Parity

Laws of physics inv't under rotations $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

R is an orthogonal matrix : $R^T R = \mathbb{1}$ so that $x^2+y^2+z^2$ is invariant.

$O(3)$ = 3×3 orthogonal matrices

$P = \begin{pmatrix} -1 & & \\ & -1 & \\ & & -1 \end{pmatrix}$ is a 3×3 orthogonal matrix that is not a rotation $\left\{ \begin{array}{l} x \to -x \\ y \to -y \\ z \to -z \end{array} \right.$

but rather reflection thru 3 mirrors

or inversion through origin $\vec{r} = -\vec{r}$

"parity operation"

$O(3)$ = rotations + inversions

$SO(3)$ = special orthogonal $\Rightarrow \det R = 1 \Rightarrow$ rotations only

Are laws of physics invariant under $O(3)$ not just $SO(3)$

answer: all but weak interactions, which violate parity

Two types of vectors

① polar vector $\vec{V} \xrightarrow{P} -\vec{V}$

eg $\vec{r} \longrightarrow -\vec{r}$

$\frac{d\vec{r}}{dt} \longrightarrow -\frac{d\vec{r}}{dt}$

$\frac{d^2\vec{r}}{dt^2} \longrightarrow -\frac{d^2\vec{r}}{dt^2}$

$\vec{F} \longrightarrow -\vec{F}$

$\vec{E} \longrightarrow -\vec{E}$

② axial vectors $\vec{V} \xrightarrow{P} +\vec{V}$

eg $\vec{L} = \vec{r} \times \vec{p} \longrightarrow (-\vec{r}) \times (-\vec{p}) = \vec{L}$

$\vec{S} \longrightarrow \vec{S}$

$\vec{\mu} \longrightarrow \vec{\mu}$

$\vec{B} \longrightarrow \vec{B}$ because $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Two types of scalars

① scalar $\phi \to \phi$

② pseudoscalar $\phi \to -\phi$ eg dot product of vector & pseudovector

$\vec{v} \cdot \vec{S} =$ helicity

Invariance of physics under parity $\xrightarrow[QM]{} [H, \Pi] = 0$ $\quad \Pi$ = parity op

$\rightarrow$ eigenstates of H (eg particles) are also eigenstates of Π

Particles have an intrinsic parity $\pi = \pm 1$ (like intrinsic angular mom.)
ie they are eigenstates of the parity operator $\hat{\Pi}|\psi\rangle = \pi|\psi\rangle$, $\pi^2 = 1$.

- photon has $\pi = -1$ $\quad$ (eg $\vec{A} \rightarrow -\vec{A}$ vector potential)

- antiparticle has $\begin{pmatrix}\text{same}\\\text{opposite}\end{pmatrix}$ parity as particle for $\begin{pmatrix}\text{bosons}\\\text{fermions}\end{pmatrix}$ $\quad$ (QFT)

- convention: quark has $\pi = +1$ $\Rightarrow$ antiquark has $\pi = -1$
  (if a pcl [type] is conserved, its intrinsic parity = matter of convention)

- parity is multiplicative so a meson has $\pi = \pi_q \pi_{\bar{q}} = -1$

actually orbital ang mom of a system contributes to parity $(-1)^l$
so meson has $\pi = (-1)^{l+1}$ but usually in ground state $(l=0)$

parity of baryon in ground state $= +1$

Great this explanation
$\left\{ \begin{array}{l} S=0 \text{ mesons are } \underline{\text{pseudoscalar}} \text{ mesons because } \pi = -1 \\ S=1 \text{ mesons are } \underline{\text{vector}} \text{ mesons because } \pi = -1 \\ \qquad\qquad (\text{pseudovector would have } \pi = +1) \end{array} \right\}$

Strong interaction conserve parity (multiplicative conservation law)

[PDG] $\quad \eta(548) \longrightarrow \pi\pi\pi$ $\quad$ BR = 55% (neutral + charged π)
$\Gamma = 1.3\text{ keV} \Rightarrow \tau = \frac{6.6E\text{-}22 \text{ s MeV}}{1.3 \text{ keV}} = 4\times10^{-19}$s (long, because not much phase space)

Why not $\eta \rightarrow \pi^0\pi^0$? $\quad$ BR $< 4\times10^{-4}$

Parity cons. if $(-1) = (-1)(-1)(-1)^l$ but $l=0$ by cons of ang mom
so parity is violated

"θ" "τ"

