Electroweak theory (Glashow Weinberg Salam 1968)

gauge theory w/ gauge group $\underbrace{SU(2)_W}_{\text{weak isospin}} \times \underbrace{U(1)_Y}_{\text{weak hypercharge}}$

$SU(2)_W$ has 3 gauge bosons $\underbrace{W^+, W^0, W^-}_{\text{adjoint of } SU(2)_W}$ } all spin 1
$U(1)_Y$ has 1 gauge boson B

All 4 bosons are "initially" massless

The Higgs mechanism gives mass $m_W \approx 80.4$ GeV to W^+, W^-

and mass $m_Z \approx 91.2$ GeV to the linear combination

$[\sin^2\theta_W = 0.2312 /_{\overline{MS}, Z^0}]$

$$Z^0 = W^0 \cos\theta_W - B \sin\theta_W \qquad \text{where} \quad \theta_W = 28.74°$$

The $W^\pm$ + Z^0 were discovered 1983 at CERN [Nobel Dreams]

The other linear combination remains massless

$$A = W^0 \sin\theta_W + B\cos\theta_W$$

remains massless + is identified as the photon

Eliminate B to obtain

$$Z^0 = W^0 \cos\theta_W - \left(\frac{A - W^0 \sin\theta}{\cos\theta}\right)\sin\theta$$

$$= \frac{W^0}{\cos\theta_W} - A^0 \tan\theta$$

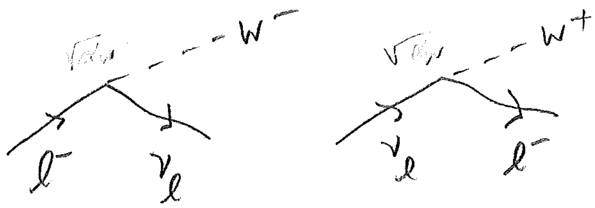
This suggests $m_Z = \frac{m_W}{\cos\theta_W} = \frac{80.4}{0.877} = 91.6$ discrepancy due to quantum corrections

The "left handed" leptons belong to doublets of $SU(2)_W$
(weak isodoublets)

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix} \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix} \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}$$

hence are coupled to the charged-gauge bosons $W^\pm$
("charged currents")

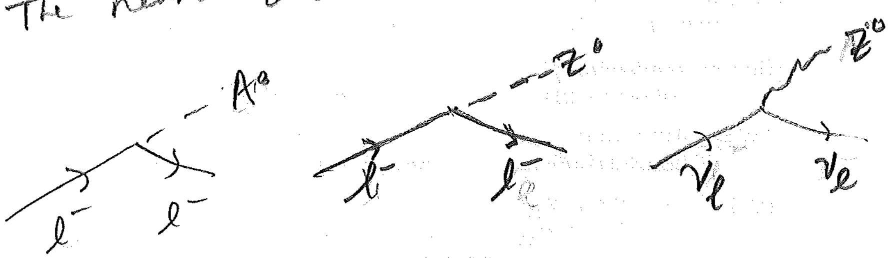
[similar to gluons & color]



charge-changing vertices

[weak int'n acts like a transformation rather than a force]

The neutral gauge bosons Z^0, A^0 only couple leptons w/ same charge

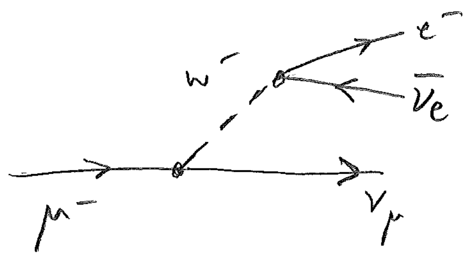


charge-preserving vertices

← (neutral currents)
observed CERN 1973

μ^- decay

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$



electroweak vertices conserve L_e, L_μ, L_τ, charge

Each vertex involving $W^\pm$ contributes a factor of $\sqrt{\alpha_W}$

[actually $\frac{\sqrt{4\pi\alpha_W}}{2\sqrt{2}} = \sqrt{\frac{\pi\alpha_W}{2}}$]

$\alpha_W = \frac{\alpha}{\sin^2\theta_W}$ where $\sin^2\theta_W = 0.23$ $\Rightarrow$ $\alpha_W \sim \frac{1}{32}$

Z^0 vertices more complicated but of same order

[$\alpha_W \gtrsim \alpha$ so why is weak interaction so weak?]

