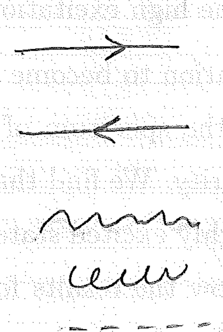
Probability of a decay or scattering event $\propto |\mathcal{M}|^2$

$\mathcal{M}$ = amplitude, represented by a sum of Feynman diagrams

- particles represented by lines)



lepton or quark

antilepton or antiquark

photon

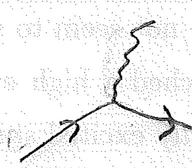
gluon

W, Z boson

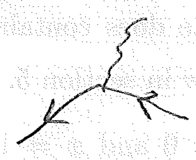
- Interactions are represented by vertices

QED (quantum electrodynamics) [Schwinger Feynman Tomonaga 1948]

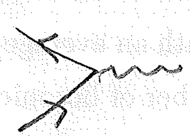
"photon couples to charged particles"



charged particle emits a photon



antiparticle emits a photon



pcl/antipcl annihilation

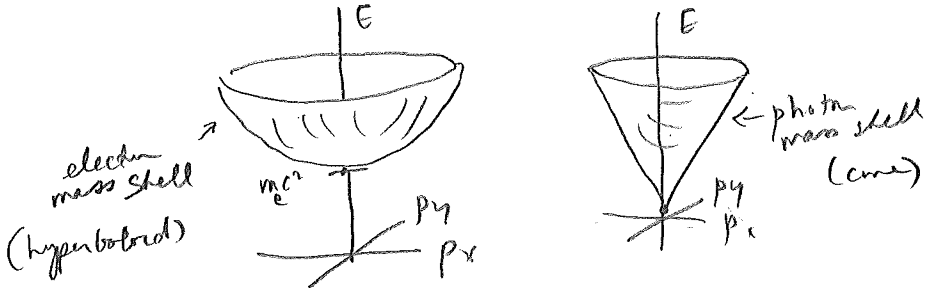
pcl/antipcl creation

$\longrightarrow$ time

F.d are not spacetime diagrams [more abstract]

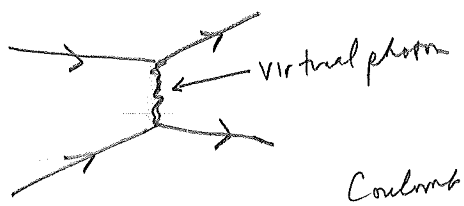
A physical particle obeys $E^2 = (c\vec{p})^2 + (mc^2)^2$

i.e. is "on the mass shell"



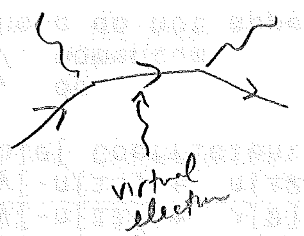
A particle off the mass shell is called virtual & only lives for a short time (by uncertainty principle)

In a QED vertex, at least one of the particles must be virtual
⇒ need at least 2 vertices in a F.d.



[explain how if initial e^- is at rest & final e^- is moving, then $p_\gamma > 0$, but $E_\gamma < 0$]

Coulomb repulsion is mediated by exchange of a virtual photon

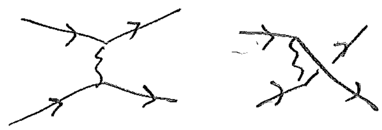


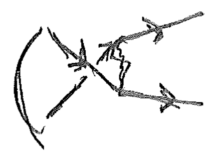
[explain how if $\gamma + e^-$ collide in cm frame, $E_e > mc^2$ while $p_e = 0$]

All external particles are real (on the mass shell)

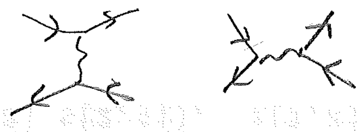
Usually several topologically distinct F.d. contribute to the amplitude for a process

$e^-e^- \to e^-e^-$
Møller scattering

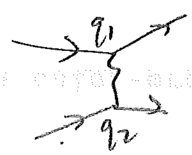


( is not distinct)

