

$C$  = Charge conjugation

actually, particle/antiparticle conjugation

$$C|p\rangle = |\bar{p}\rangle$$

$$C^2|p\rangle = C|\bar{p}\rangle = |p\rangle \quad \text{so } C^2 = 1$$

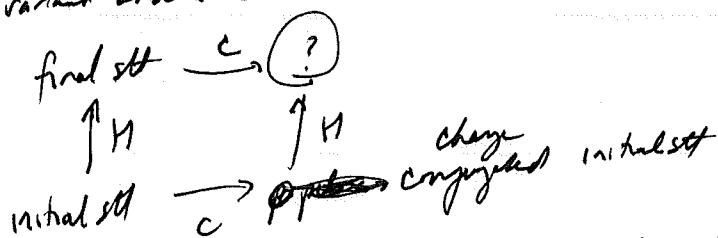
only Eigenstates of  $C$  are neutral particles ( $\text{re } p = \bar{p}$ )  $\rightarrow \gamma, \pi^0, \eta, \eta', \omega, \phi$

2 possibilities:

$$C|p\rangle = |p\rangle$$

$$\text{or } C|p\rangle = -|p\rangle$$

Are laws of physics invariant under  $C$ ?



re is  $p$  or  $\bar{p}$  distinct just a convention?

If  $[H, C] = 0$ , the eigenvalue under  $C$  is a conserved quantity

$$C_\gamma = -1$$

If charge is reversed, electric field is reversed

not neutral except for  $C$  itself.  $\rightarrow [C_{\text{gluon}} = -1]$   
 $n$  photons:  $C = (-1)^n$

$C$  = charge conjugation (  $particle \leftrightarrow antiparticle$  conjugation )

$$\hat{C}|\psi\rangle = |\text{anti-}\psi\rangle$$

If a particle is its own antiparticle (  $\therefore$  completely neutral )  
it is an eigenstate of  $\hat{C}$

$$\hat{C}|\psi\rangle = c|\psi\rangle$$

Since  $\hat{C}^2 = 1 \Rightarrow c = \pm 1$  depends on the particle

$$c = -1 \text{ for } \pi^0 \quad \left( \hat{C}: q \rightarrow -q \Rightarrow \vec{E} \rightarrow -\vec{E} \Rightarrow \vec{A} \rightarrow -\vec{A} \right)$$

$C$  is (multiplicatively) conserved for strong & electromagnetic  
but not for weak interactions

forbidden  
modes

$$\begin{aligned} \pi^0 &\rightarrow \gamma\gamma \\ \eta &\rightarrow \gamma\gamma \\ \eta' &\rightarrow \gamma\gamma \end{aligned}$$

suggests  $C = 1$  for pseudoscalar mesons

$$\begin{aligned} \pi^0 &\rightarrow \gamma\gamma\gamma \\ \eta &\rightarrow \gamma\gamma\gamma \\ \eta' &\rightarrow \gamma\gamma\gamma \end{aligned}$$

prohibited by  $C$  ( $BR < 3 \times 10^{-8}$ )

[PDG]

$$\begin{aligned} \rho^0 &\rightarrow \pi^0 \gamma \\ \omega &\rightarrow \pi^0 \gamma \end{aligned}$$

} suggests that  $C = -1$  for vector mesons

However  $\omega \rightarrow \eta \pi^0, \pi^0 \pi^0 \pi^0, \pi^0 \gamma \gamma$  not allowed

Aside

$$C_{\pi} = (-1)^{L+1}$$

C only applies to neutral particles  
(limited in usefulness)

Isospin conserved by strong interactions.

C takes  $\pi^+$  to  $\pi^-$ , isospin rotation takes  $\pi^-$  to  $\pi^+$   
so charged mesons are eigenstates of combined  
charge conjugation & isospin rotation

This is called "G-parity"

$$G = C_{\text{neutral meson}} (-1)^I \quad \text{where } I = \text{isospin}$$

$$\begin{array}{c} \pi^+ \pi^0 \pi^- \\ \uparrow \\ C=1 \\ I=1 \end{array} \quad \text{has } G = -1$$

$$\begin{array}{c} \rho^+ \rho^0 \rho^- \\ \uparrow \\ C=-1 \\ I=1 \end{array} \quad \text{has } G = +1$$

$$\begin{array}{c} \eta \\ \uparrow \\ C=1 \\ I=0 \end{array} \quad \text{has } G = 1$$

$$\begin{array}{c} \phi \\ \uparrow \\ C=-1 \\ I=0 \end{array} \quad \text{has } G = -1$$

Summary

|                | <u>S</u> | <u>I</u> | <u>P</u> | <u>C</u> | <u>G</u> |                                 |
|----------------|----------|----------|----------|----------|----------|---------------------------------|
| $\pi^0$        | 0        | 1        | -1       | 1        | -1       | $\Rightarrow (-1)^{\# \pi}$     |
| $\eta, \eta'$  | 0        | 0        | -1       | 1        | 1        |                                 |
| $\rho$         | 1        | 1        | -1       | -1       | 1        | $\rightarrow (\text{see } 1PB)$ |
| $\phi, \omega$ | 1        | 0        | -1       | -1       | -1       |                                 |
| $\delta$       | 1        |          | -1       | -1       |          |                                 |