Weak interaction is weak because $W^{\pm}$ is so massive.

[In μ^- decay, W^- is very virtual.]

virtual W^- : $\Delta E \sim m_W c^2$

$$\Delta t \sim \frac{\hbar}{\Delta E}$$

[wrote this in IN 4]

range $r_0 \sim c\,\Delta t \sim \frac{\hbar c}{m_W c^2} = \frac{200\text{ MeV fm}}{80{,}000\text{ MeV}} \sim 0.0025\text{ fm}$

[very short]

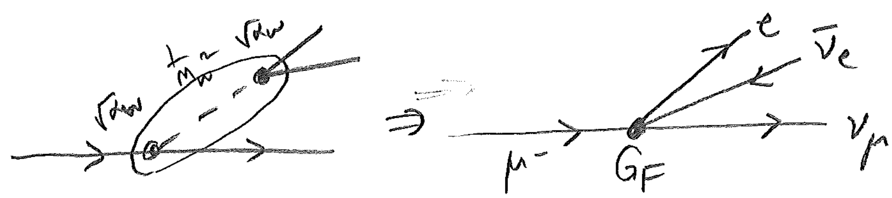
recall $V(r) \sim \frac{e^{-r/r_0}}{r}$

QFT: The virtual W^- suppresses the amplitude by a factor of $\frac{1}{M_W^2}$

$$\mathcal{M} \sim \sqrt{\alpha_W}\,\frac{1}{M_W^2}\,\sqrt{\alpha_W} \sim \frac{1}{32}\cdot\frac{1}{(80\text{ GeV})^2} \sim 5\times10^{-6}\text{ GeV}^{-2}$$

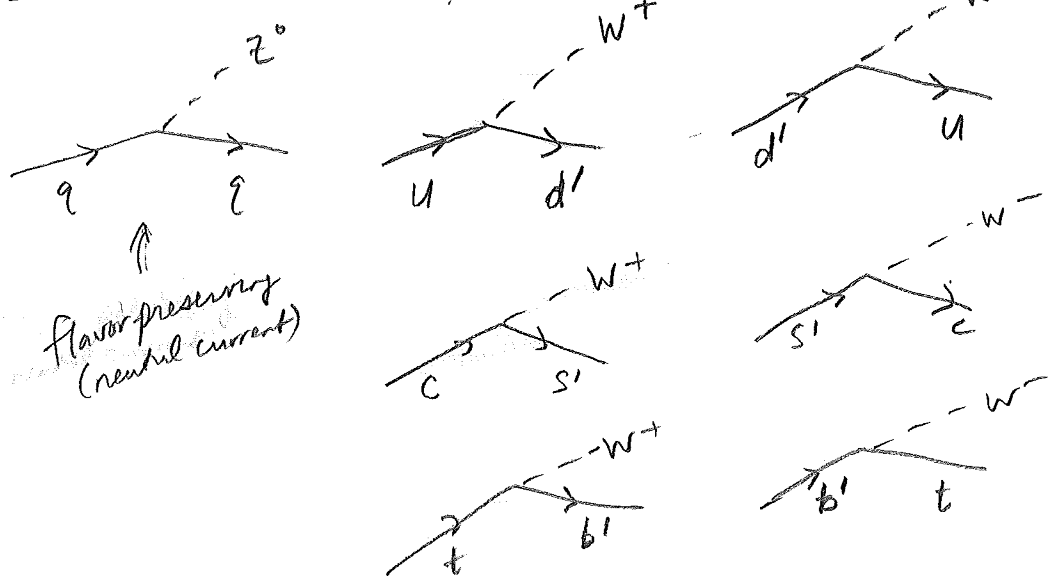
Recall $G_F \sim 1.1\times10^{-5}\text{ GeV}^{-2}$

calculation $\Rightarrow$ $G_F = \frac{\pi}{\sqrt{2}}\,\frac{\alpha_W}{M_W^2}$



Quark vertices

flavor-changing (charged current)



[plus equivalent vertices]

$\begin{pmatrix} u \\ d' \end{pmatrix}$ $\begin{pmatrix} c \\ s' \end{pmatrix}$ $\begin{pmatrix} t \\ b' \end{pmatrix}$ are doublets of weak isospin: $SU(2)_{\text{weak}}$

[like $\begin{pmatrix} \nu \\ \ell^- \end{pmatrix}$]

[What are d', s', b'?]

d', s', b' are almost but not quite d, s, b.