$e^+e^- \to e^+e^-$
Bhabha scattering



Each vertex contributes a factor of electric charge to the amplitude
(⇒ neutral particles cannot emit γ)

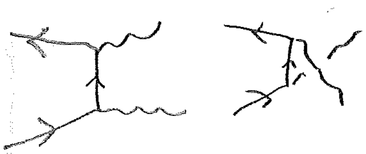


$\mathcal{M} \sim q_1 q_2 \quad \Rightarrow \quad V = \frac{K q_1 q_2}{r}$ (Coulomb potential)

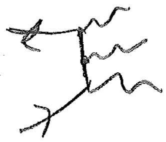
Positronium cannot annihilate into a single photon
[$E_\gamma > 0$, $p_\gamma = 0$ in cm frame]

$e^+e^- \not\to \gamma$

$e^+e^- \to \gamma\gamma$



$e^+e^- \to \gamma\gamma\gamma$



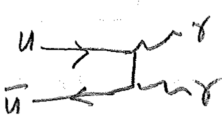
etc.

$$\frac{\mathcal{M}_{3\gamma}}{\mathcal{M}_{2\gamma}} \sim e \Rightarrow \frac{\Gamma_{3\gamma}}{\Gamma_{2\gamma}} \sim e^2 \sim \frac{Ke^2}{\hbar c} = \alpha \sim \frac{1}{137} \qquad \frac{BR(3\gamma)}{BR(2\gamma)} \sim 1\%$$

Rule of thumb: each additional vertex ⇒ BR ~ 0.01 [but there are other factors → angular momentum cons laws]
⇓
$\mu^- \to e^- \bar{\nu}_e \nu_\mu$ 100%
$\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma$ 1.4%

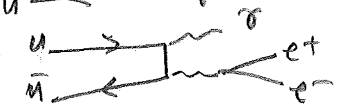
[$\pi^0 \to \gamma\gamma\gamma$ forbidden by C
$\pi^0 \to e^+e^-$ (6×10^{-8}) suppressed]

$\pi^0 \to \gamma\gamma$



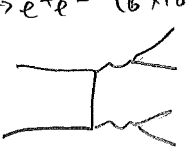
~100%

$\pi^0 \to \gamma e^+e^-$



~ 0.012

$\pi^0 \to e^+e^-e^+e^-$

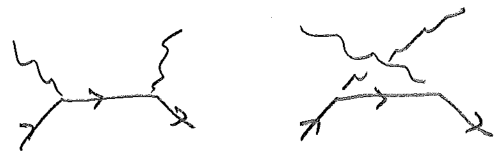


$\sim 3\times10^{-5}$

Interaction of photons w/ matter

1). Compton scattering

$\gamma e^- \to \gamma e^-$

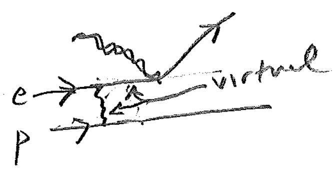


2) photoelectric effect: γ absorbed by atom, e^- liberated

$\gamma e^- \not\to e^-$

A free electron cannot absorb a γ

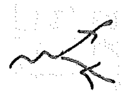
$\gamma \underbrace{e p}_{\text{hydrogen}} \to e^- p$



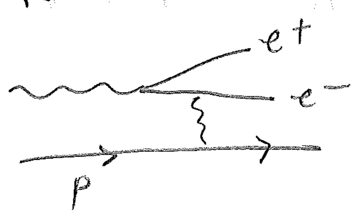
An electron in an atom can!

