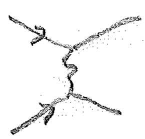
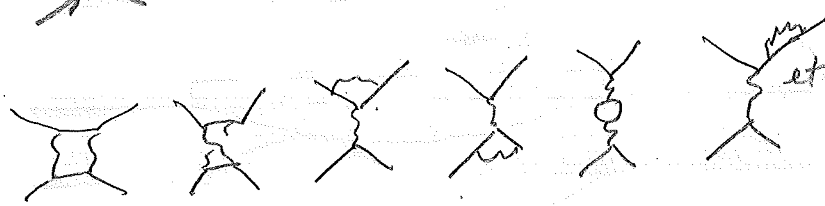
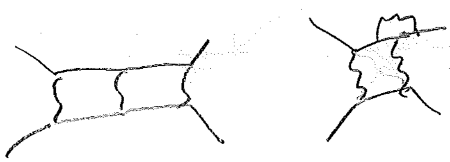
Higher order corrections to QED amplitudes.

$e^-\mu^- \to e^-\mu^-$

$$\mathcal{M} = \left(\begin{array}{c}\text{diagrams w/}\\ \text{2 vertices}\end{array}\right) + \left(\begin{array}{c}\text{diagrams w/}\\ \text{4 vertices}\end{array}\right) + \left(\begin{array}{c}\text{diagrams w/}\\ \text{six vertices}\end{array}\right) + \dots$$

2 vertices  (tree) $\mathcal{M} \sim \alpha$

4 vertices  etc (1 loop) $\mathcal{M} \sim \alpha^2$

6 vertices  etc. $\mathcal{M} \sim \alpha^3$ (2 loops)

Since $\alpha \ll 1$, diagrams w/ more vertices give smaller contributions to the amplitude $\Rightarrow$ perturbative series

Unfortunately loop amplitudes generally diverge ($\to \infty$) and need to be "renormalized"

~~QED~~ 29

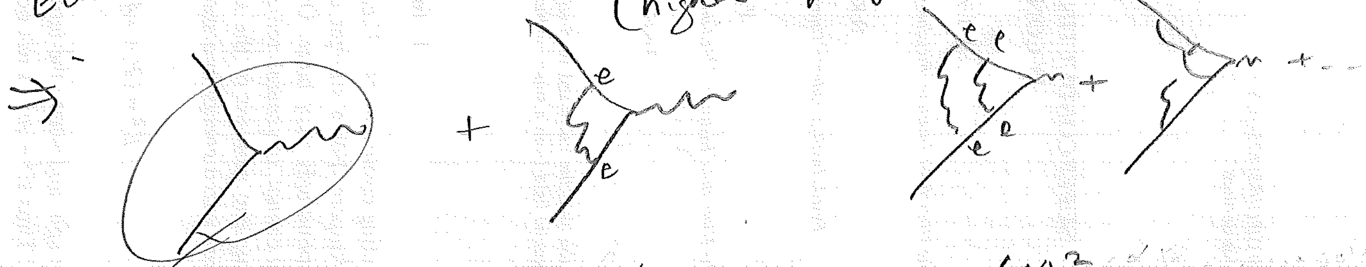
REDU

Magnetic moment of electron $\mu = \frac{g}{2}\mu_B$ $\mu_B = \frac{e\hbar}{2m_e}$ RC

Dirac eqn predicts $g=2$.
QED gives corrections.

