_0 = 0, \quad b_1 = 1, \quad b_2 = 0, \quad b_3 = -1$$

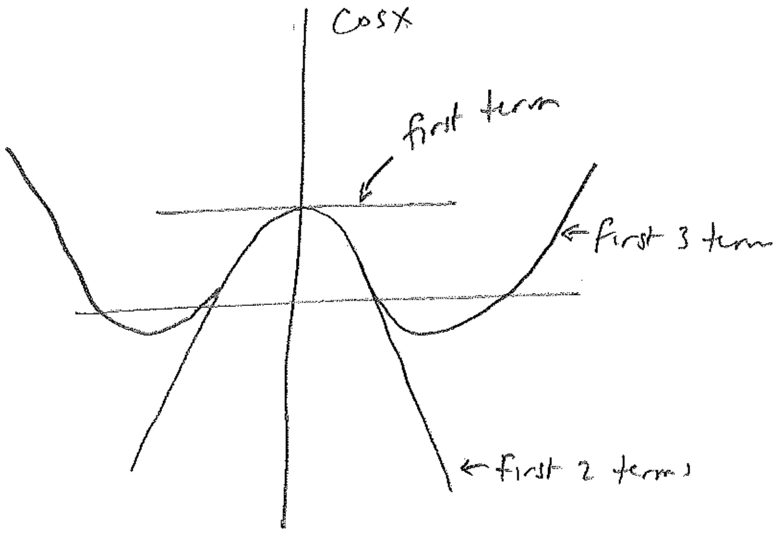
$$\sin x = x - \frac{1}{3!}x^3 + \dots$$

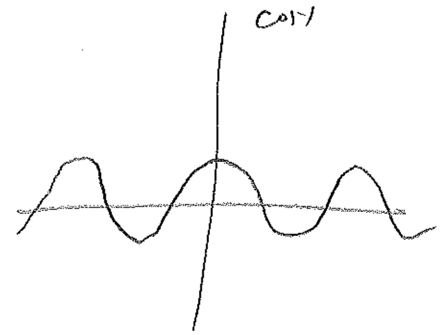
estimate $\sin(10^\circ) = \sin\left(\frac{10}{57.3}\text{rad}\right) \approx \frac{10}{57.3} = 0.1745$

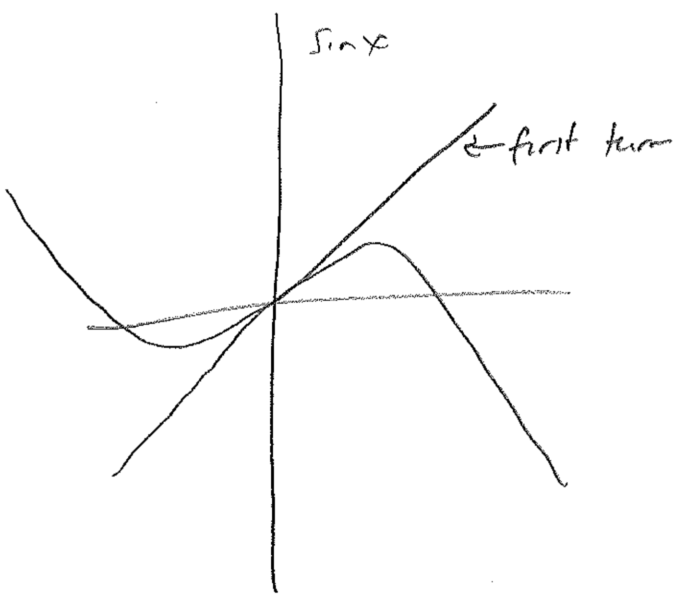
estimate error $\frac{1}{3!}(0.17)^2 \approx 0.001$