If they were, then weak interactions would conserve strangeness because no lighter quark to decay into.

Approximation: restrict to 1st 2 generations

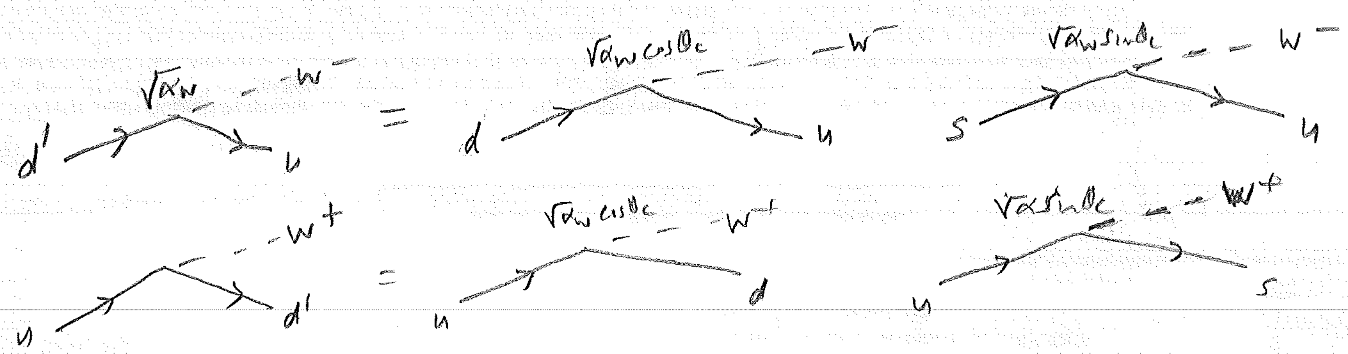
⇒ d', s' are linear combinations of d, s

Cabibbo mixing

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

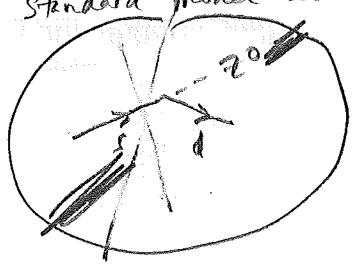
$\theta_c \cong 12^\circ$ (expt)
mixing angle

$$= \begin{pmatrix} 0.98 & 0.2 \\ -0.2 & 0.98 \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$



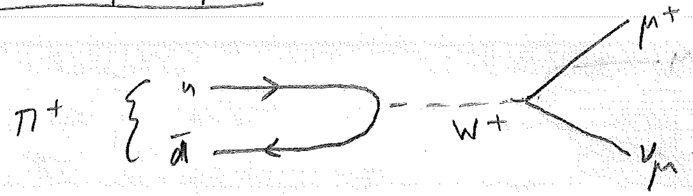
intergenerational mixing vertices
(responsible for violation of
conservation of strangeness)

Note: standard model does not allow FCNC vertices



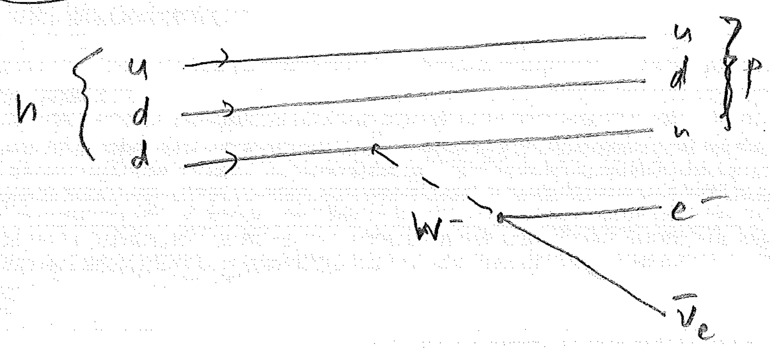
Strangeness preserving weak decays ($\Delta S = 0$)