Electron interacting with external magnetic field
(higher loop dgms contribute to anomalous magnetic moment)

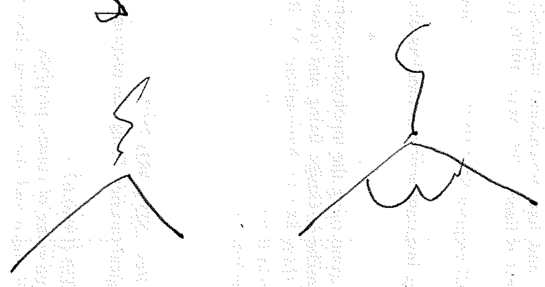
$\Rightarrow$



$$g = 2\left(1 + \frac{\alpha}{2\pi} - 0.328\left(\frac{\alpha}{\pi}\right)^2 + \ldots\right)$$

$\frac{\alpha}{2\pi}$: 0.0011614 $0.328\left(\frac{\alpha}{\pi}\right)^2$: 0.00000177

$\underbrace{\quad\quad\quad}_{1.001159652181}$ $\leftarrow$ need α^4 corrections to get 12 sig figs



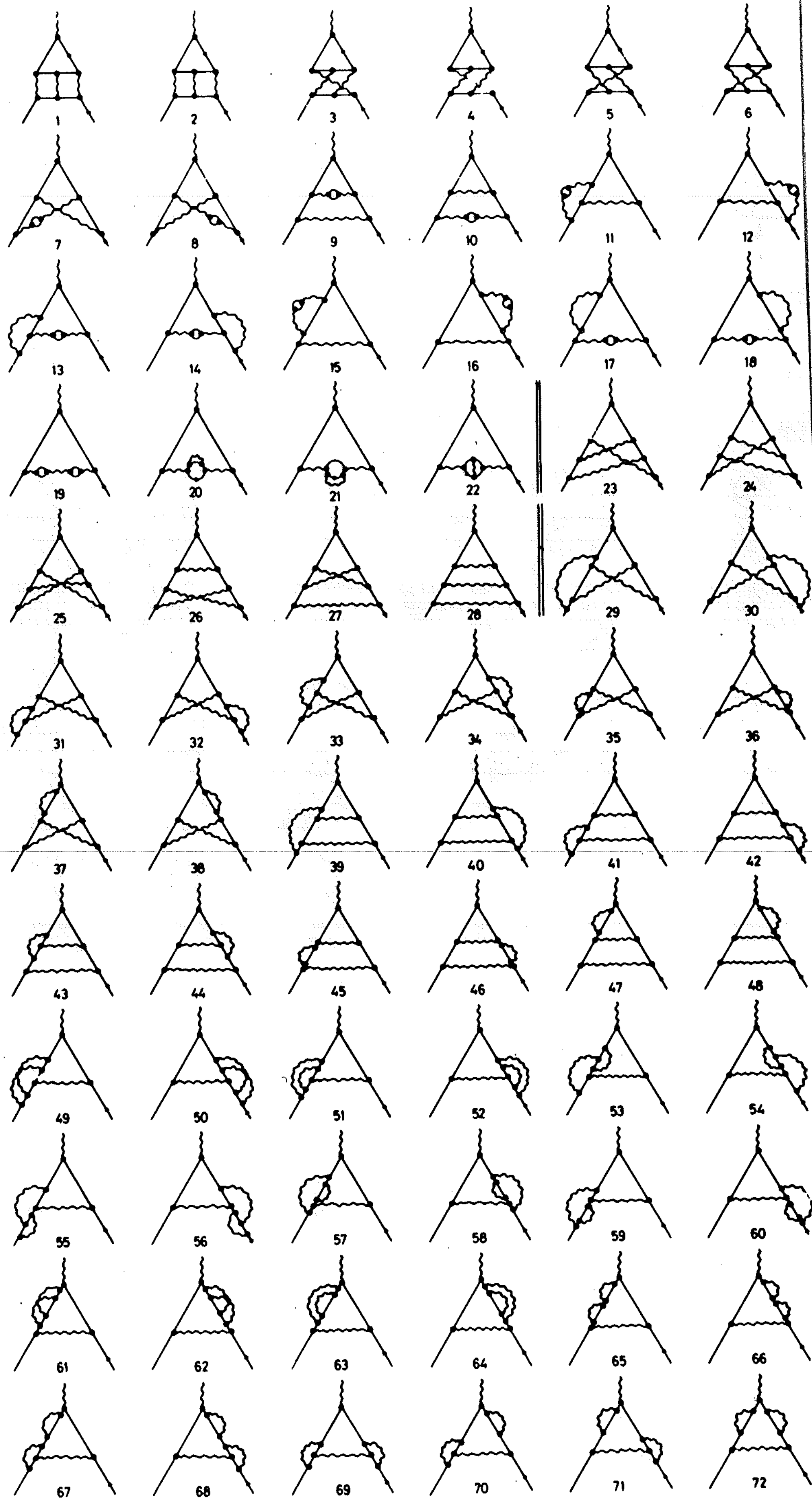


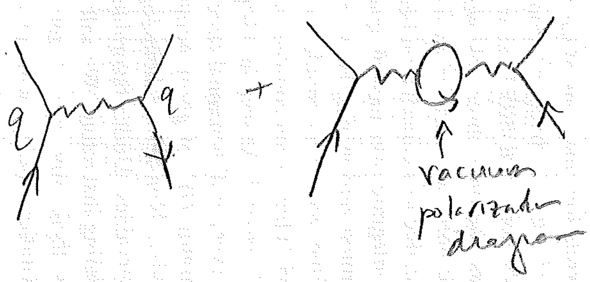
Fig. 8.2 The Feynman graphs which have to be evaluated in computing the α^3 corrections the lepton magnetic moments (after Lautrup *et al.* 1972).

QED-7/10

RC

Running of the coupling constant

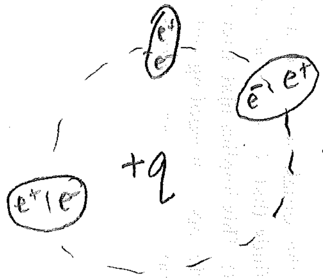