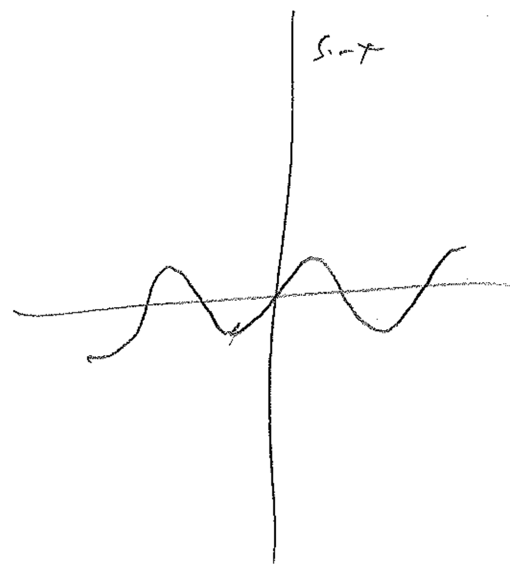
(exact: 0.17365 ...)

graphical approach




cos x is an <u>even</u> function in x

- power series contains only even powers
- symmetric under reflection across y-axis: $f(-x)=f(x)$

sin x is an <u>odd</u> function

- odd powers
- antisymmetric $f(-x) = -f(x)$

Any function can be decomposed into even & odd functions

$$f(x) = f_{even}(x) + f_{odd}(x)$$

$$f_{even}(x) \equiv \frac{1}{2}[f(x) + f(-x)]$$
$$f_{odd}(x) = \frac{1}{2}[f(x) - f(-x)]$$

- $f_{even}$ is even
- $f_{odd}$ is odd
- sum adds up to $f$

$$f(x) = \sum_{n=0}^{\infty} b_n x^n$$

$$\Rightarrow \begin{cases} f_{even} = \sum_{n\ even} b_n x^n \\ f_{odd} = \sum_{n\ odd} b_n x^n \end{cases}$$

Define <u>hyperbolic trig functions</u>

$$\cosh x = (\text{even part of } e^x) \equiv \frac{1}{2}(e^x + e^{-x}) = 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4$$

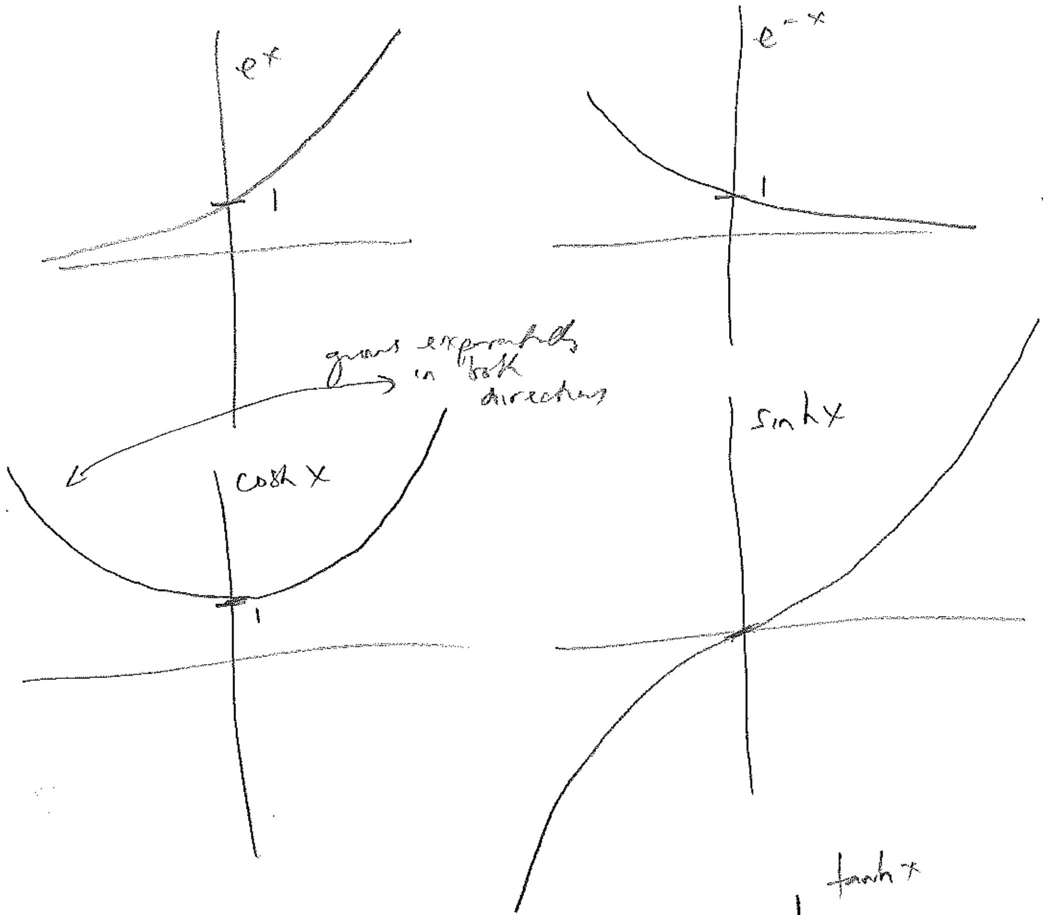
Similar to $\cos x$ except signs all positive