$K^+ \to \pi^+\pi^0$ 21%

$\pi^+\pi^+\pi^-$ 5.6%

$\pi^+\pi^0\pi^0$ 1.7%

$s=0$ $s=0 \Rightarrow l=0$

τ/θ puzzle : or parity not conserved in weak interactions

1956 Lee, Yang proposed weak interacts does not conserve parity

1956 C.S. Wu $^{60}Co \to$

spin polarized

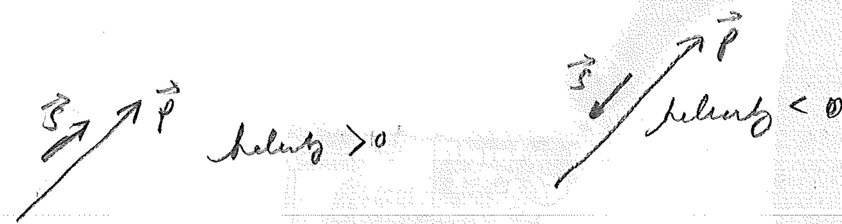
$\vec{S}$

e^- preferentially emitted in direction of spin

violates parity

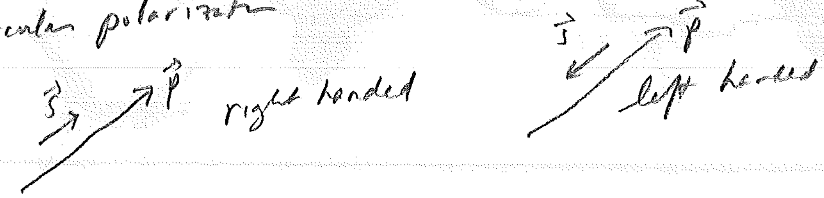
Consider a moving particle w/ spin $\vec{S}$

Its "helicity" is $\vec{S}\cdot\vec{p}$ = spin along direction of motion



Helicity depends on IRF!: not invariant under boosts.
except for massless particles for which helicity is invariant.

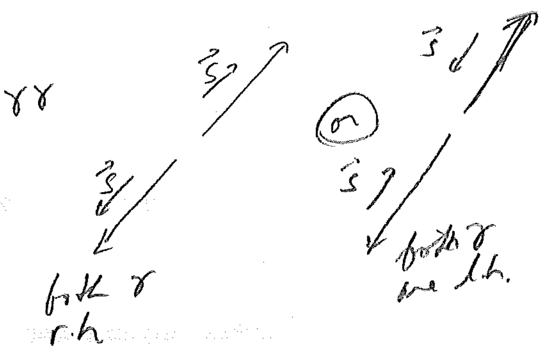
Light: circular polarization



$\vec{S}\cdot\vec{p}$ = pseudoscalar so left ↔ right under parity!

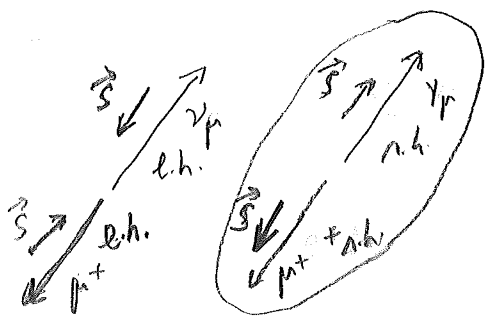
$\pi^0 \to \gamma\gamma$

spin 0



Since EM conserves parity, both are equally likely

$\pi^+ \to \mu^+ \nu_e$

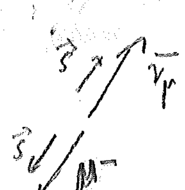


never occurs ⇒ parity violation

Can't measure easily helicity of ν but can measure spin of the other particle.

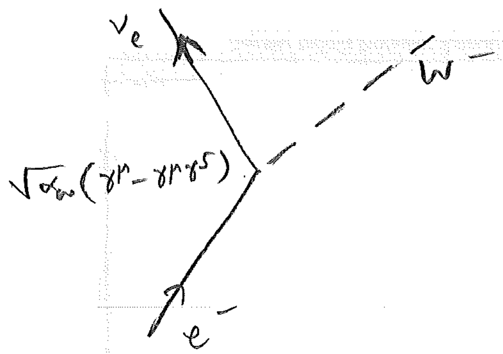
weak interactions only couples to left-handed neutrinos

$\pi^- \to \mu^- \bar{\nu}_e$



weak interactions only couple to right-handed $\bar{\nu}$.

Weak interaction vertex violates parity.



(1) Dirac matrices

γ^μ = 4-vector

γ^5 = pseudoscalar

$\gamma^\mu\gamma^5$ = axial vector

So vertex is of the (V−A) form.