strength of the electromagnetic coupling $\alpha = \frac{Ke^2}{\hbar c}$ is not actually constant but increases logarithmically w/ energy (not exponentially!) due to screening



$q \qquad e^- \; e^+ \qquad -q$

virtual pair acts as a dipole which is polarized by the charges.

$\Rightarrow$ vacuum acts as a dielectric, reducing the electric field & $\therefore$ the force between the charges. The vacuum partially screens the charges from each other



$\leftarrow$ effective charge $q_{eff} < q$ (reduced by negative charge inside the gaussian surface)

$$E = \frac{K q_{eff}}{r^2} = \frac{K q(r)}{r^2}$$

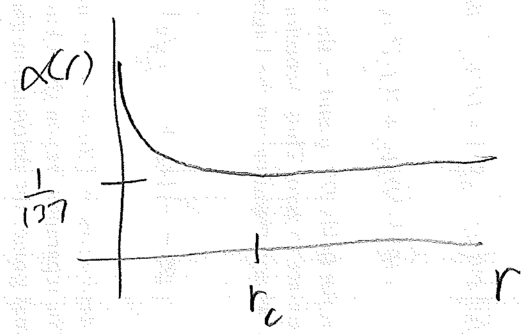
Screening stops for $r \gtrsim \frac{\hbar}{m_e c}$ = Compton wavelength

$$\alpha = \frac{K e_{eff}^2}{\hbar c} = \frac{1}{137}$$

The charges we observe are the effective charge at $r \sim r_c$.

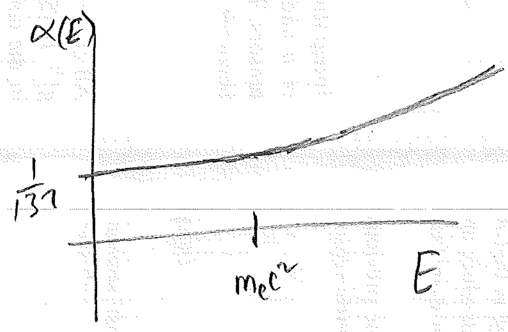
The charge for $r < r_c$ is actually larger

