

[non commutative]

Nonabelian gauge theories SU(N)

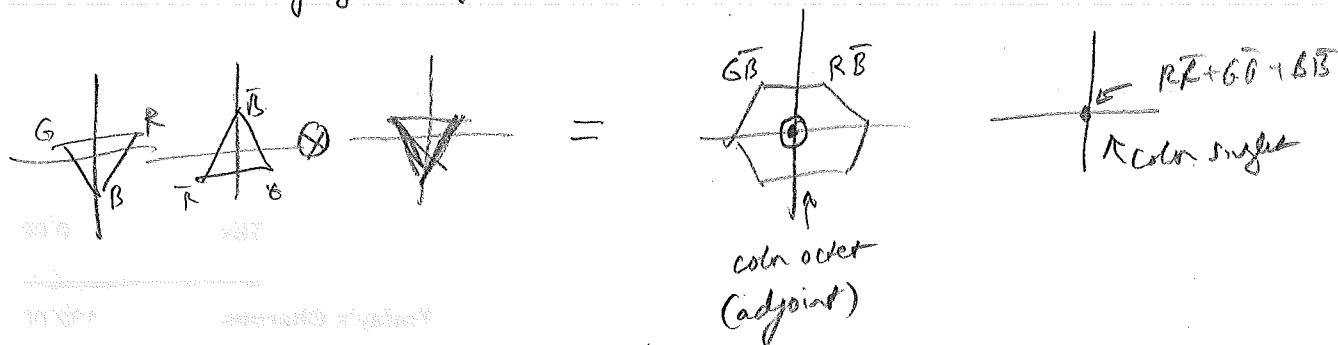
Contains a set of $N^2 - 1$ spin-1 gauge bosons
belonging to the adjoint rep of SU(N)

adjoint rep is contained in the product of fund. $\otimes$ fund

$$N \otimes \bar{N} = \underbrace{(N^2 - 1)}_{\text{adjoint}} \oplus \underbrace{1}_{\text{singlet}}$$

eg: SU(3)_c gauge theory

$$3 = \begin{pmatrix} R \\ G \\ B \end{pmatrix} \quad \bar{3} = \begin{pmatrix} \bar{R} \\ \bar{G} \\ \bar{B} \end{pmatrix}$$



8 gauge bosons = gluons

Since gluons do not belong to a color singlet, they are
not observable in isolation (confinement)

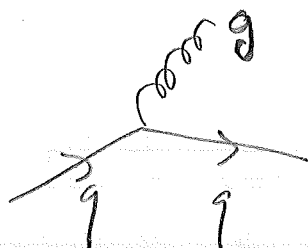
~~Because~~ Gluons only interact w/ objects w/ color

Quantum chromodynamics (QCD)

(Politzer Gross Wilczek 1977)

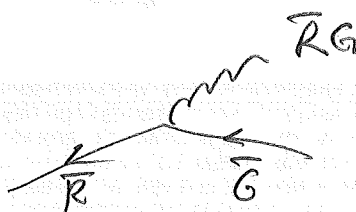
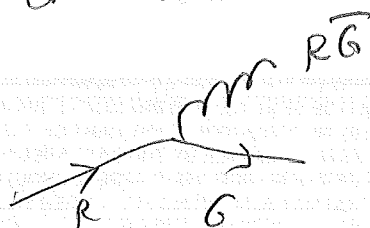
= $SU(3)_c$ gauge theory $\left\{ \begin{array}{l} \text{8 gluons (in the adjoint rep)} \\ \text{plus quarks (in the fundamental rep)} \end{array} \right.$

QCD vertex



Color is conserved at each vertex;
gluons allow quarks to change color

[use colored chalk]