$\pi^+ \to \mu^+ \nu_\mu$



$\mathcal{M} \sim \alpha_W \cos\theta_c$

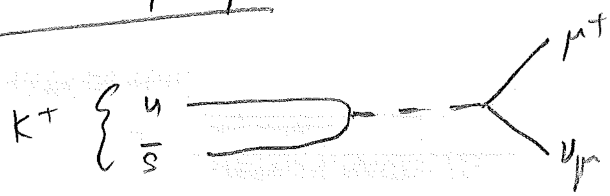
$n \to p e^- \bar{\nu}_e$



$\mathcal{M} \sim \alpha_W \cos\theta_c$

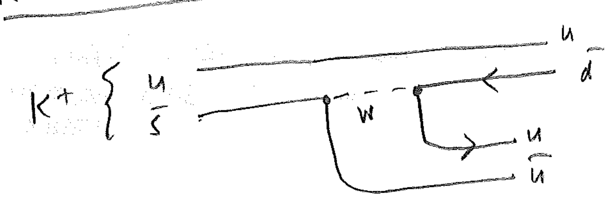
Strangeness violating weak decays ($\Delta S \neq 0$)

$K^+ \to \mu^+ \nu_\mu$ [64%]



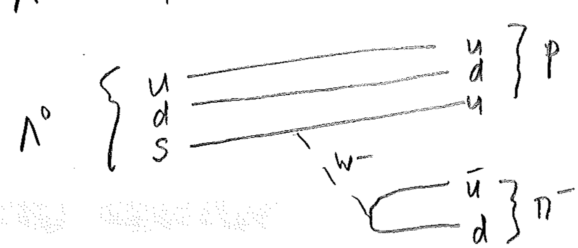
$\mathcal{M} \sim \alpha_W \sin\theta_c$

$K^+ \to \pi^+ \pi^0$ [21%]



$\mathcal{M} \sim \alpha_W \sin\theta_c \cos\theta_c$

$\Lambda^0 \to p \pi^-$ [64%]



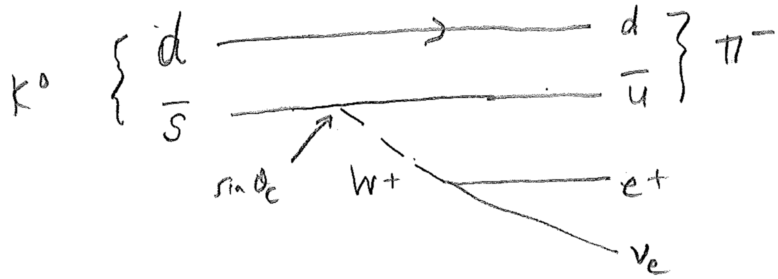
$\mathcal{M} \sim \alpha_W \sin\theta_c \cos\theta_c$

$\Delta S \neq 0$ ~~probabilities suppressed~~ rates by a factor of $\sin^2\theta_c \sim 0.04$

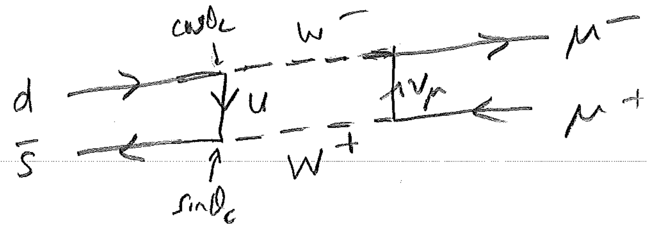
Why is θ_c small? No one knows.

$\boxed{K^0_L}$

$K^0 \longrightarrow \pi^- e^+ \nu_e$ [40%]



$K^0 \longrightarrow \mu^+\mu^-$ [6.8×10^{-9}]



