$\hat{C}$ = charge conjugation

actually, particle/antiparticle conjugation

$$C|p\rangle = |\bar{p}\rangle$$

$$C^2|p\rangle = C|\bar{p}\rangle = |p\rangle \quad \text{so} \quad C^2 = 1$$

only Eigenstates of C are neutral particles (ie $\ni p = \bar{p}$) $\longrightarrow \gamma, \pi^0, \eta, \eta'$, ρ^0, ω, ϕ

2 possibilities:

$$C|p\rangle = |p\rangle$$

or $C|p\rangle = -|p\rangle$

Are laws of physics invariant under C?

$$\begin{array}{ccc} \text{final state} & \xrightarrow{C} & ? \\ \uparrow H & & \uparrow H \\ \text{initial state} & \xrightarrow{C} & \text{charge conjugated initial state} \end{array}$$

ie is ptcl/antiptcl distinction just a convention?

If so, $[H, C] = 0$, the eigenvalue under C is a multiplicatively conserved quantity

$$C_\gamma = -1$$

If charge is reversed, electric field is reversed

not neutral except for Cartan subalg. $\longrightarrow$ $[\, C_{gluon} = -1 \,]$

n photons : $C = (-1)^n$

C = charge conjugation (pcl $\leftrightarrow$ antipcl conjugation)

$$\hat{C}|\psi\rangle = |\text{anti-}\psi\rangle$$

If a particle is its own antiparticle (i.e. completely neutral) it is an eigenstate of $\hat{C}$

$$\hat{C}|\psi\rangle = c|\psi\rangle$$

Since $\hat{C}^2 = 1 \Rightarrow c = \pm 1$ depending on the particle

$c = -1$ for γ $\left(\hat{C}: q \to -q \Rightarrow \vec{E} \to -\vec{E} \Rightarrow \vec{A} \to -\vec{A}\right)$

C is (multiplicatively) conserved for strong & electromagnetic but not for weak interactions

four fn mesons
$\pi^0 \to \gamma\gamma$
$\eta \to \gamma\gamma$
$\eta' \to \gamma\gamma$

suggests $C = 1$ for pseudoscalar mesons

$\downarrow$

$\pi^0 \to \gamma\gamma\gamma$ prohibited by C $(BR < 3 \times 10^{-8})$ [PDG]
$\eta \to \gamma\gamma\gamma$ "
$\eta' \to \gamma\gamma\gamma$

$$\left.\begin{array}{l} \rho^0 \to \pi^0\gamma \\ \omega \to \pi^0\gamma \end{array}\right\}$$ suggests that $C = -1$ for vector mesons

Hence $\omega \to \eta\pi^0, \pi^0\pi^0\pi^0, \pi^0\gamma\gamma$ not allowed

[aside
$C_{q\bar{q}} = (-1)^{l+s}$]

C only applies to neutral particles
(limited in usefulness)

Isospin conserved by strong interactions.

C takes π^+ to π^-, isospin rotation takes π^- to π^+
so charged mesons are eigenstates of combined
charged conjugation & isospin rotation

This is called "G-parity"

$$G = C_{\substack{\text{neutral}\\ \text{meson}}} (-1)^I \quad \text{where } I = \text{isospin}$$

$\pi^+ \pi^0 \pi^-$ has $G = -1$

$C = 1$, $I = 1$

$\rho^+ \rho^0 \rho^-$ has $G = +1$

$C = -1$, $I = 1$

η, η' have $G = 1$

$C = 1$, $I = 0$

ϕ, ω has $G = -1$

$C = -1$, $I = 0$

Summary

	S	I	P	C	G	
π^0	0	1	-1	1	-1	$\Rightarrow (-1)^{\# \pi}$
η, η'	0	0	-1	1	1	
ρ	1	1	-1	-1	1	$\rightarrow$ (see IPB)
ϕ, ω	1	0	-1	-1	-1	
γ	1		-1	-1		