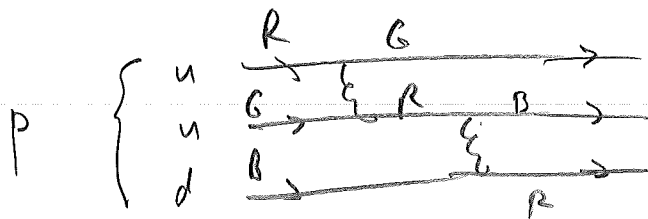
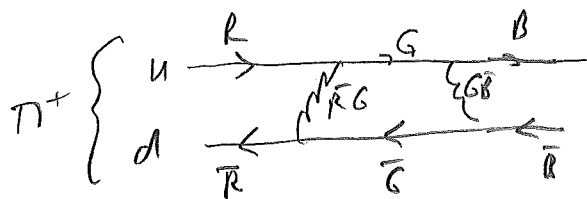
Recall only hadrons that

QCD 3

are color-singlets (ie 1 dim rep of $SU(3)_c$)
are observable in isolation

mesons: $\frac{1}{\sqrt{3}} (R\bar{R} + G\bar{G} + B\bar{B})$

baryons: $\frac{1}{\sqrt{6}} (RGB \pm \text{perm})$



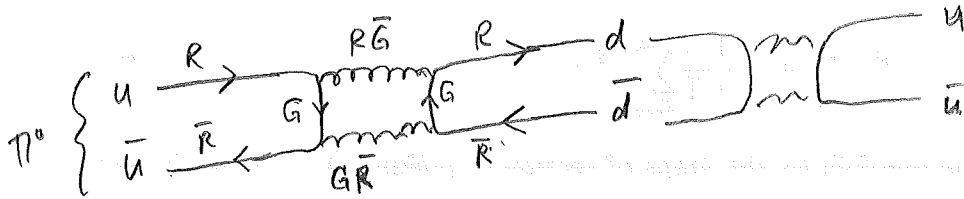
gluons do not allow quark to change flavor
so (eg) strangeness is conserved in strong interactions

$$\pi^+ = u\bar{d}$$

$$\pi^- = d\bar{u}$$

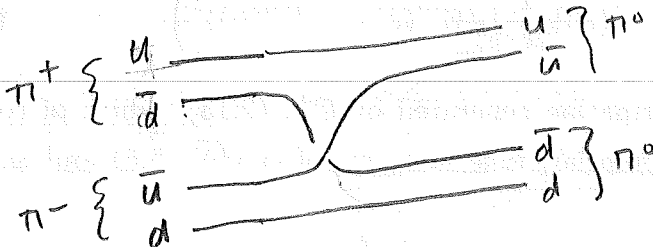
$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

GCD-24

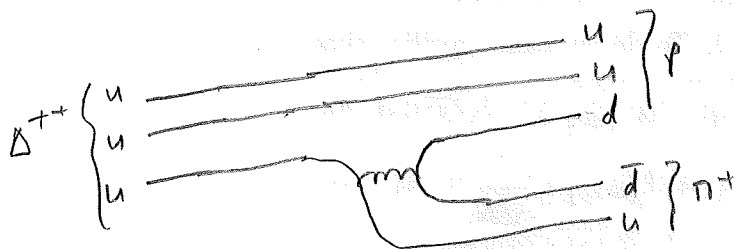


Strong interactions decay + scattering involve exchange
and/or creation of quarks

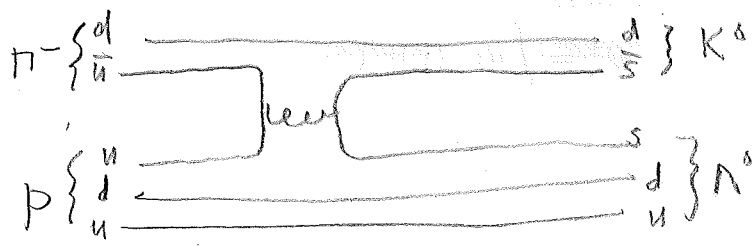
$$\pi^+\pi^-\rightarrow\pi^0\pi^0$$



$$\Delta^{++}\rightarrow p\pi^+$$



Associated production: $\pi^-p\rightarrow\Lambda^0K^0\Rightarrow$ Strangeness conserved