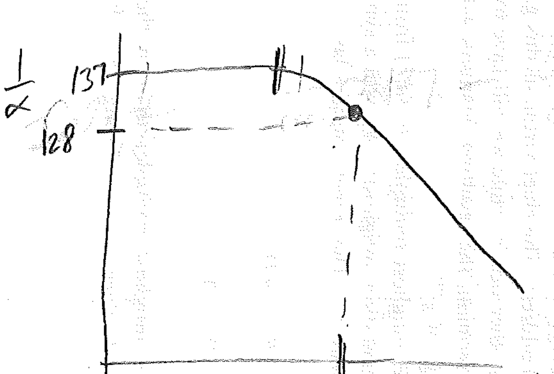
$$\alpha(r) = \frac{K e^2(r)}{\hbar c}$$



higher energy particles have shorter de Broglie wavelengths & so probe shorter scale

$$E \sim \frac{\hbar c}{r}$$





$$\frac{1}{\alpha} \sim -\log E$$

[try to scale this]

RC

$$e^2 = \frac{e_c^2}{1 + \frac{e_c^2}{3\pi} \log\left(\frac{\Lambda^2}{m^2}\right)}$$

BLP QFD
133.4

Halzen Martin
7.57

$$\frac{1}{e_m^2} = \frac{1}{e_\Lambda^2} + \frac{1}{3\pi} \log\left(\frac{\Lambda^2}{m^2}\right)$$

2.5

$$\frac{1}{\alpha(Q)} = \frac{1}{\alpha(\mu)} - \frac{n}{3\pi} \log\left(\frac{Q^2}{\mu^2}\right)$$

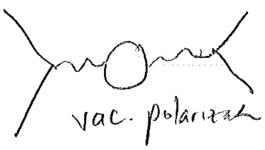
$137 = 128$

$85 = \frac{Q}{\mu} = 2E18$

n d

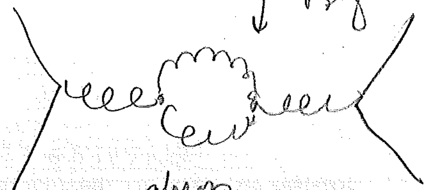
QCD-7

[we're saved by one thing] [Griffiths 2.3]



⇒ cause α to increase as $E \uparrow$

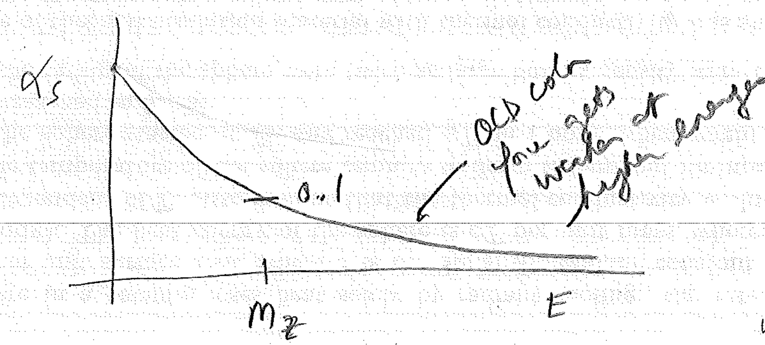
opposite sign!



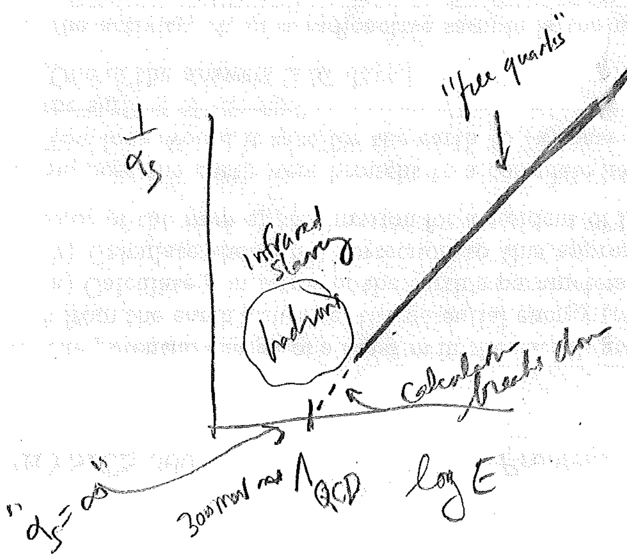
⇒ causes α_s to decrease as $E \uparrow$