~~[requires 2 vertices (not FCNC allowed)]~~

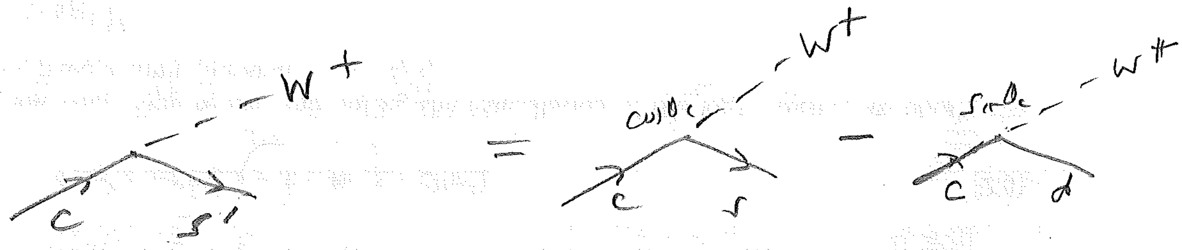
[Even though diagram is suppressed, it predicts too high a branching ratio]

~~GW 9~~
Ew-8

Solution: GIM mechanism (1970)

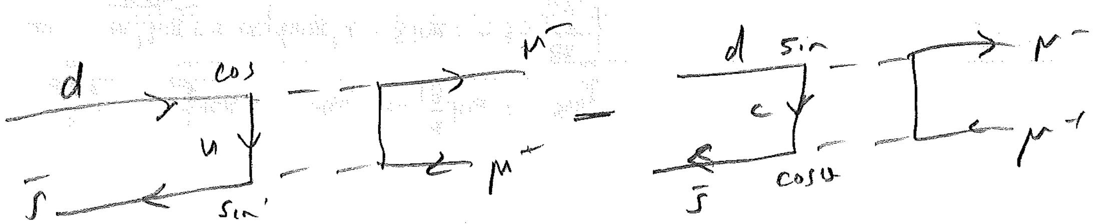
(Glashow-Iliopoulos-Maiani)

Postulate a 4th quark c which belongs to a weak isodoublet $\begin{pmatrix} c \\ s' \end{pmatrix}$



