[only did this 2005] → and partially in 2013

HM-1

28

1

~~Field~~

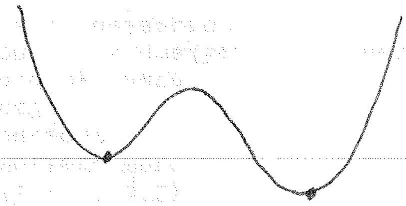
Lagrangian $L = K - V$ ⟹ euler lagrange eqns

$(\nabla\phi)^2 + m^2\phi^2$

⇓

pcls of
$E^2 = p^2 + m^2$

$V(x)$

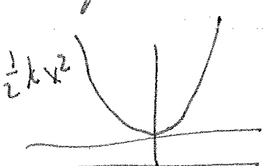


local minima of $V(x)$ → stable equilibria

system does small oscillations about equilibrium

→ quantize $E = (n + \frac{1}{2})\hbar\omega$

$V = \frac{1}{2}kx^2$



For unit mass $\omega = \sqrt{\frac{k}{m}}$ ⟹ $\boxed{E = (n+\frac{1}{2})\sqrt{k}}$

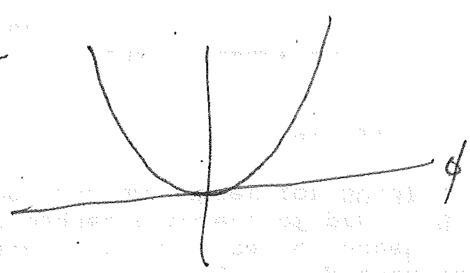
Set $\hbar = 1$

Field theory

field $\phi(\vec{r}, t)$ ⟹ spin 0 particle

Lagrange density $\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)}_{\text{kinetic + gradient energy}} - V(\phi)$

let $V(\phi) = \frac{1}{2}m^2\phi^2$

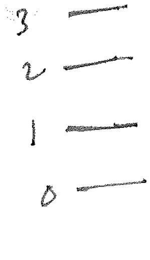


field for small oscillations about minimum of $V(\phi)$

$E = (n + \cancel{\frac{1}{2}})m$

ignore

$E = nm$



So energy levels are reinterpreted as states of n particles of mass m

ground state ⟹ no particles

1st exc. ⟹ 1 particle

2nd exc. ⟹ 2 particles } bosons

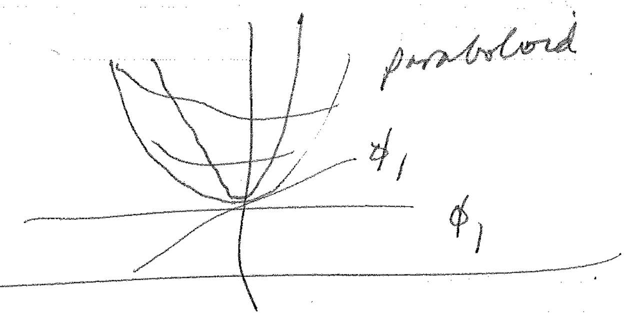
HM 2

2

2. Scalar fields: $\phi_1, \phi_2$

$$\mathcal{L} = \frac{1}{2}(\partial\phi_1)^2 + \frac{1}{2}(\partial\phi_2)^2 - \underbrace{\frac{1}{2}m^2(\phi_1^2 + \phi_2^2)}_{V(\phi_1,\phi_2)}$$

O(2) Symmetry $\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \longrightarrow \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$ $\Rightarrow V(\phi_1,\phi_2)$ unchanged




Let $\phi = (\phi_1 + i\phi_2)$

$\phi^* = (\phi_1 - i\phi_2)$

~~Then O(2) symmetry ⇒ U(1) symmetry~~ ~~$\phi \to e^{i\theta}\phi$~~

$$\mathcal{L} = \frac{1}{2}(\partial\phi)(\partial\phi)^* - \frac{1}{2}m^2\phi^*\phi$$

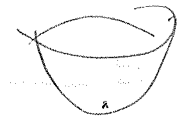
O(2) symmetry ⇒ U(1) symmetry $\quad \phi \to e^{i\theta}\phi$

→ global symmetry because $\theta$ is indep of $x$...