"anti-screening" ⇒ strong force becomes weak at short distances

when quarks are really near they do not have much effect on each other



$\frac{1}{\alpha_s} \sim \log E$ ⇒ asymptotic freedom

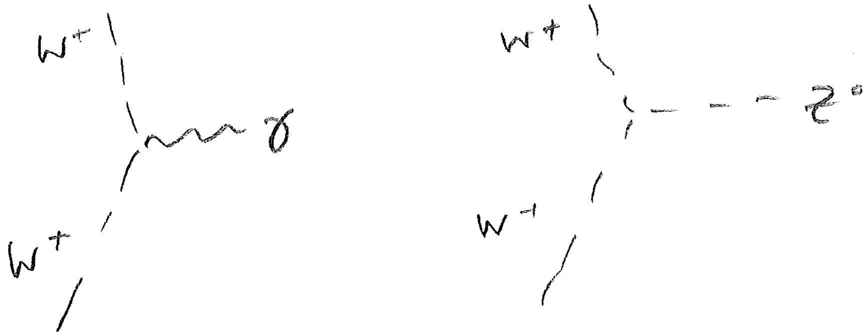


perturbative calculations become reasonable at high energies (& in fact are why we believe in QCD)

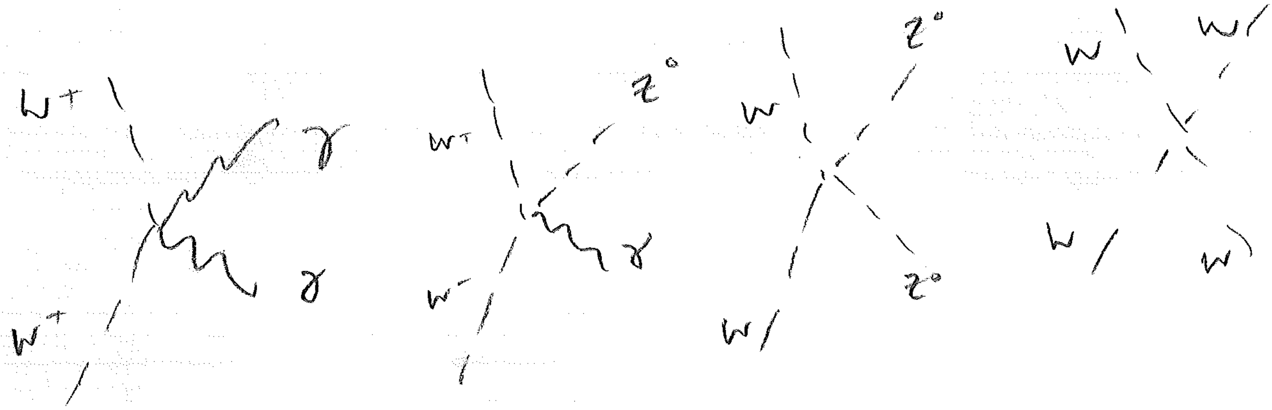
~~IW-W8~~ W8 5/4/16

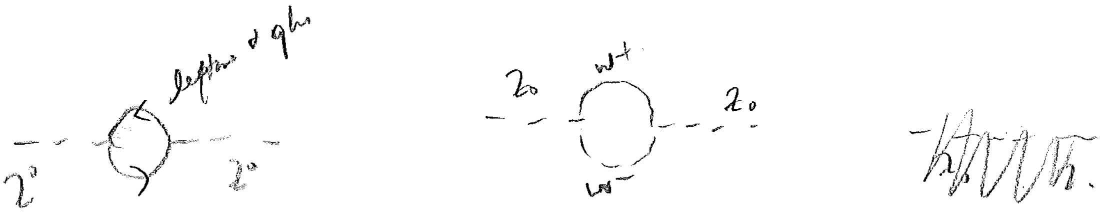
RC

vertices involving weak + electromagnetic gauge bosons



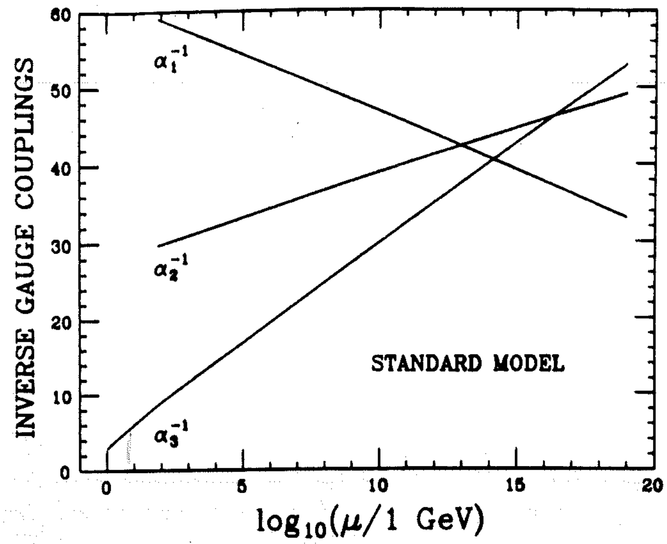
no WWW vertices (doesn't conserve charge)