$d' = d\cos\theta_c + s\sin\theta_c$

$s' = s\cos\theta_c - d\sin\theta_c$

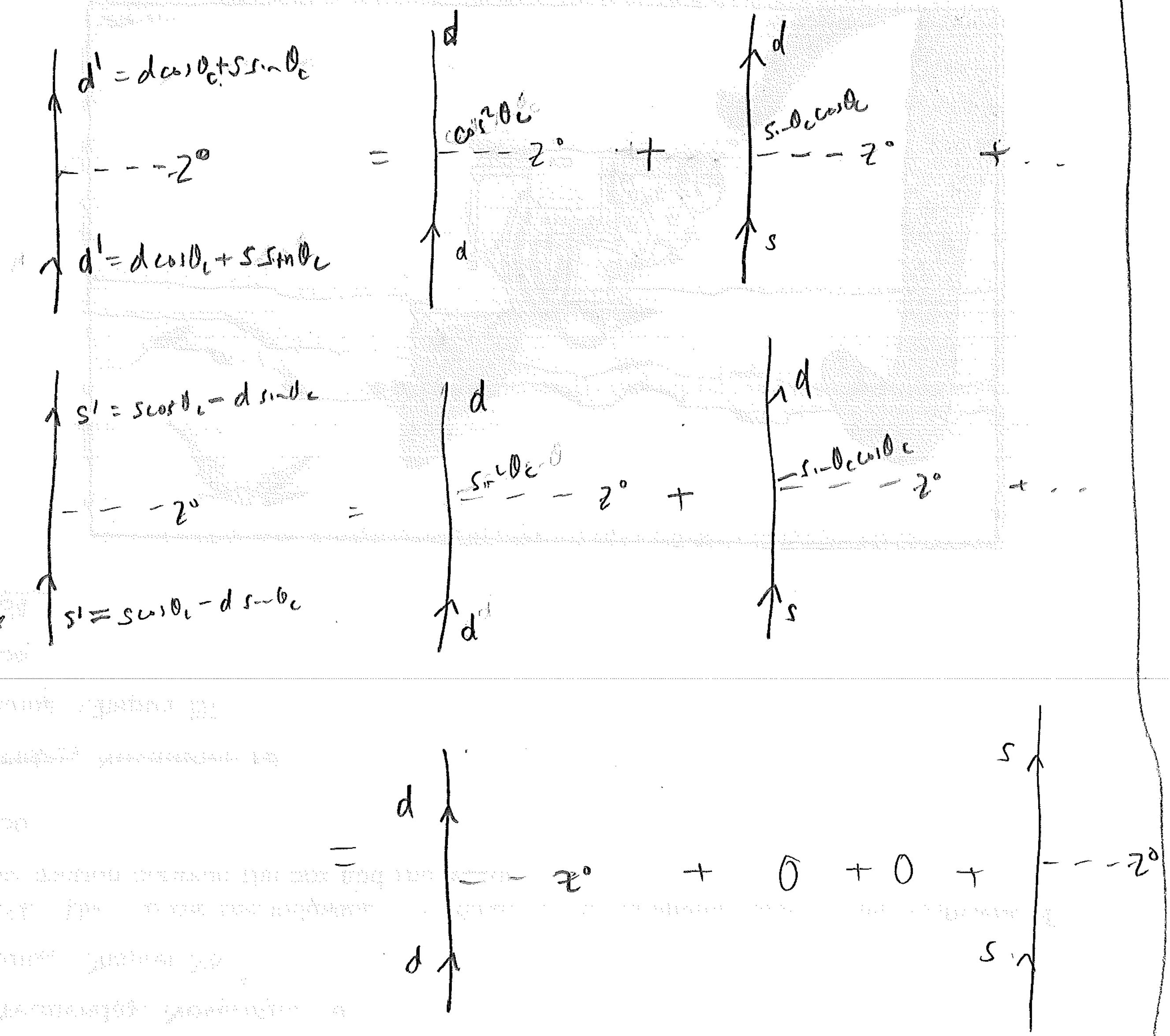


if $m_c = m_u$ these would cancel, suppressing $K^0 \to \mu^+\mu^-$

since $m_c \gg m_s$, cancellation is not complete allowing $K^0 \to \mu^+\mu^-$ at a small rate

[If asked?]

Why no FCNC vertices involving Z^0?

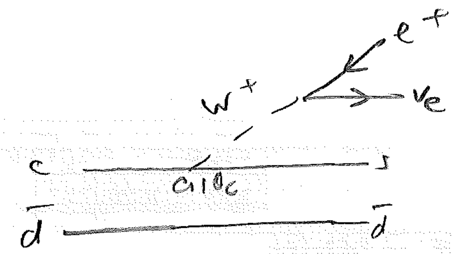


$$\begin{matrix} u & & c \\ d & \swarrow & \downarrow s \end{matrix}$$

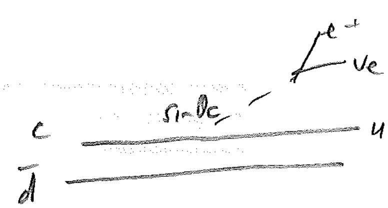
Because θ_c is small, decays are more likely to occur within generations as between them

eg

$D^+ = c\bar{d}$



		BR
$D^+ \to \bar{K}_0 e^+ \nu_e$		0.088
$\to \pi^0 e^+ \nu_e$		0.004

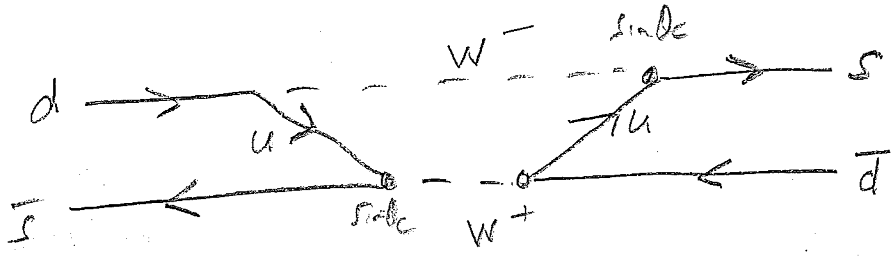


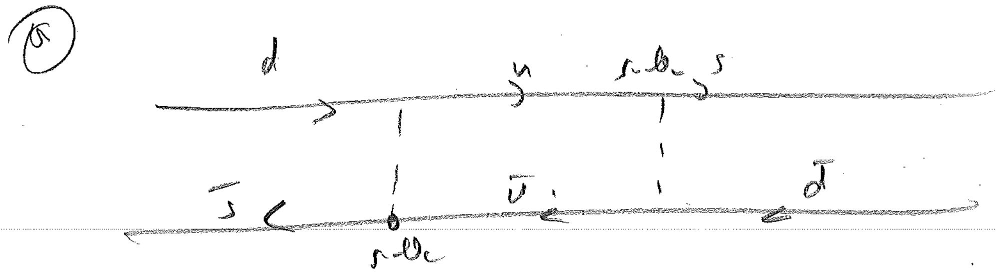
$$\text{Ratio} = \left|\frac{\sin\theta_c}{\cos\theta_c}\right|^2 = 0.045$$