~~This is how we describe charged particles.~~

same rotat at each pt in space + time

HM (6)



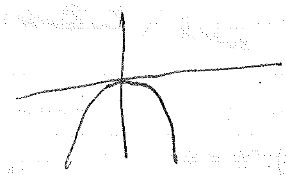
$V(\phi) = \frac{1}{2} m^2 \phi^2$

ground state of theory ("vacuum") $\Rightarrow$ minimum of $V(\phi)$ $\Rightarrow$ $\phi = 0$

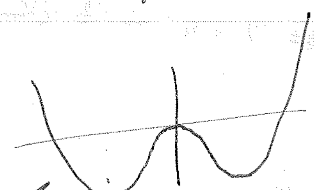
The point $\phi = 0$ is invariant under $U(1)$ symmetry

What if $\phi = 0$ not gnd state? ~~e.g.~~ i.e. local maximum rather than minimum

$V(\phi) = -\frac{1}{2} m^2 \phi^2$



add $\phi^4$ term



$\cancel{V(\phi) = \frac{m^2}{4v^2}\phi^4 - \frac{1}{2} m^2 \phi^2}$

$\cancel{= \frac{m^2}{4v^2}(\phi^2 - v^2)^2 - \frac{m^2 v^2}{4}}$

$\cancel{= \frac{m^2}{4v^2}(\phi + v)^2(\phi - v)^2 \approx m^2(\phi - v)^2}$

$V(\phi) = \frac{1}{4}\lambda^2 \phi^4 - \frac{1}{2}\mu^2 \phi^2 = \frac{1}{4}\lambda^2\left(\phi^2 - \frac{\mu^2}{\lambda^2}\right)^2 - \frac{\mu^4}{4\lambda^2}$

expand near minimum: $\phi = \frac{\mu}{\lambda} + \eta$

$V = \frac{1}{4}\lambda^2\left(\phi + \frac{\mu}{\lambda}\right)^2\left(\phi - \frac{\mu}{\lambda}\right)^2 - \frac{\mu^4}{4\lambda^2}$

$= \frac{1}{4}\lambda^2\left(\frac{2\mu}{\lambda} + \eta\right)^2 \eta^2$

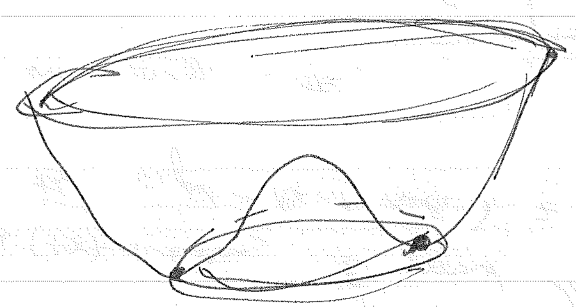
$= \underline{\mu^2 \eta^2} + \underline{\mu\lambda\eta^3} + \underline{\frac{1}{4}\lambda^2\eta^4}$

$\frac{1}{2}\underbrace{(2\mu^2)}_{M_H^2}\eta^2$

$m_H = \sqrt{2}\mu$

$$V(\phi) = \lambda |\phi|^4 - \frac{1}{2} m^2 |\phi|^2$$

mexican hat



minima at $|\phi| = v e^{i\lambda}$

once minimum is chosen system no longer has U(1) symmetry.

Important

Lagrang has U(1) symmetry but ground state of theory breaks the symmetry

→ Spontaneous symmetry breaking

$$\mathcal{L} = \cancel{\frac{1}{2}(\partial_\mu\phi + iA_\mu\phi)(\partial_\mu\phi^* - iA_\mu\phi^*) - \frac{1}{4}\lambda^2\phi^*\phi + \frac{1}{2}m^2\phi^*\phi}$$

$$\cancel{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

$$\mathcal{L} = \frac{1}{2}(\partial\phi_1)^2 + \frac{1}{2}(\partial\phi_2)^2 - \frac{1}{4}\lambda^2\left(\phi_1^2 + \phi_2^2 - \frac{\mu^2}{\lambda^2}\right)^2$$

SB: $\phi_1 = \frac{\mu}{\lambda} + \eta$

$\phi_2 = \xi$

$$\frac{1}{2}(\partial\eta)^2 + \frac{1}{2}(\partial\xi)^2 - \frac{1}{4}\lambda^2\Big(\underbrace{\left[\frac{\mu}{\lambda}+\eta\right]^2 + \xi^2 - \frac{\mu^2}{\lambda^2}}_{}\Big)^2$$

$$\underbrace{\left(\frac{2\mu}{\lambda}\eta + \eta^2 + \xi^2\right)^2}_{}$$

$$-\mu^2\eta^2 + (\eta^3, \eta^4, \eta\xi^2, \eta^2\xi^2, \xi^4 \text{ terms})$$

$\xi$ is massless

$\eta$ is massive

HM 3

(3)

vector field $A_\mu$ $\longrightarrow$ spin 1 particle (photons, W, Z)

kinetic + gradient

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \text{field strength} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Theory has local U(1) gauge invariance: $A_\mu \longrightarrow A_\mu - \partial_\mu \theta$.

$\theta$ depends on x, t. $\Rightarrow$ "local" symmetry.