· Violates parity because under parity $(V-A) \to -(V+A)$

When acting on massless fermions $\gamma^5 \nu_L = -\nu_L$ for l.h. particles

$\gamma^5 \nu_R = +\nu_R$ for r.h. particles.

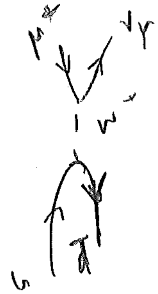
Hence $(\gamma^\mu - \gamma^\mu\gamma^5)\nu_L = 2\gamma^\mu\nu_L$

$(\gamma^\mu - \gamma^\mu\gamma^5)\nu_R = 0$ So vertex vanishes for r.h. neutrinos.

Does this mean that r.h. massless neutrinos don't exist?

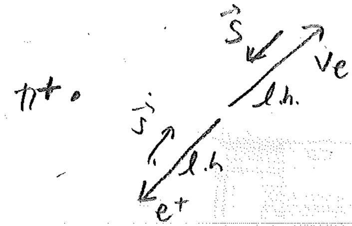
They don't couple to our world (except gravitationally!)

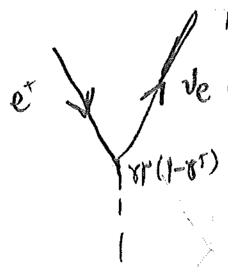
In standard model, neutrinos are only left-handed.



$\pi^+ \to \mu^+ \nu_\mu$ 99.99%

$\pi^+ \to e^+ \nu_e$ 1.23×10^{-4} despite much more phase space





$m_e \ll m_\pi$

Suppose e^+ were massless, then weak interactions would only couple to r.h. e^+ (and l.h e^-) & so this would vanish → forbidden by parity (& angular mom cons)

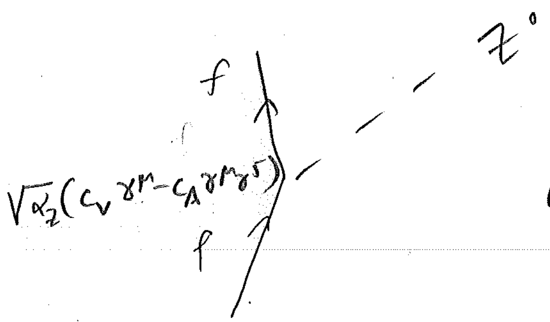
Since $m_e \neq 0$, $\exists$ small coupling to l.h. e^+. [only in massless limit does $\sigma = 1-\gamma_5$ mean left handed]

in fact $$\frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^- \to \mu^+ \nu_\mu)} \sim \left(\frac{m_e}{m_\mu}\right)^2 \frac{1}{(m_\pi^2 - m_\mu^2)^2} \sim 1.28 \times 10^{-4}$$

(Griffiths 10.84)

neutral current vertices also violate parity

$\sin^2\theta_W = 0.2312$



$$\alpha_Z = \frac{\alpha}{\sin^2\theta_W \cos^2\theta_W} \approx \frac{1}{24}$$

f	c_V	c_A
ν_e ν_μ ν_τ	$1/2$	$1/2$
e μ τ	$-1/2 + 2\sin^2\theta_W$	$-1/2$
u c t	$1/2 - 4/3 \sin^2\theta_W$	$1/2$
d s b	$-1/2 + 2/3 \sin^2\theta_W$	$-1/2$

ie $\pm\left(\frac{1}{2} + 2Q\sin^2\theta_W\right)$

~~Is parity con~~

Are the laws of physics unchanged under parity transformation?

~~so~~ If did inversion + ran things forward, + then inverted back, and final state be same? ~~a + f ...~~

$$\begin{array}{ccc} \text{final state} & \xleftarrow{P} & \text{inverted final state} \\ \uparrow H & & \uparrow \text{Hamiltonian (time evolution operator)} \\ \text{initial state} & \xrightarrow{P} & \text{inverted initial state} \end{array}$$

It certainly seems so.

$[H, P] = 0$?