[Prob 25 $D^0 = c\bar{u} \to K^-\pi^+$ 0.039
$\to \pi^-\pi^+$ 0.00140
$\to K^+\pi^-$ 0.00015]

HW problem

$$\boxed{K^0 \longrightarrow \bar{K}^0}$$

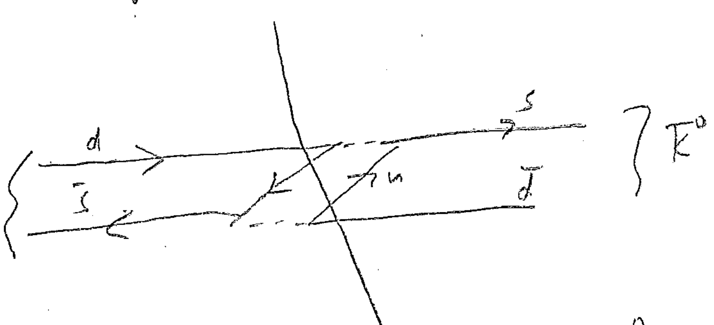




$K^0 \to \pi^0 \to \bar{K}^0$

could write this

K^0



$\bar{K}^0$

This cut is $K^0 \to \pi^+\pi^- \to \bar{K}^0$

[Repeat]

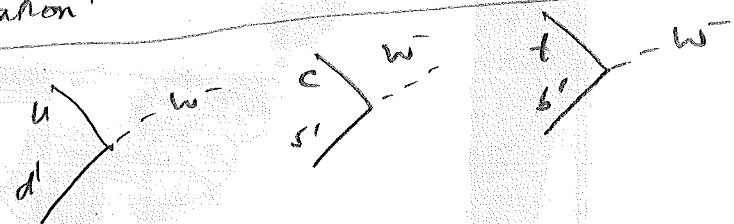
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

↑ weak interaction eigenstate

↑ mass eigenstate

"Cabibbo rotation"

Actually w/ 3 generations

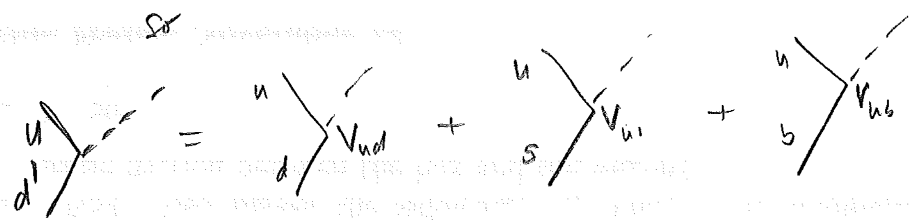


$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{CKM (Cabibbo Kobayashi Maskawa) matrix}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

↑ weak eigenstate

↑ mass eigenstate

so



experimentally ← see PDP CKM mtx

$$\underline{\underline{V_{ud}}} \gg \underline{\underline{V_{us}}} \gg V_{ub}$$

0.97   0.23   0.004 ⟶ intergenerational mixing between 1st + 3rd generations is suppressed even more than bw 1st + 2nd

Roughly $\begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$

so qk decay proceeds in a cascade

$$\begin{tikzcd}
t \arrow[r, "V_{tb}"'] \arrow[dr, dashed, "V_{ts}"'] \arrow[ddr, dashed, "V_{td}"'] & b \arrow[r, "V_{cb}"'] \arrow[dr, dashed, "V_{us}"'] & c \arrow[r, "V_{cs}"'] \arrow[dr, dashed, "V_{cd}"'] & s \arrow[r, "V_{us}"'] & u \\
& s & u & d & \\
& d & & &
\end{tikzcd}$$

~~q_I C_V C_A~~

??