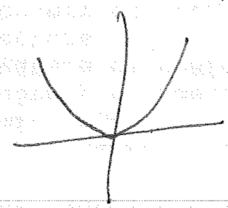
i.e. do a diff rotation at each pt in space/time

$$F_{\mu\nu} \longrightarrow \partial_\mu(A_\nu - \partial_\nu \theta) - \partial_\nu(A_\mu - \partial_\mu \theta)$$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu - \partial_\mu \partial_\nu \theta + \partial_\nu \partial_\mu \theta$$

$$= F_{\mu\nu}$$

what about a potential energy for $A_\mu$?



$$\frac{1}{2} m^2 A_\mu A^\mu$$

under local $U(1)$: $\frac{1}{2} m^2 (A_\mu - \partial_\mu \theta)(A^\mu - \partial^\mu \theta)$

$$\neq \frac{1}{2} m^2 A_\mu A^\mu$$

not gauge invariant

$\Rightarrow$ unbroken $U(1)$ ga inv $\Rightarrow$ photon is massless

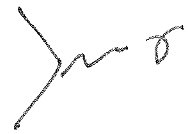
also $W^\pm$ $Z^0$

Yet $W^+, W^-, Z$ are massive!

HM 4 ~~8~~ 9

$V(\phi)$ invariant under $\phi \to \phi\, e^{i\theta}$

~~charged spin 0 particle also~~



a spin 0 part w/ chg couples to photon
& ∴ also has U(1) local gauge invariance

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^*(\partial\phi) \cancel{- \frac{1}{2} m^2 \phi^*\phi}$$

$$\phi \to e^{i\theta(x,t)}\phi \quad \Rightarrow \quad \phi^*\phi \text{ unchgd}$$

but $$\partial_\mu\phi \Longrightarrow \partial_\mu(e^{i\theta}\phi) = e^{i\theta}\partial_\mu\phi + i(\partial_\mu\theta)e^{i\theta}\phi$$

$|\partial\phi|^2$ is <u>not</u> invariant because of

we have not included coupling to photon

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi + iA_\mu\phi)(\partial_\mu\phi^* - iA_\mu\phi^*) + \frac{1}{2}m^2\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

The all 3 terms have local gauge invc.

$$\partial_\mu\phi + iA_\mu\phi \longrightarrow \partial_\mu(e^{i\theta}\phi) + i(A_\mu - \partial_\mu\theta)\phi$$

$$= e^{i\theta}\partial_\mu\phi + \underline{i\partial_\mu\theta\, e^{i\theta}\phi} + iA_\mu\phi e^{i\theta} - \underline{i\partial_\mu\theta\, e^{i\theta}\phi}$$

$$= e^{i\theta}(\partial_\mu\phi + iA_\mu\phi)$$

$\partial_\mu\phi + iA_\mu\phi \equiv D_\mu\phi$ = gauge covariant derivative

$$\mathcal{L} = \frac{1}{2}(D_\mu\phi)(D_\mu\phi)^* + \frac{1}{2}m^2|\phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

HM 9

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$-\frac{1}{4}\lambda^2\left( \left[\frac{\mu}{\lambda}+\eta\right]^2+\xi^2-\frac{\mu^2}{\lambda^2}\right)^2$$
$$\frac{1}{2}\left(\partial\eta+i\partial\xi+iA_\mu\frac{\mu}{\lambda}\right)\left(\partial\eta-i\partial\xi-iA_\mu\frac{\mu}{\lambda}\right)$$

$$=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}\lambda^2\left(\frac{2\mu\eta}{\lambda}+\ldots\right)^2$$
$$+\frac{1}{2}(\partial\eta)^2+\underbrace{\frac{1}{2}(\partial\xi)^2+\frac{1}{2}A_\mu A^\mu\left(\frac{\mu}{\lambda}\right)^2+\left(\frac{\mu}{\lambda}\right)A_\mu(\partial^\mu\xi)}$$

$$\frac{1}{2}\left(\partial_\mu\xi+\frac{\mu}{\lambda}A_\mu\right)^2$$

let $A_\mu \to A'_\mu - \frac{\lambda}{\mu}(\partial_\mu\xi)$

$$=\underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}}\underbrace{\left(\frac{\mu}{\lambda}\right)^2}_{M_{\text{gauge boson}}}A_\mu A^\mu+\underbrace{\frac{1}{2}(\partial\eta)^2-\frac{1}{2}\underbrace{(2\mu^2)}_{M^2_{\text{Higgs}}}\eta^2}_{\text{Higgs particle}}$$

1 massive gauge boson

$$\frac{M_{\text{Higgs}}}{M_{\text{gauge boson}}}=\frac{\sqrt{2}\,\mu}{\left(\frac{\mu}{\lambda}\right)}=\sqrt{2}\,\lambda$$

undetermined

gauge boson eats the goldstone boson & becomes massive

goldstone boson becomes longitudinal component of $W^\pm$