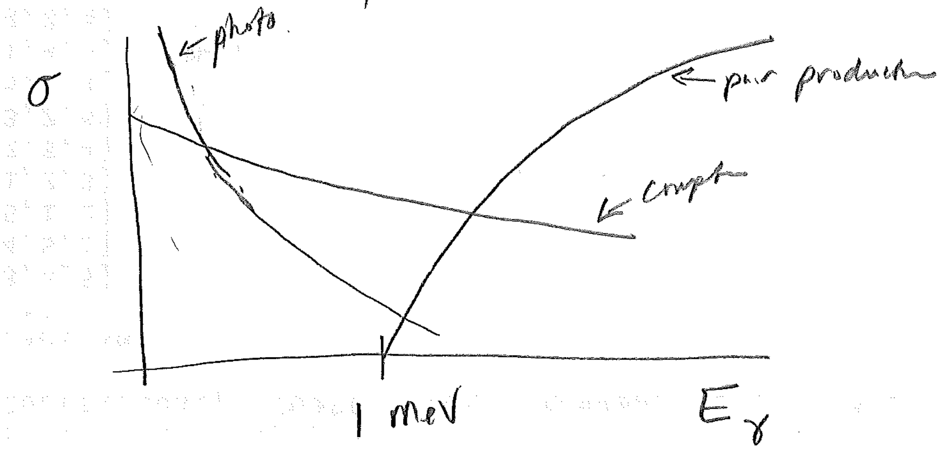
3) pair production in presence of matter (if $E_\gamma > 1$ MeV)

$\gamma \not\to e^+ e^-$

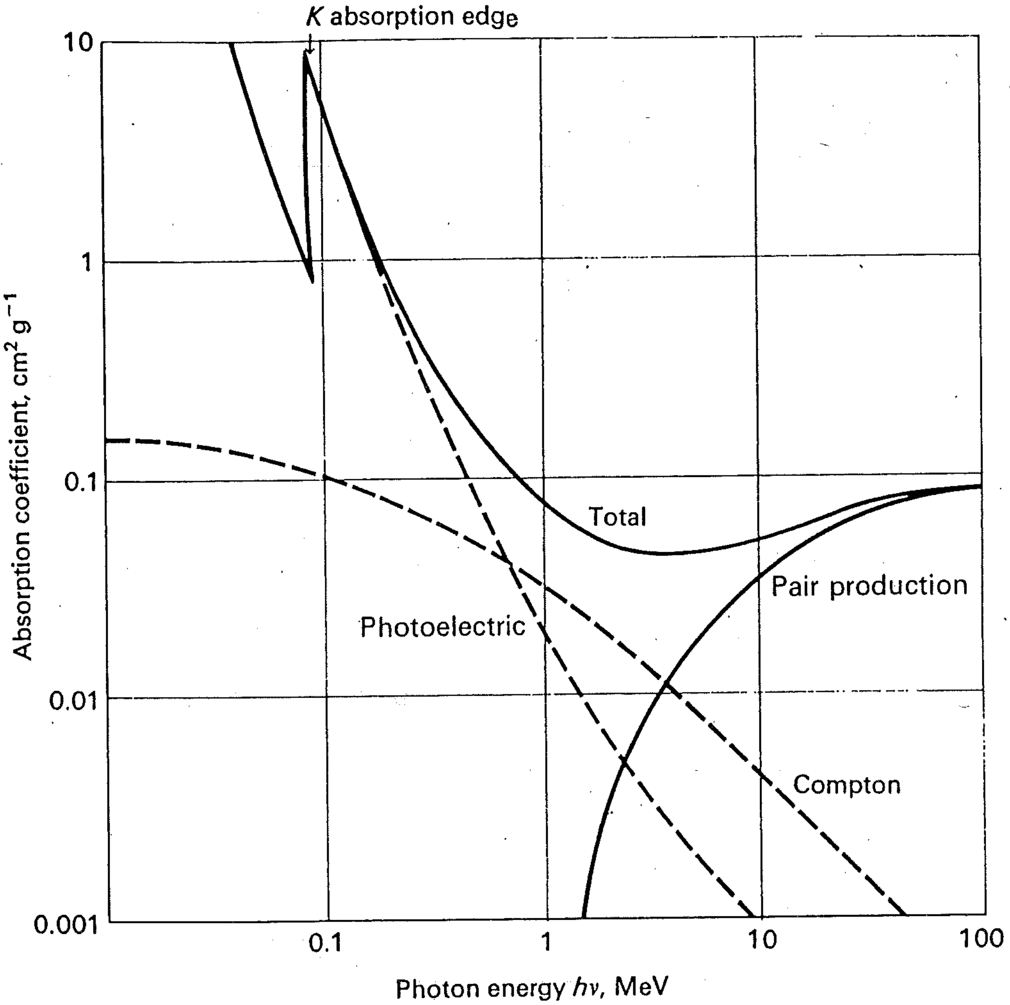


$\gamma p \to e^+ e^- p$





[show plot]



The absorption coefficient per g cm^{-2} of lead for γ-rays as a function of energy.

