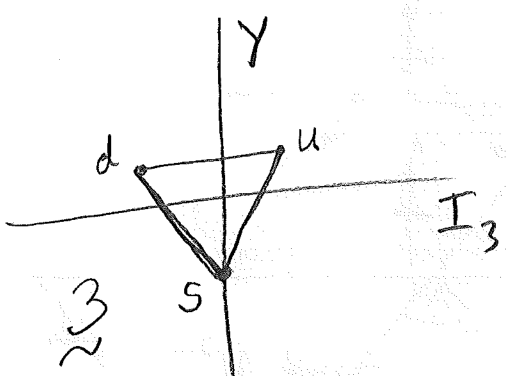
Quark model

[Gell-Mann / Ne'eman recognized the baryons + ~~mesons~~ as filling out representations of SU(3) ($\underset{\sim}{10}$, $\underset{\sim}{8}$) be built out of simpler representations ($\underset{\sim}{3}$).
Is this merely a mathematical construct, or does the fund. rep correspond to something physical? ie are baryons built out of 3 more fundamental particles?]

Gell-Mann + Zweig postulated existence of entities belonging to the fundamental rep of SU(3) flavor

Zweig: "aces"
Gell-Mann: "quarks" → [Finnegan's wake]



	I_3	Y	A	S	Q
u	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$2/3$
d	$-\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$-1/3$
s	0	$-\frac{2}{3}$	$\frac{1}{3}$	-1	$-1/3$

Since baryons are built of 3 qks, quarks have $A = \frac{1}{3}$

$$Y = A + S$$

$$Q = I_3 + \frac{1}{2} Y$$

[non observation of qks, + their fractional charges led Gell Mann to be tentative about their existence]
[illegible] scatt confirmed charges

FINNEGANS WAKE

James Joyce

New York: The Viking Press

1939

riverrun, past Eve and Adam's, from swerve of shore to bend of bay, brings us by a commodius vicus of recirculation back to Howth Castle and Environs.

Sir Tristram, violer d'amores, fr'over the short sea, had passencore rearrived from North Armorica on this side the scraggy isthmus of Europe Minor to wielderfight his penisolate war: nor had topsawyer's rocks by the stream Oconee exaggerated themselse to Laurens County's gorgios while they went doublin their mumper all the time: nor avoice from afire bellowsed mishe mishe to tauftauf thuartpeatrick: not yet, though venissoon after, had a kidscad buttended a bland old isaac: not yet, though all's fair in vanessy, were sosie sesthers wroth with twone nathandjoe. Rot a peck of pa's malt had Jhem or Shen brewed by arclight and rory end to the regginbrow was to be seen ringsome on the aquaface.

The fall (bababadalgharaghtakamminarronnkonnbronntonnerronntuonnthunntrovarrhounawnskawntoohoohoordenenthurnuk!) of a once wallstrait oldparr is retaled early in bed and later on life down through all christian minstrelsy. The great fall of the offwall entailed at such short notice the pftjschute of Finnegan, erse solid man, that the humptyhillhead of humself prumptly sends an unquiring one well to the west in quest of his tumptytumtoes: and their upturnpikepointandplace is at the knock out in the park where oranges have been laid to rust upon the green since devlinsfirst loved livvy.

sad and weary I go back to you, my cold father, my cold mad father, my cold mad feary father, till the near sight of the mere size of him, the moyles and moyles of it, moananoaning, makes me seasilt saltsick and I rush, my only, into your arms. I see them rising! Save me from those therrble prongs! Two more. Onetwo moremens more. So. Avelaval. My leaves have drifted from me. All. But one clings still. I'll bear it on me. To remind me of. Lff! So soft this morning ours. Yes. Carry me along, taddy, like you done through the toy fair. If I seen him bearing down on me now under whitespread wings like he'd come from Arkangels, I sink I'd die down over his feet, humbly dumbly, only to washup. Yes, tid. There's where. First. We pass through grass behush the bush to. Whish! A gull. Gulls. Far calls. Coming, far! End here. Us then. Finn, again! Take. Bussoftlhee, mememormee! Till thousendsthee. Lps. The keys to. Given! A way a lone a last a loved a long the

PARIS,
1922-1939.

— Three quarks for Muster Mark!
Sure he hasn't got much of a bark
And sure any he has it's all beside the mark.
But O, Wreneagle Almighty, wouldn't un be a sky of a lark
To see that old buzzard whooping about for uns shirt in the dark
And he hunting round for uns speckled trousers around by Palmer-
stown Park?
Hohohoho, moulty Mark!
You're the rummest old rooster ever flopped out of a Noah's ark
And you think you're cock of the wark.
Fowls, up! Tristy's the spry young spark
That'll tread her and wed her and bed her and red her
Without ever winking the tail of a feather
And that's how that chap's going to make his money and mark!