(S, S-bar created in pairs)

gluons have color & can couple to gluons

QCD
REDO

gluons

$$g \otimes g = g \oplus \dots$$

[doesn't happen 7 photons]

gluons

QED is linear, QCD is non-linear
(superposition)

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$$

QED
 $e \sim \sqrt{\alpha}$

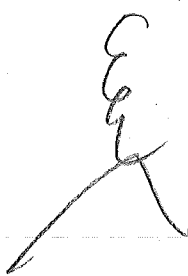
QCD
 α_s

QCD-5
~~25~~
 Fermi
 QED

$$\alpha \approx \frac{1}{137} \Rightarrow \text{weak}$$

$$\alpha_s \approx 1-10$$

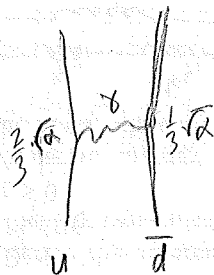
[Haken
 + Martin]



2 consequences

① force between quarks much stronger

[therefore force between 2 q's by gluon exchange is much larger than Coulomb force]



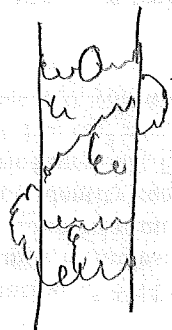
$$\Rightarrow \frac{2}{3} \frac{1}{3} \frac{d}{r^2} \text{ (repulsive)}$$



$\Rightarrow$ strong attract

~~color introduced and~~
~~new quarks possible~~
~~fermion statistics~~
~~but it also a source~~
~~of gluons & new force~~

② higher order diagrams just as important as (or more so) ~~lowest~~ tree diagrams $\rightarrow \infty$ # of diagrams impossible to calculate



$\rightarrow$ unlike hydrogen, no one has computed binding of u & $\bar{d}$ to give π^+

$\rightarrow$ It is believed, but no one has proved, that ~~free~~ quarks can never be isolated due to exchange of gluons (Confinement hypothesis)

$\rightarrow$ 2.11 Since

experimental: no q's observed

$\rightarrow$ Does QCD predict confinement?

$\rightarrow$ Does QCD describe reality?

REDO

k_{in}

~~QCD~~
on

one model suggests that gluon exchange

leads to $F = F_0$ (const) (rather to $\frac{1}{r^2}$)

so $U = F_0 r$

~~so that pulling q's apart~~

as if q's were connected by a string
of constant tension

experiment $\Rightarrow F_0 \approx 16 \text{ tons} = 16 \left(\frac{2000 \frac{\text{lbs}}{\text{ton}}}{(2.2 \frac{\text{lbs}}{\text{kg}})} \right) \left(10 \frac{\text{m}}{\text{ft}} \right) = 1.5 \text{E5 N}$

$= \frac{(1.5 \text{E5}) \text{ J/m}}{(1.6 \text{E-13} \text{ J/meV})} = 18 \frac{\text{meV}}{\text{fm}}$