ρ^0 decays $\Gamma = 150\,\text{MeV} \Rightarrow \tau \sim 10$ sec

100% $\rho^0 \longrightarrow \pi^+\pi^-$
- Parity ok
- G-parity ok

$\rho^0 \not\longrightarrow \pi^0\pi^0$
- Clebsch Gordan coeff vanishes (Isospin)
- Parity ok
- Violates C
- G-parity ok

not listed $\rho^0 \longrightarrow \pi^0\pi^0\pi^0$
- violates C, violates G-parity

$\approx 1 \times 10^{-4}$ $\rho^0 \longrightarrow \pi^+\pi^-\pi^0$
- violates G-parity [but still goes somewhat]

ω decay $\Gamma = 8\,\text{MeV}$

89% $\omega \longrightarrow \pi^+\pi^-\pi^0$
- ok w/ G-parity

$< 3 \times 10^{-4}$ $\omega \longrightarrow \pi^0\pi^0\pi^0$
- G-parity ok
- violates C

1.7% $\omega \longrightarrow \pi^+\pi^-$
- Parity ok
- violates G [still goes, but much less than phase space would expect]

not listed $\omega \longrightarrow \pi^0\pi^0$
- Parity ok
- violates G-parity
- violates C

[why not listed? other reason]

Problem 4.37

don't do in class

η decay $\Gamma = 1.29$ keV ($\Rightarrow$ not strong)

23% $\eta \rightarrow \pi^+\pi^-\pi^0$ • violates G-parity!

33% $\eta \rightarrow \pi^0\pi^0\pi^0$ • violate G-parity } electromagnetic
• does not violate C

$< 3\times10^{-4}$ $\eta \rightarrow \pi^+\pi^-$ • does not violate G-parity
• isospin ok
• violate parity

$< 4\times10^{-4}$ $\eta \rightarrow \pi^0\pi^0$ • G-parity ok
• isospin ok
• charge conjugate ok
• parity violated

so prefers to violate G-parity over parity
but ~~[illegible]~~ w/ small width.

[cf prob. 4.37]

39% $\eta \rightarrow \gamma\gamma$

5% $\eta \rightarrow \pi\pi\gamma$

C only applies to completely neutral particles

Combine C and isospin to define "G-parity"

both conserved by strong interact

$\rightarrow G = C_{neutral}(-1)^I$

Omit

$G \equiv C \cdot$ (rotation about 2 axis in isospin space) $180°$

$= C \cdot e^{i\pi I_2}$

~~If part is neutral~~

$e^{i\pi I_2} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

$G|\pi^0\rangle = C|\pi^0\rangle = |\pi^0\rangle$

$G|\pi^+\rangle = e^{i\pi I_2}|\pi^-\rangle = |\pi^+\rangle$

I_3, I_2, I_1

$I_3 \rightarrow -I_3$

π^- π^0 π^+

Omit

~~$G = C_{neutral}(-1)^I$~~

~~All non-strange~~

All mesons made of u + d qks + antiqks are G-eigenstates

$G_{\pi^+,\pi^0,\pi^-} = -1$ isospin triplet

$G_{\eta,\eta'} = +1$ isospin singlet

$G_{\rho^+,\rho^0,\rho^-} = +1$

$G_{\phi,\omega} = -1$

[Hence η, η', ρ can go to even # π

ϕ, ω can go to odd # π]

$$J_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$J_x^2 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$J_x^3 = J_x$$

$$J_x^4 = J_x^2$$

$$e^{i\pi J_x} = 1 + (i\pi) J_x - \frac{\pi^2}{2} J_x^2 - i\frac{\pi^3}{6} J_x^3$$

$$= 1 + \left[i\pi\,(J) + \frac{(i\pi)^3}{6!} \cdots \right] J_x$$

$$- \frac{\pi^2}{2} J_x^2 + \cdots$$

$$= (\sin\pi) J_x + (\cos\pi) J_x^2 + 1 - J_x^2$$

$$= 1 - 2J_x^2 = 1 - \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} & & -1 \\ & -1 & \\ -1 & & \end{pmatrix}$$

$$(e^{i\pi J_x})\,|\pi_\pm\rangle = -|\pi_\mp\rangle$$

$$(e^{i\pi J_x})\,|\pi_0\rangle = -|\pi_0\rangle$$