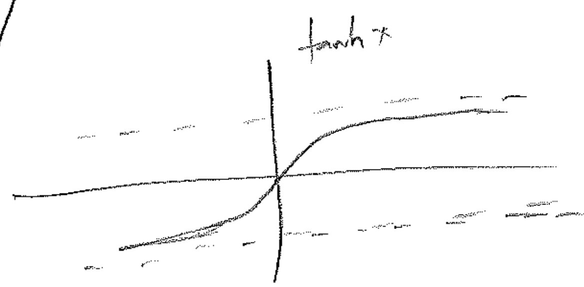
$$\sinh x = (\text{odd part of } e^x) = \frac{1}{2}(e^x - e^{-x}) = x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \ldots$$

Use definitions to show:

$$\begin{cases} \cosh^2 x - \sinh^2 x = 1 \\ \sinh(2x) = 2\sinh x \cosh x \\ \cosh(2x) = \cosh^2 x + \sinh^2 x \\ \frac{d}{dx}\sinh x = \cosh x \\ \frac{d}{dx}\cosh x = \sinh x \end{cases}$$

A5




$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

Consider $(a+b)^{\frac{1}{2}}$

estimate this when $a \gg b$

First, factor out the $a$:

$$a^{\frac{1}{2}}\left(1+\frac{b}{a}\right)^{\frac{1}{2}} \Rightarrow x = \frac{b}{a} \ll 1$$

Next, use power series in $x$ for $(1+x)^{\frac{1}{2}}$ "binomial expansion"

(prob set 1) $\Rightarrow$ $a^{\frac{1}{2}}\left(1+\frac{1}{2}\left(\frac{b}{a}\right)-\frac{1}{8}\left(\frac{b}{a}\right)^2+\dots\right)$

---

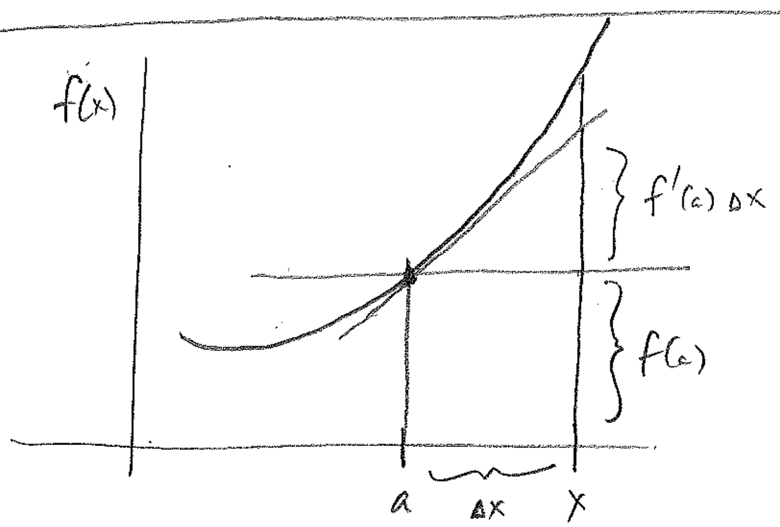
In general, can expand around any point $x=a$ not just $x=0$ (provided $f$ is analytic at $x=a$)