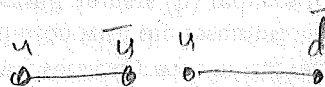
$= 1000 \frac{\text{meV}}{\text{fm}}$



extended by factor of fm $\Rightarrow$ requires 10^5 's of meV

~~the energy is so large that~~

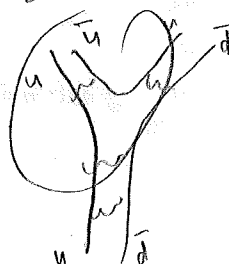
$\rightarrow$ create $u\bar{u}$ pair



string snaps



$\pi^+ \rightarrow \pi^+ \pi^0$



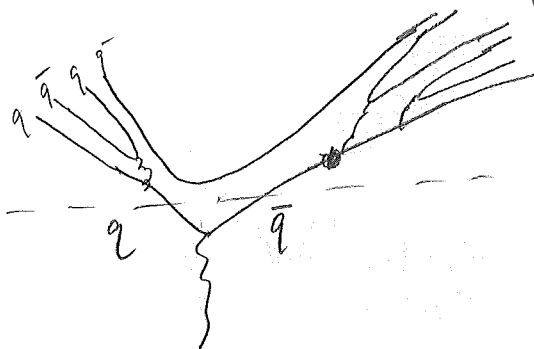
QCD
1978-1985

Left to right

REDO

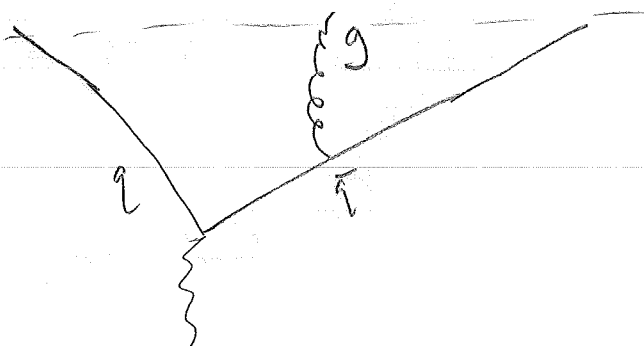
red
jet

Hadronization



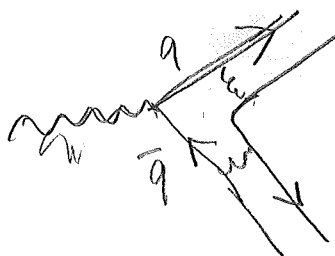
$\Rightarrow$ 2-jets

of pions



$\Rightarrow$ 3-jets

Perkins 8.28



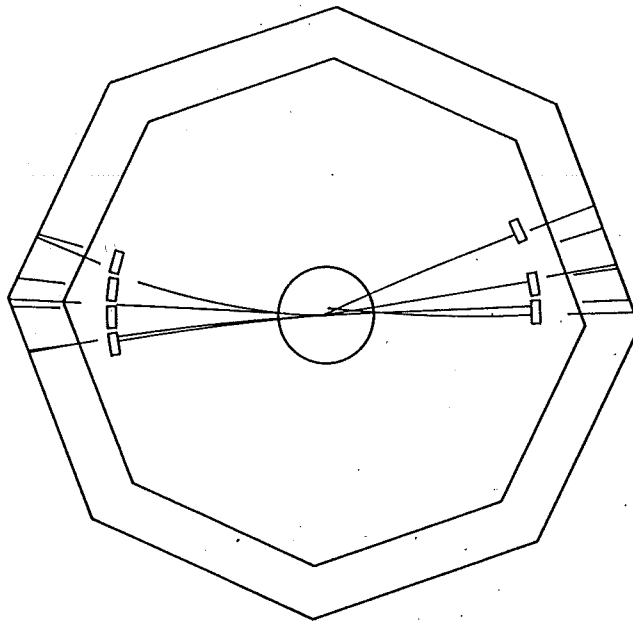


Figure 8.1 A typical two-jet event. (Courtesy J. Dorfan, SLAC.)