$f^0$  decays

$\Gamma = 150 \text{ MeV} \Rightarrow \tau \sim 10^{-10} \text{ sec}$

100%  $f^0 \rightarrow \pi^+ \pi^-$

- Parity ok
- G-parity ok

$f^0 \not\rightarrow \pi^0 \pi^0$

- Clebsch-Gordan coeff vanishes (1.1.4 p. 14)
- Parity ok
- violates C
- G-parity ok

not listed  $f^0 \rightarrow \pi^0 \pi^+ \pi^-$

- violates C, violates G-parity

$\sim 1 \times 10^{-4}$

$f^0 \rightarrow \pi^+ \pi^- \pi^0$

- violates G-parity [but still goes somewhat]

$\omega$  decay

$\Gamma = 8 \text{ MeV}$

89%

$\omega \rightarrow \pi^+ \pi^- \pi^0$

- ok  $\forall$  G-parity

$< 3 \times 10^{-4}$

$\omega \rightarrow \pi^0 \pi^0 \pi^0$

- G-parity ok
- violates C

1.7%

$\omega \rightarrow \pi^+ \pi^-$

- Parity ok
- violates G

[still goes, but much less than phase space would expect]

not listed

$\omega \rightarrow \pi^0 \pi^0$

- Parity ok
- violates G-parity
- violates C

[why not listed?]  
other reason

Problem 4.37

don't do in class

10/11/11

$\eta$  decay  $m = 1.29 \text{ keV}$  ( $\Rightarrow$  not strong)

23%  $\eta \rightarrow \pi^+ \pi^- \pi^0$  • violates G-parity!  
 33%  $\eta \rightarrow \pi^0 \pi^0 \pi^0$  • violates G-parity  
• does not violate C

} electromagnetic

$< 3 \times 10^{-4}$   $\eta \rightarrow \pi^+ \pi^-$  • does not violate G-parity  
• isospin ok  
• violates parity

$< 4 \times 10^{-4}$   $\eta \rightarrow \pi^0 \pi^0$  • G-parity ok  
• isospin ok  
• charge conjugation ok  
• parity violated

so prefers to violate G-parity over parity  
 but ~~not~~ small width.

[cf prob. 4.37]

39%  $\eta \rightarrow \pi \pi$   
 5%  $\eta \rightarrow \pi \pi \pi$

C only applies to completely neutral particles

Combine C and  $\psi$  to define "G-parity"

both conserved  
by strong interact

$$G = C_{\text{neutral}} (-1)^I$$

180° rotation about 2 axis in isospin space

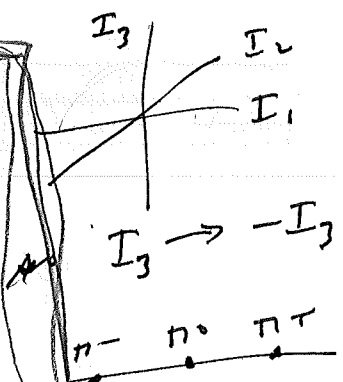
$$G \equiv C \cdot e^{i\pi I_2}$$

~~isospin is conserved by strong interact~~

$$e^{i\pi I_2} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$G |\pi^0\rangle = C |\pi^0\rangle = + |\pi^0\rangle$$

$$G |\pi^+\rangle = e^{i\pi I_2} |\pi^+\rangle = - |\pi^+\rangle$$



~~All mesons made of u + d quarks + antiquarks are G-eigenstates~~

$$G_{\pi^+, \pi^0, \pi^-} = -1$$

$$G_{\rho^+, \rho^0, \rho^-} = +1$$

$$G_{\eta, \eta'} = +1$$

$$G_{\phi, \omega} = -1$$

isospin triplet  
isospin singlet

Hence  $\eta, \eta', \rho$  can go to even #  $\pi$   
 $\phi, \omega$  can go to odd #  $\pi$



$$J_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$J_x^2 = \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$J_x^3 = J_x$$

$$J_x^4 = J_x^2$$

$$e^{inJ_x} = 1 + (in)J_x - \frac{\pi^2}{2} J_x^2 - i\frac{\pi^3}{6} J_x^3$$

$$= 1 + [in(J) + \frac{(in)^3}{6!} \dots] J_x$$

$$- \frac{\pi^2}{2} J_x^2 + \dots$$

$$= (\sin \pi) J_x + (\cos \pi) J_x^2 + 1 - J_x^2$$

$$= 1 - 2J_x^2 = 1 - \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$(e^{inJ_x}) |n_{\pm}\rangle = -|n_{\mp}\rangle$$

$$(e^{inJ_x}) |n_0\rangle = -|n_0\rangle$$