$f(x) = f(a+\Delta x)$ where $\Delta x = x - a$

$$= \sum_{n=0}^{\infty} b_n (\Delta x)^n \quad \text{where} \quad b_n = \frac{1}{n!}\frac{d^n f}{dx^n}\bigg|_{\Delta x = 0,\ \text{i.e. } x=a}$$

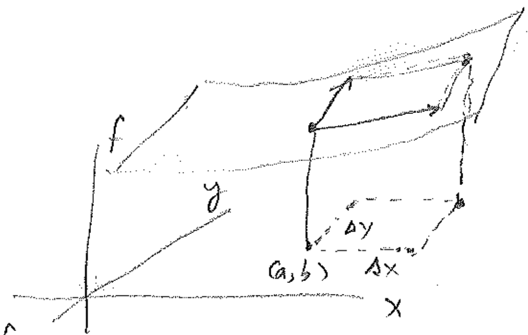
$$= f(a) + \frac{df}{dx}(a)\,\Delta x + \frac{1}{2!}\frac{d^2 f}{dx^2}(a)\,(\Delta x)^2 + \dots$$

---




$f''(a) \approx$ curvature

Omit




can defer until later

## Taylor series in 2 variables

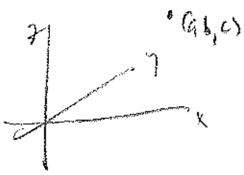
$$f(a+\Delta x, b+\Delta y) = f(a,b) + \frac{\partial f}{\partial x}(a,b)\,\Delta x + \frac{\partial f}{\partial y}(a,b)\,\Delta y$$

$$+ \frac{1}{2!}\left[\frac{\partial^2 f}{\partial x^2}(a,b)\,\Delta x^2 + \frac{\partial^2 f}{\partial y^2}(a,b)\,\Delta y^2 + 2\frac{\partial^2 f}{\partial x \partial y}(a,b)\,\Delta x\,\Delta y\right]$$

$$+ \dots + \frac{1}{n!}\left[\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y}\right]^n f + \dots$$

## Taylor series in 3 variables

[hard to visualize: eg temperature field]




$$f(a+\Delta x, b+\Delta y, c+\Delta z) = f(a,b,c) + \underbrace{\frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z}_{\underbrace{\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)}_{\text{grad } f \,=\, \vec{\nabla} f} \cdot \underbrace{(\Delta x, \Delta y, \Delta z)}_{\Delta\vec{r} \,=\, \text{vector displacement}}} + \dots$$

Physical meaning of gradient?

First order change $\Delta f$ is $\vec{\nabla} f \cdot \Delta\vec{r} = |\vec{\nabla} f|\,|\Delta\vec{r}|\cos\theta$

For a displacement of fixed length $|\Delta\vec{r}|$,
$\Delta f$ is maximized when $\cos\theta = 1$ $\Rightarrow \Delta\vec{r} \parallel \vec{\nabla} f$
$\Rightarrow \vec{\nabla} f$ points in direction of steepest increase (gradient)

Since $\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

we can define $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ = del = nabla