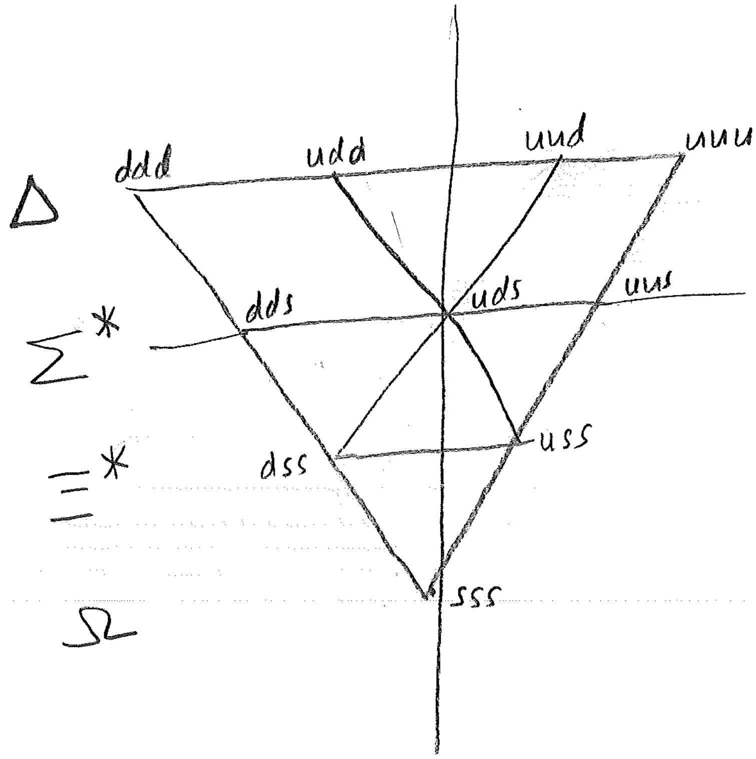
Overhoved, shrillgleescreaming. That song sang seaswans. The winging ones. Seahawk, seagull, curlew and plover, kestrel and capercallzie. All the birds of the sea they trolled out rightbold when they smacked the big kuss of Trustan with Usolde.

And there they were too, when it was dark, whilest the wild-caps was circling, as slow their ship, the winds aslight, upborne the fates, the wardorse moved, by courtesy of Mr Deaubaleau Downbellow Kaempersally, listening in, as hard as they could, in Dubbeldorp, the donker, by the tourneyold of the wattarfalls, with their vuoxens and they kemin in so hattajocky (only a