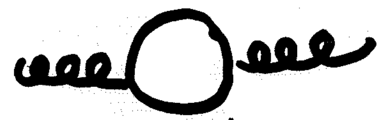




weak coupling to decrease w/ energy
These cause weak interaction to be asymptotically free

Recent measurements (~1990) indicate that standard model couplings don't quite meet!

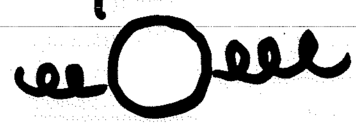


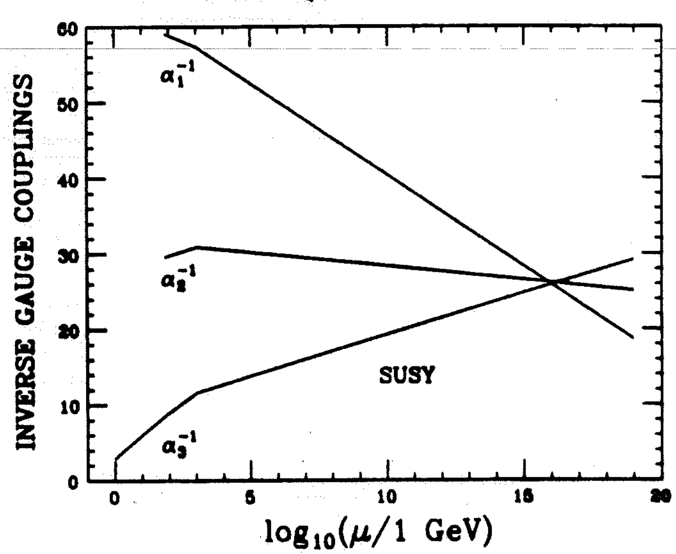


quarks

The "supersymmetric standard model" has additional particles which change the slopes

quarks





"squarks"

(1) The grand unified scale is raised (10^{16} GeV)

$\Rightarrow$ Protons decay more slowly.

(2) $\tau \sim 10^{33} - 10^{35}$ years

(3) Predicts new particles!

GUTS

proton decay