If so, then ~~state~~ eigenstates of energy are also eigenstates of parity

(1) states have a definite parity! → either +1 or −1 (2) And parity is conserved

~~Parity is multiplicative, not additive!~~

e.g. hydrogen atom have functions: $R(r) Y_{lm}(\theta, \phi)$

$l = 0 \Rightarrow$ S-state parity = 1 spherical sym

$l = 1 \Rightarrow$ P-state parity = −1

in general $\underline{\underline{\pi = (-1)^l}}$

→ particles also have intrinsic parity, eg
γ has $\pi = -1$ ("vector meson", $A_\mu \to -A_\mu$)
also $W^\pm, Z^0$

$\langle \psi_f | \vec{p} | \psi_i \rangle$

In ~~atomic~~ electric dipole transitions, parity must change, (eg $3p \to 2p$ not allowed)
"selection rule"
since photon has intrinsic parity $\pi_\gamma = -1$

Frauenfelder + Henley

Laporte 1924: atoms have classes of transitions b/w them

Wigner 1927: selection rules following invc under parity

Selection rules obeyed by electromagnetic + strong interactions (eg nuclear transitions)

G 130n
could let $\vec{E} \to +\vec{E}$
$\vec{B} \to -\vec{B}$
$q \to -q$

~~Parity~~

~~States have a definite parity, either +1 or -1.~~

ie under Parity operation; wave function get multiplied by the constant λ $\quad P|\psi\rangle = \pi|\psi\rangle$

Do it twice get back original so $\pi^2 = 1$ so $\lambda = \pm 1$

If $P|\psi\rangle = +|\psi\rangle$ we say positive parity

If $P|\psi\rangle = -|\psi\rangle$ we say negative parity

- parity is conserved: $P_{init} = P_{final}$

Parity of a system of ptcls is the product of the individual parities ~~times the parity of the relative motion~~

eg

$P|a\rangle = \pi_a|a\rangle$

$P|b\rangle = \pi_b|b\rangle$

$P|ab\rangle = \underline{\pi_a \pi_b}|ab\rangle$

Actually, relative motion of ptcls ina a sys can give a contribution to parity too.

~~Parity~~ Parity is a multiplicative not additive, conserv law. ~~System of particles has parity = $\Pi \Pi$~~ system 2 pcl A+B has parity $\Pi_A \Pi_B (-1)^l$

If a particle is conserved, its intrinsic parity is a matter of convention. [photon can be destroyed, so not convention]

$A B \to C D$

$\Pi_A \Pi_B (-1)^l = \Pi_C \Pi_D (-1)^{l'}$

eg quark: define $\Pi_q = +1$

[QFT: antipcls have same intrinsic parity as pcls for bosons ($\gamma = \bar{\gamma}$)
~~antipcls~~ antipcls opposite " " " " fermions

$\Rightarrow \Pi_{\bar{q}} = -1$

Parity of composite pcls = product of parities of constituents (for ground state, ie $l = 0$)

$\Pi_{\text{Baryon}} = (1)^3 = 1$

$\Pi_{\text{mesons}} = (1)(-1) = -1$ in general: $(-1)^{l+1}$

"pseudoscalar mesons" (vector mesons: change sign under parity)

$\eta \to \pi^+\pi^-\pi^0$ 23%
$\eta \to 3\pi^0$ 33%
$\eta \to 2\gamma$ 39%

don't talk about unless necessary

$\eta \to \gamma\gamma$
$\pi^0 \to \gamma\gamma$

(-1) $(-1),(-1) \Rightarrow l = 1$

$S = 0$ $J = 1 + 1 + L$

$\Delta^{++} \to p\pi^0$

$J = \frac{3}{2}$ $J = \frac{1}{2} \Rightarrow L = 1, 2$

$(+1)$ $(+1)(-1)(-1)^l \Rightarrow L = 1$

$\eta \to \pi\pi$

$S = 0$ $S = 0 \Rightarrow L = 0$

(-1) $(-1)(-1)(-1)^{L=0} \Rightarrow$ violates parity $\Rightarrow$ not seen: $BR < 3 \times 10^{-4}$

$\eta \to \pi\pi\pi$ parity conserved

$\rho \to \pi\pi$

$J = 1$ $S = 0, L = 1$

(-1) $(-1)(+1)(-1)$

FtM

$l = 0$
but total $J = 1$

isospin 1 (sym), $l = 1 \Rightarrow$ spin $= 1$ (sym)
If ~~spin = 0, then l = 0~~ $\Rightarrow$ parity $= -1$

$d\pi^- \to nn$ $\quad \Pi_{\pi_0} = -1$

-1 $\quad d\pi^- \to nn\gamma$

$d\pi^- \not\to nn\pi^0$ $\quad l = 1,\ s = 0$ or $s = 1,\ l = 0$

a rather involved explanation

~~Nnnnnde~~ I hated them

$d = pn$ w/ spin 1, $l = 0 \Rightarrow \Pi_d = \Pi_p \Pi_n = 1$

iso spin 0, Pauli

$\Pi(p, n, d) = 1$, $\Pi(^3H, ^3He, ^4He) = 1$

~~$\Pi(^3H) = +1$~~
~~$\Pi(^3He) = +1$~~