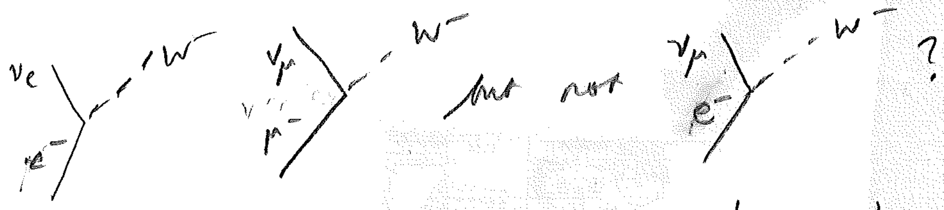
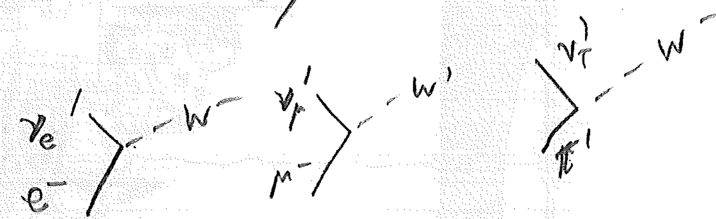
Why no intergenerational mixing in lepton sector?

ie why electron #, μ #, τ # conserved seperately?

Why



Why doesn't W^- couple to

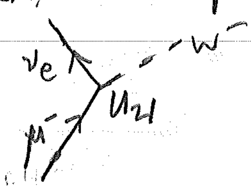


where

$$\begin{pmatrix} \nu_e' \\ \nu_\mu' \\ \nu_\tau' \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

weak intn eigenstates — mass eigenstates

Then Mixing between lepton generations can occur if matrix is not diagonal



But if neutrinos are massless or have equal masses then $\nu_e', \nu_\mu', \nu_\tau'$ are also mass eigenstates!

so what we mean by ν_e is ν_e', ie the ν that couples to the electron ∴ there isn't any ~~...~~ intergenerational mixing.

But if ν's are massive + have different masses ... then there can be intergenerational mixing, or rather ν oscillations...

Assume mixing between only 1st + 2nd generations (for simplicity)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

weak interaction eigenstates, i.e. ν_e couples to e^- + ν_μ " to μ^- (lepton # eigenstate)

mass eigenstate w/ masses m_1 + m_2

If ν_e is emitted at the origin, what fraction will have been converted to ν_μ after a distance x?

$$\psi(0,0) = \nu_e = (\cos\theta)\,\nu_1 + (\sin\theta)\,\nu_2$$

For an energy (mass) eigenstate $\psi(x,t) = \psi(0,0)\, e^{\frac{ipx}{\hbar} - \frac{iEt}{\hbar}}$

(Nearly) massless particles travel x in time $t \approx \frac{x}{c}$

$$\psi(x,t) = \psi(0,0)\, e^{\frac{i}{\hbar}\left(p - \frac{E}{c}\right)x}$$

$$E = \sqrt{(cp)^2 + (mc^2)^2} = cp\sqrt{1 + \left(\frac{mc}{p}\right)^2}$$

$$\approx cp\left(1 + \frac{1}{2}\left(\frac{mc}{p}\right)^2\right) = c\left(p + \frac{m^2c^2}{2p}\right)$$

$$\psi(x,t) \approx \psi(0,0)\, e^{-\frac{i\, m^2c^2}{2\hbar p}x}$$

$$\Psi(x,t) = (\cos\theta)\,\nu_1\, e^{-\frac{i m_1^2 c^2}{2\hbar p}x} + (\sin\theta)\,\nu_2\, e^{-\frac{i m_2^2 c^2}{2\hbar p}x}$$

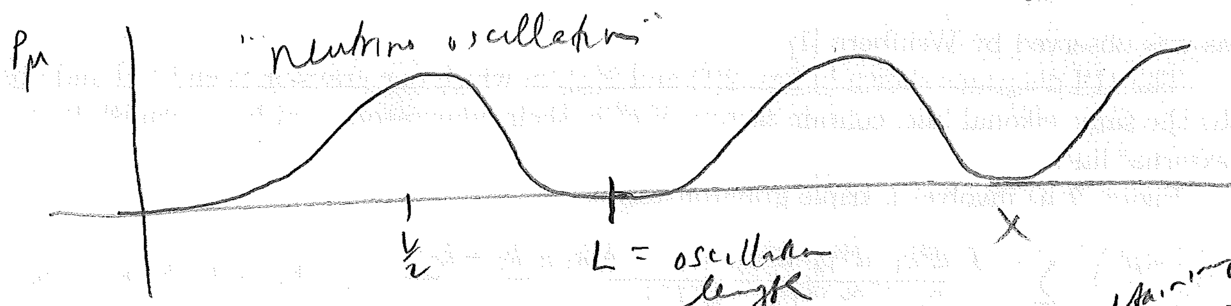
Now $\nu_\mu = -\sin\theta\,\nu_1 + \cos\theta\,\nu_2$. So amplitude that the ν is a μ-neutrino is

$$\langle \nu_\mu | \Psi \rangle = -\sin\theta \langle \nu_1 | \Psi \rangle + \cos\theta \langle \nu_2 | \Psi \rangle$$

$$= -\sin\theta\cos\theta \Big[\underbrace{e^{-\frac{i m_1^2 c^2}{2\hbar p}x} - e^{-\frac{i m_2^2 c^2}{2\hbar p}x}}_{e^{-i\frac{(m_1^2+m_2^2)c^2 x}{4\hbar p}} \cdot 2i \sin\left(\frac{(m_2^2 - m_1^2)c^2 x}{4\hbar p}\right)} \Big]$$

$$P_\mu = |\langle \nu_\mu | \Psi \rangle|^2 = \underbrace{4\sin^2\theta\cos^2\theta}_{(\sin 2\theta)^2}\, \sin^2\left(\frac{(\Delta m^2) c^2 x}{4\hbar p}\right)$$

oscillation depends on differences of m^2



$$\sin^2\left(\frac{(\Delta m^2) c^2 L}{4\hbar p}\right) = 0$$

$$\Rightarrow \frac{(\Delta m^2) c^2 L}{4\hbar p} = \pi \quad \Rightarrow \quad L = \frac{4\pi\hbar p}{(\Delta m^2) c^2}$$

Prob of obtaining ν_μ maximized at $\frac{L}{2}$.

Alternatively

$$(\Delta m^2)\, c^4 = (m_2 c^2)^2 - (m_1 c^2)^2 = \frac{4\pi\hbar c\,(cp)}{L}$$

$$= \frac{(2.5\times10^{-6}\ \mathrm{m\cdot eV})\ cp}{L}$$

Suppose there is significant oscillation between sun + earth

$L \approx 10^{11}\,\mathrm{m}$ $\qquad \Delta (mc^2)^2 = 2.5\times10^{-11} (\mathrm{eV})^2$

$cp \approx 1\ \mathrm{MeV}$

eg. if $m_1 = 0$ then $m_2 c^2 \approx 5\times10^{-6}\,\mathrm{eV}$

so $m_2 c^2 > 10^{-6}\,\mathrm{eV}$.