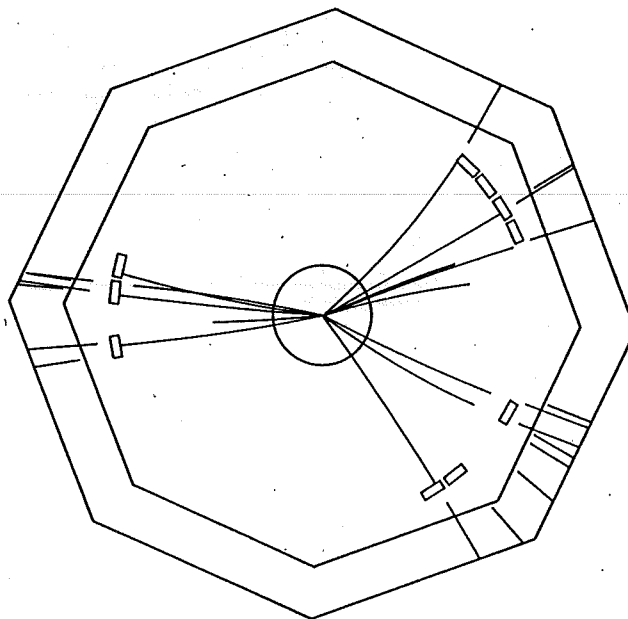


Figure 8.2 A three-jet event. (Courtesy J. Dorfan, SLAC.)

Griffiths

HW problem

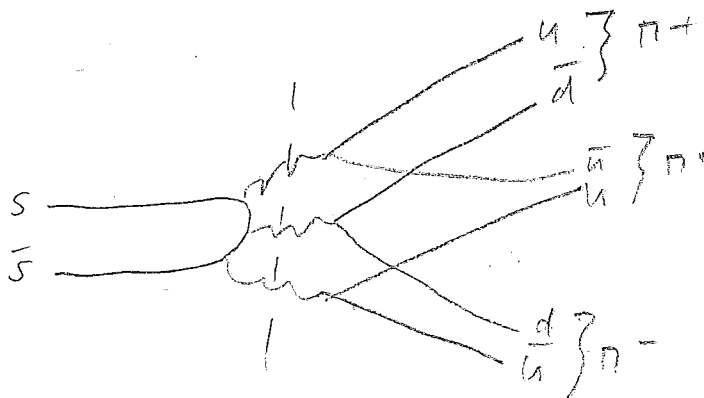
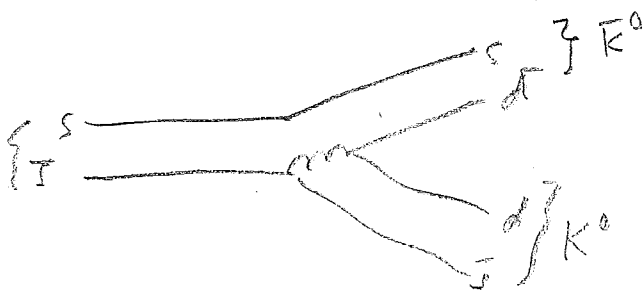
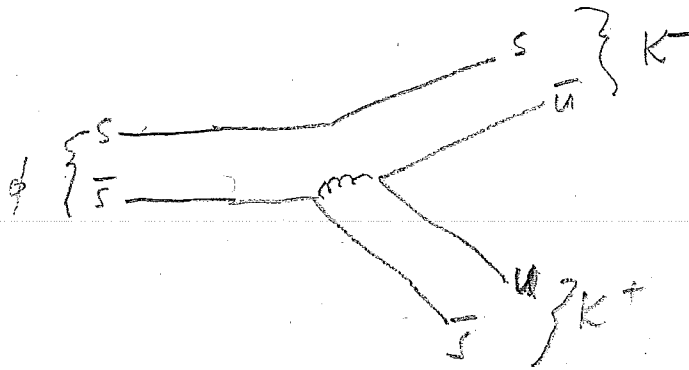
$$\phi(1020) = s\bar{s}$$

$$m = 1020 \text{ MeV}$$

$$\tau = 1.5 \times 10^{-22} \text{ s} \quad (\text{strong decay})$$

	<u>BR</u>	<u>Q</u>
$\phi \rightarrow K^+ K^-$	49%	30 MeV
$\rightarrow K_L^0 K_S^0$	34%	
$\rightarrow \pi^+ \pi^0 \pi^-$	15%	600 MeV

$\phi \rightarrow \pi^+ \pi^-$ violates
G parity



($\phi \rightarrow \pi^+ \pi^-$ suppressed)