Perkins

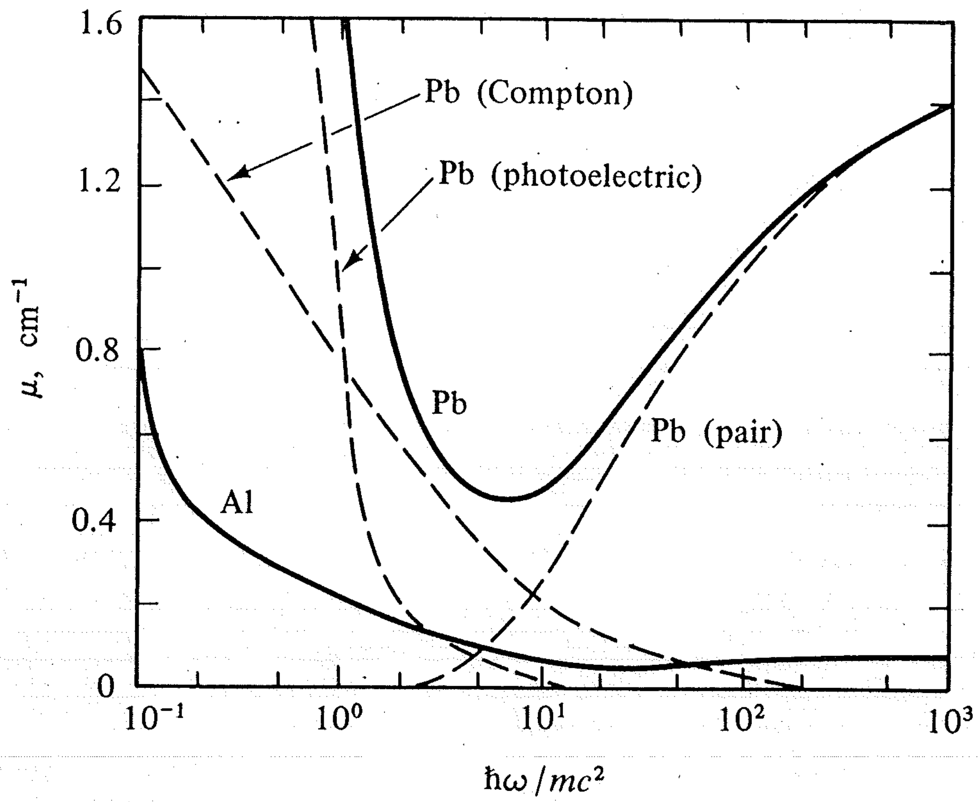


Fig. 3.7. Total absorption coefficients of γ rays by lead and aluminum as a function of energy (solid lines). Photoelectric absorption of aluminum is negligible at the energies considered here. Dashed lines show separately the contributions of photoelectric effect, Compton scattering, and pair production for Pb. Abscissa, logarithmic energy scale; $\hbar\omega/mc^2 = 1$ corresponds to 511 keV. (From W. Heitler, *The Quantum Theory of Radiation*, The Clarendon Press, Oxford, 1936, p. 216.)

Frauenfelder & Henley

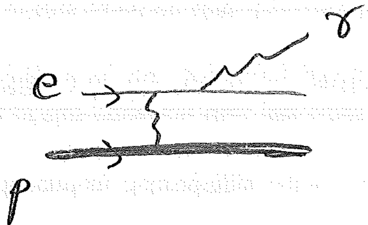
Bremsstrahlung (inverse photo electric)