$\boxed{Q}$ should we do ξ vertices

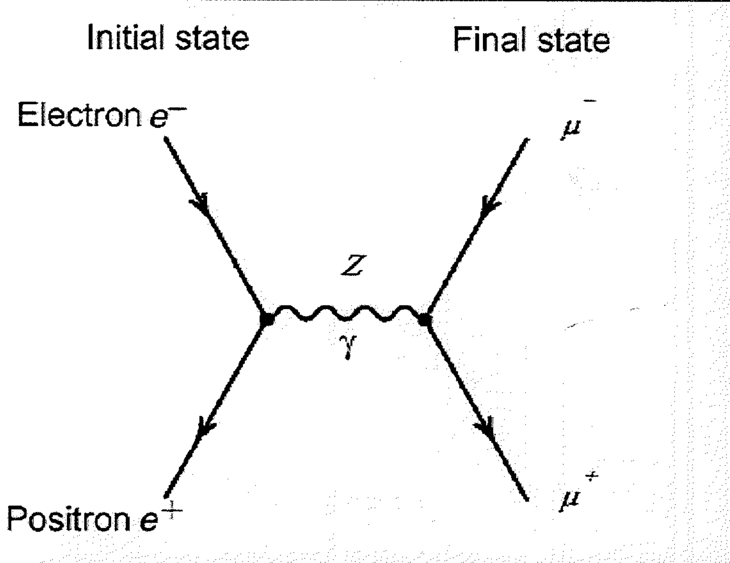
$\xi(C_V - C_A \delta_5)$

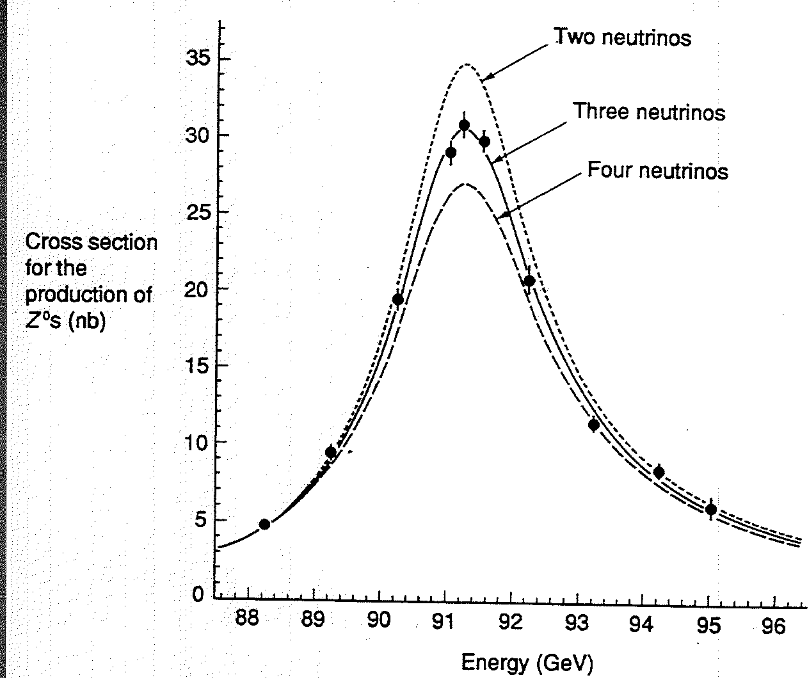
$\downarrow$

∴ predict decay of ξ_0?

problem is: need to talk abt chirality

How Many Neutrinos?





$$Z^0 \rightarrow q\bar{q}(u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b})$$

$$Z^0 \rightarrow l\bar{l}(e^-e^+, \mu^-\mu^+, \tau^-\tau^+)$$

$$Z^0 \rightarrow \nu\bar{\nu}(\nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau)$$

$\nu\bar{\nu}$

Γ_Z measured

$\Gamma_{had}, \Gamma_l, \Gamma_\nu$ calculated

$$\Gamma_Z = \Gamma_{had} + 3\Gamma_l + N_\nu$$

$$N = 2.99 + 0.02$$

otal width $\Gamma \sim$ decay probability (~1/lifetime)

artial $\Gamma_i \sim$ branching rate (channel i)