**Table 8.** Circular and hyperbolic functions.

| Circular functions | Hyperbolic functions |
|---|---|
| **DEFINITIONS** | |
| 1. $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$ | 1. $\sinh\theta = \dfrac{e^{\theta} - e^{-\theta}}{2}$ |
| 2. $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ | 2. $\cosh\theta = \dfrac{e^{\theta} + e^{-\theta}}{2}$ |
| 3. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ | 3. $\tanh\theta = \dfrac{\sinh\theta}{\cosh\theta}$ |
| 4. $\sin\theta = \theta - \dfrac{\theta^3}{3!} + \dfrac{\theta^5}{5!} - \dfrac{\theta^7}{7!} + \cdots$ | 4. $\sinh\theta = \theta + \dfrac{\theta^3}{3!} + \dfrac{\theta^5}{5!} + \dfrac{\theta^7}{7!} + \cdots$ |
| 5. $\cos\theta = 1 - \dfrac{\theta^2}{2!} + \dfrac{\theta^4}{4!} - \dfrac{\theta^6}{6!} + \cdots$ | 5. $\cosh\theta = 1 + \dfrac{\theta^2}{2!} + \dfrac{\theta^4}{4!} + \dfrac{\theta^6}{6!} + \cdots$ |
| 6. $\tan\theta = \theta + \dfrac{\theta^3}{3} + \dfrac{2}{15}\theta^5 + \cdots$ | 6. $\tanh\theta = \theta - \dfrac{\theta^3}{3} + \dfrac{2}{15}\theta^5 - \cdots$ |
| **RELATIONS** | |
| 7. $\sin(-\theta) = -\sin\theta$ | 7. $\sinh(-\theta) = -\sinh\theta$ |
| 8. $\cos(-\theta) = \cos\theta$ | 8. $\cosh(-\theta) = \cosh\theta$ |
| 9. $\tan(-\theta) = -\tan\theta$ | 9. $\tanh(-\theta) = -\tanh\theta$ |
| 10. $\boxed{\cos^2\theta + \sin^2\theta = 1}$ | 10. $\boxed{\cosh^2\theta - \sinh^2\theta = 1}$ |
| 11. $\sin(\theta_1 + \theta_2) = \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2$ | 11. $\sinh(\theta_1 + \theta_2) = \sinh\theta_1\cosh\theta_2 + \cosh\theta_1\sinh\theta_2$ |
| 12. $\cos(\theta_1 + \theta_2) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$ | 12. $\cosh(\theta_1 + \theta_2) = \cosh\theta_1\cosh\theta_2 + \sinh\theta_1\sinh\theta_2$ |
| 13. $\tan(\theta_1 + \theta_2) = \dfrac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1\tan\theta_2}$ | 13. $\tanh(\theta_1 + \theta_2) = \dfrac{\tanh\theta_1 + \tanh\theta_2}{1 + \tanh\theta_1\tanh\theta_2}$ |

## POOR MAN'S QUICK RECIPES

**Circular functions**

For small $\theta$: $\sin\theta \approx \theta$, $\tan\theta \approx \theta$

*Example:* $\theta = 0.1$

| | |
|---|---|
| Poor man's recipe | $\sin\theta \approx 0.1$ |
| | $\tan\theta \approx 0.1$ |
| Accurate values | $\sin\theta = 0.0998$ |
| | $\tan\theta = 0.1003$ |

**Hyperbolic functions**

For small $\theta$: $\sinh\theta \approx \theta$, $\tanh\theta \approx \theta$

*Example:* $\theta = 0.1$

| | |
|---|---|
| Poor man's recipe | $\sinh\theta \approx 0.1$ |
| | $\tanh\theta \approx 0.1$ |
| Accurate values | $\sinh\theta = 0.1002$ |
| | $\tanh\theta = 0.0997$ |

For large $\theta$: $\sinh\theta \approx e^{\theta}/2$, $\cosh\theta \approx e^{\theta}/2$

*Example:* $\theta = 3$ $\quad e^{\theta} \approx 20$

| | |
|---|---|
| Poor man's recipe | $\sinh\theta \approx 10$ |
| | $\cosh\theta \approx 10$ |
| Accurate values | $\sinh\theta = 10.018$ |
| | $\cosh\theta = 10.068$ |