↳ an accelerated e^- emits EM radiation

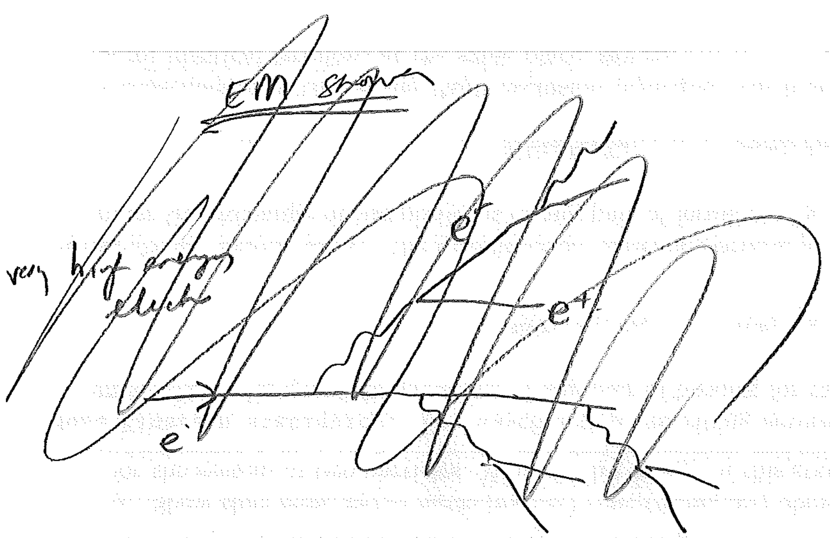
$e^- \not\to e^- \gamma$

not possible

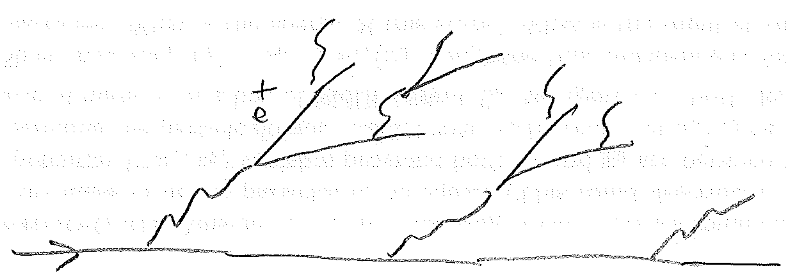
$e^- p \to e^- p \gamma$

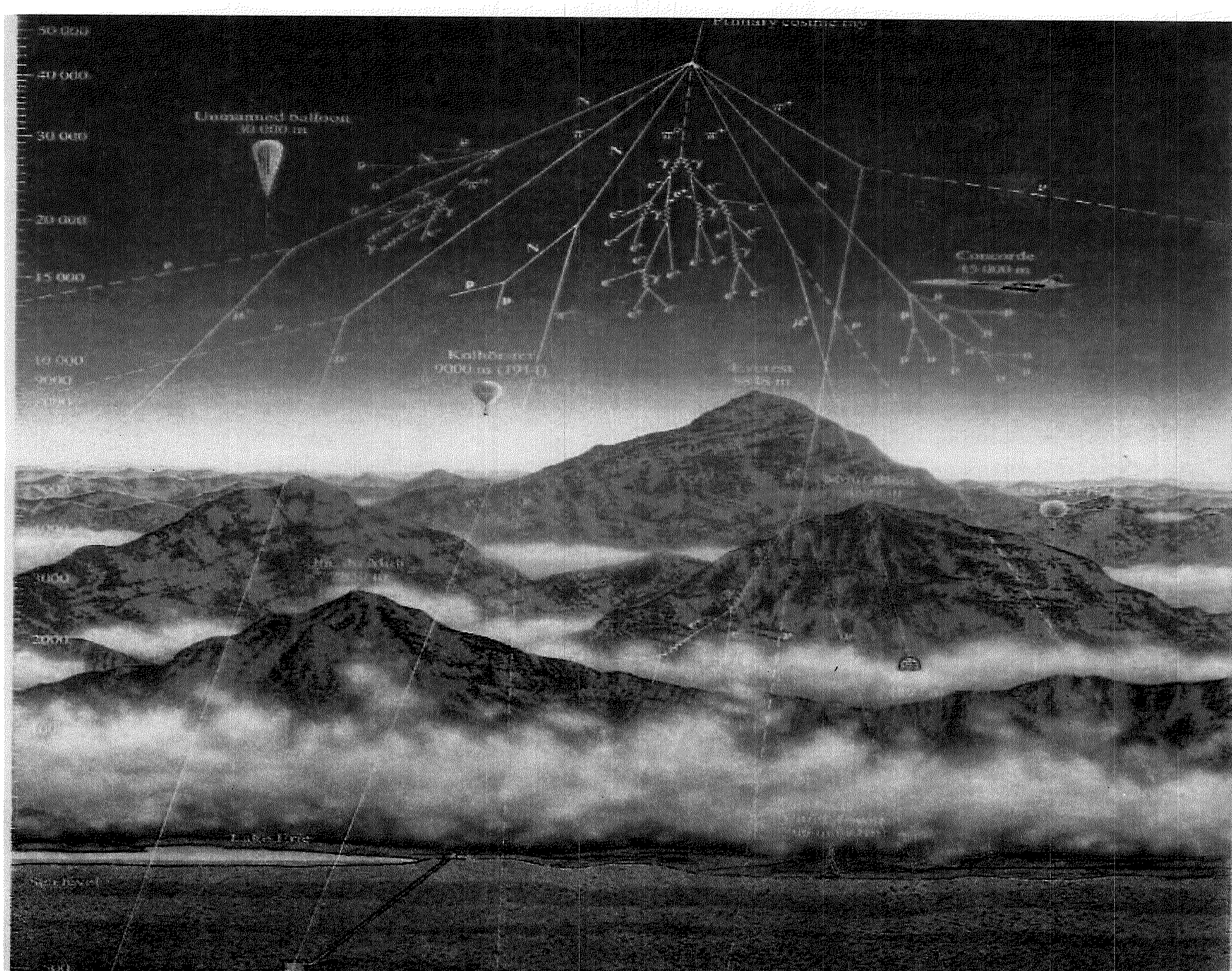


bremsstrahlung

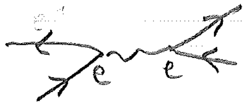


EM shower



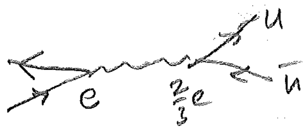

Primary cosmic ray
50 000
40 000
30 000
20 000
15 000
10 000
Unmanned balloon
30 000 m
Concorde
15 000 m
Kolhörster
9000 m (1914)
Lake Erie
Sea level

$e^+e^- \longrightarrow \mu^+\mu^-$



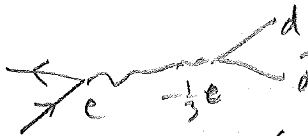