Baryons are built of 3 quarks

$3 \times 3 \times 3 = 10 + 8 + 8 + 1$

Decuplet



$S = 0$

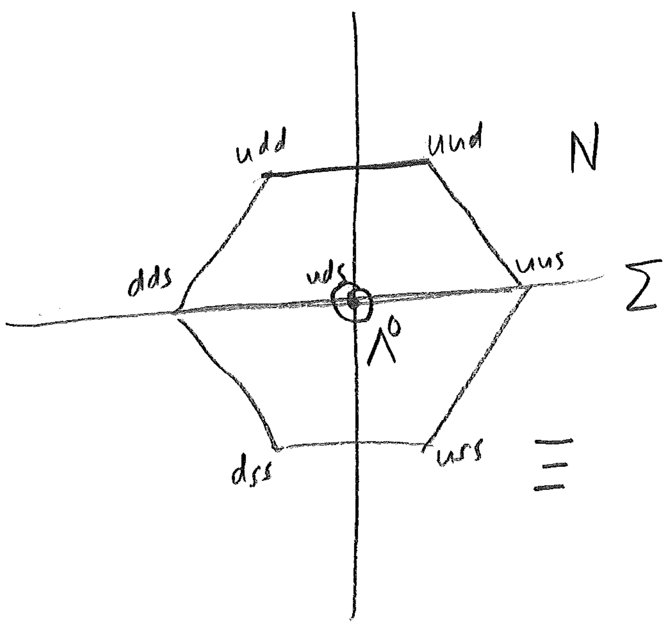
$S = -1$

$S = -2$

$S = -3$

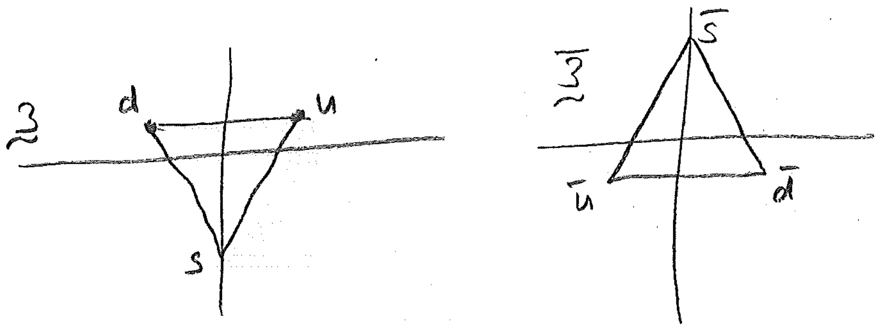
Increasing mass due to $m_s >> m_u, m_d$

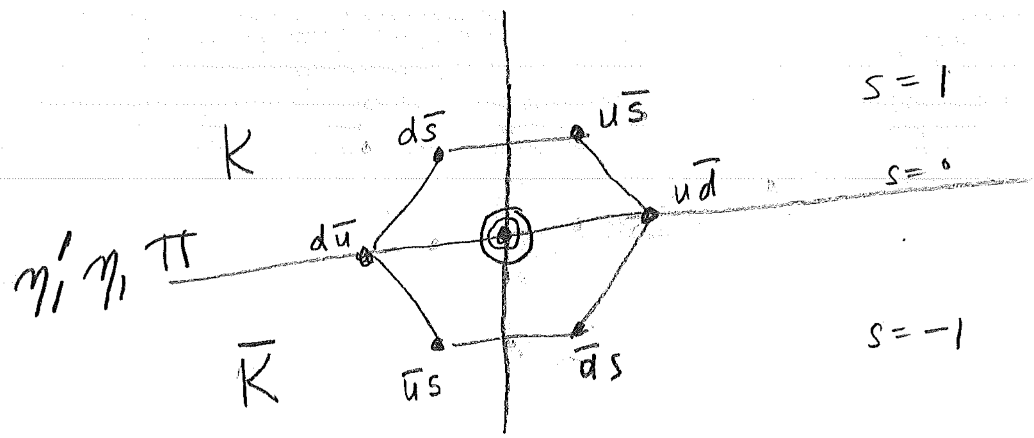
$m_s \sim 150$ MeV heavier than m_u, m_d



What about the 1?

mesons are built from quark + antiquark





Quarks: $A = \frac{1}{3}$

Antiquarks $A = -\frac{1}{3}$

$\Rightarrow$ Baryons $A = 1$

$\Rightarrow$ mesons $A = 0$

Baryon # cons. is just <u>quark #</u> conservation

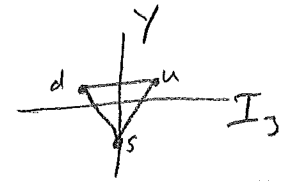
Standard model conserves # qks + # leptons

Weak interactions allows qks to change generations

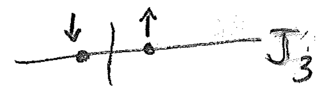
In standard model, leptons don't change generations

[Both Δ^+ and p consist of uud. What's the difference? $J=\frac{3}{2}$ vs $\frac{1}{2}$]

Quarks belong to fund. rep $\underset{\sim}{3}$ of $SU(3)_{flavor}$



Quarks have $J=\frac{1}{2}$ i.e.
belong to fund rep $\underset{\sim}{2}$ of $SU(2)_{spin}$



Quarks belong to $(\underset{\sim}{3}, \underset{\sim}{2})$ of $SU(3)_{flavor} \times SU(2)_{spin}$

Baryon spin $\quad 2\otimes 2\times 2 = (3_S \oplus 1_A)\otimes 2 = 4_S \oplus 2_S \oplus 2_A$

totally symm $\quad \underset{\sim}{4} = \begin{cases} \uparrow\uparrow\uparrow & m=3/2 \\ \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) & m=1/2 \\ \vdots & \vdots \end{cases}$ Baryon Decuplet

symm in 1st 2 $\quad 2_S = \left\{ \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \right. \quad m=\frac{1}{2}$

antisymm in 1st 2 $\quad 2_A = \left\{ \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \right. \quad m=\frac{1}{2}$

Baryon octet

meson spin $\quad 2\otimes 2 = 3\oplus 1$

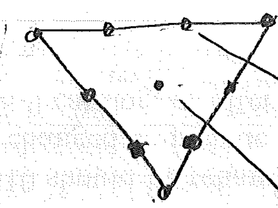
$3 = \begin{cases} \uparrow\uparrow \\ \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ \downarrow\downarrow \end{cases}$ Vector mesons