$\mathcal{M} \sim e^2 \sim \alpha \Rightarrow \sigma \sim |\mathcal{M}|^2 \sim \alpha^2$

$e^+e^- \longrightarrow u\bar{u}$

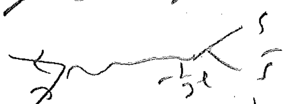


$\mathcal{M} \sim \frac{2}{3}e^2 \sim \frac{2}{3}\alpha \Rightarrow \sigma \sim \frac{4}{9}\alpha^2$

$\mathcal{M} \sim eQ$



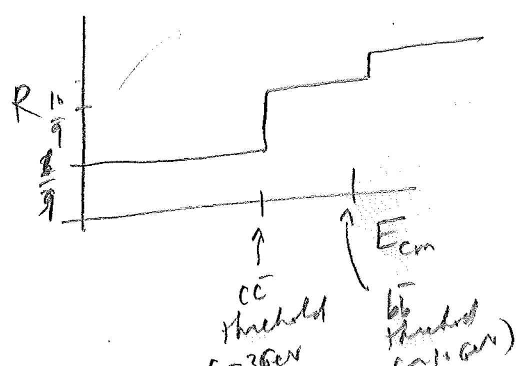
$\sigma \sim \frac{1}{9}\alpha^2$



$\sigma \sim \frac{1}{9}\alpha^2$

$e^+e^- \to$ hadrons $\longrightarrow \sigma \sim \frac{6}{9}\alpha^2$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{2}{3}$$



[show R]

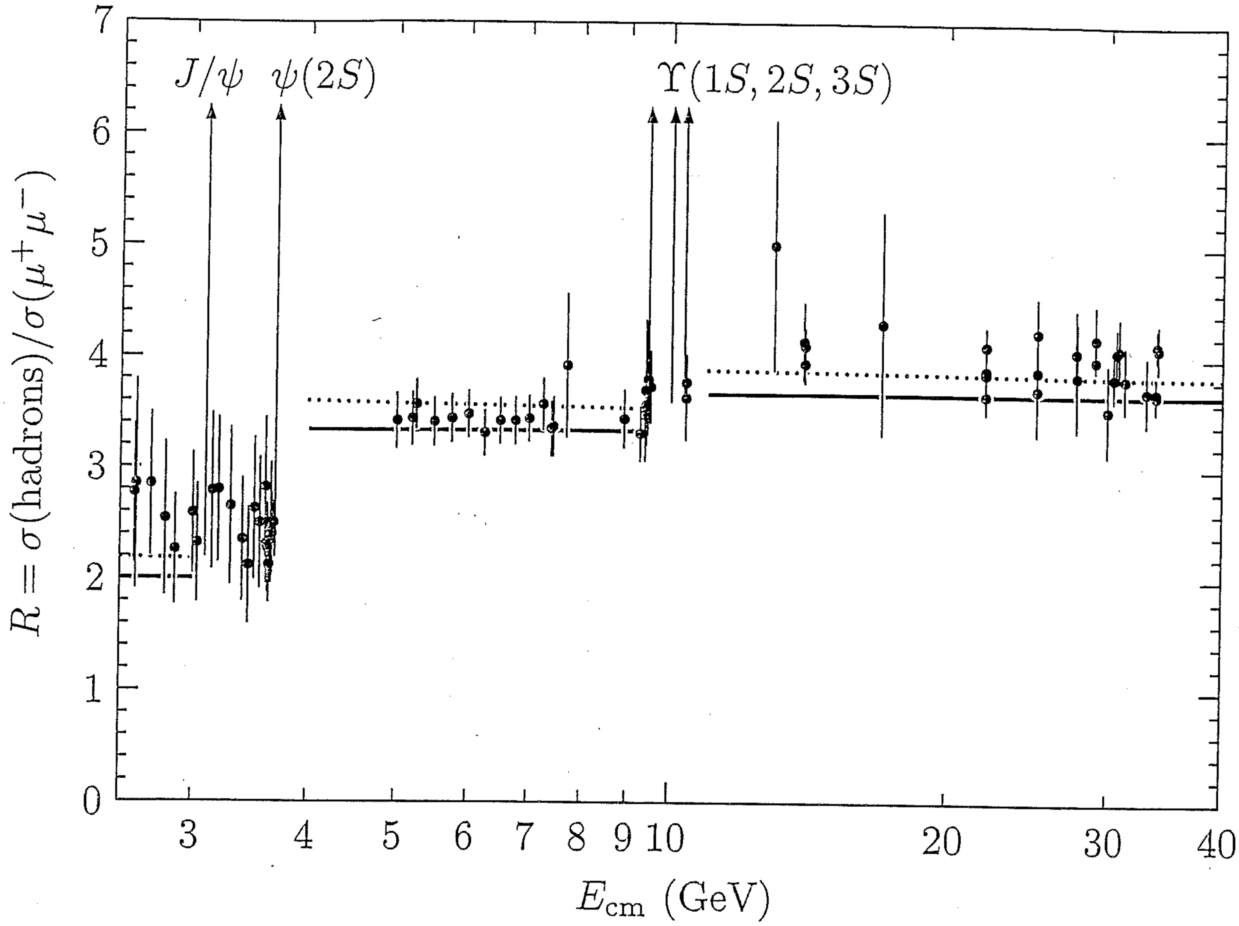


Figure 5.3. Experimental measurements of the total cross section for the reaction $e^+e^- \to$ hadrons, from the data compilation of M. Swartz, *Phys. Rev.* D (to appear). Complete references to the various experiments are given there. The measurements are compared to theoretical predictions from Quantum Chromodynamics, as explained in the text. The solid line is the simple prediction (5.16).

Peskin