$1 = \left\{ \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \right.$ Scalar mesons

SU(3) representation of baryons

$$\underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \underset{\sim}{3} = (\underset{\sim}{6}_S \otimes \underset{\sim}{\bar{3}}_A) \otimes \underset{\sim}{3}$$

$$= 10 \oplus 8_S \oplus 8_A \oplus 1_A$$

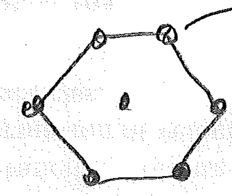
totally sym $\underset{\sim}{10} =$ 

$\Delta^{++} = uuu$

$\Delta^{+} = \frac{1}{\sqrt{3}}(uud + udu + duu)$

$\vdots$

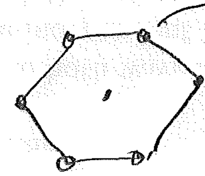
$\Sigma^{0*} = \frac{1}{\sqrt{6}}(uds + usd + dus + dsu + sud + sdu)$

sym in 1st 2 qks $\underset{\sim}{8}_S =$ 

$\frac{1}{\sqrt{6}}(2\,uud - udu - duu)$

[which is the proton? complicated]

$\frac{1}{\sqrt{2}}(udu - duu)$

anti in 1st 2 qks $\underset{\sim}{8}_A =$ 

totally anti 1_A

$\frac{1}{\sqrt{6}}(uds - usd + dsu - dus + sud - sdu)$

[why don't we have this for spin]

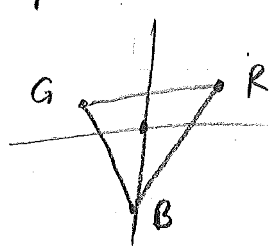
Problem of baryon decuplet $\underset{\sim}{10}$ which has $S = 3/2$
Symmetric in both spin and in flavor
Violate Pauli exclusion

eg Δ^{++} : $(uuu)(\uparrow\uparrow\uparrow) = u\uparrow u\uparrow u\uparrow$ is sym'c under exchange of 2 ptcles.

$\Longrightarrow (u_R\uparrow\, u_G\uparrow\, u_B\uparrow \pm \text{perms})$

Solution: introduce a new quantum number! color
each quark carries one of 3 colors (R G B)
in addition to one of 3 flavors (u d s)
" " " " " " 2 spins ($\uparrow$ or $\downarrow$).

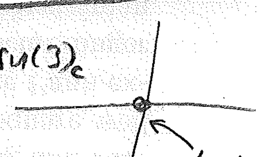
Introduce a new group $SU(3)_{color}$ [O.W. Greenberg 1964]



quarks transform in $\underset{\sim}{3}$ of $SU(3)_c$

Baryons transform in $\underset{\sim}{3}\times\underset{\sim}{3}\times\underset{\sim}{3} = \underset{\sim}{10} + \underset{\sim}{8} + \underset{\sim}{8} + \underset{\sim}{1}$

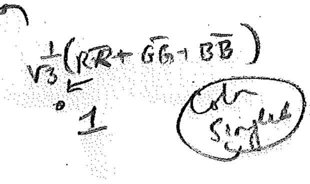
Confinement postulate: observable particle ~~must be color neutral~~
$\Longrightarrow$ must transform in the $\underset{\sim}{1}$ of $SU(3)_c$
(we call this "color neutral") Singlet



$$\underset{\sim}{1} = \frac{1}{\sqrt{6}}(RGB - RBG + GBR - GRB + BRG - BGR)$$

Note: $\underset{\sim}{1}$ is completely antisymmetric in color

Mesons: color $\underset{\sim}{3}\otimes\underset{\sim}{\bar{3}} = \underset{\sim}{8}\oplus 1$



	$SU(3)_f$	$SU(2)_{spin}$	$SU(3)_c$
quarks	$(\underset{\sim}{3}$,	$\underset{\sim}{2}$,	$\underset{\sim}{3})$
baryon decuplet	$(\underset{\sim}{10}_S$,	$\underset{\sim}{4}_S$,	$\underset{\sim}{1}_A)$

eg Δ^{++} w/ $m = \frac{3}{2}$: $(uuu)(\uparrow\uparrow\uparrow) \frac{1}{\sqrt{6}}(RGB - RBG + \ldots)$

$$= \frac{1}{\sqrt{6}} (u_R\uparrow \, u_G\uparrow \, u_B\uparrow - u_R\uparrow \, u_B\uparrow \, u_G\uparrow + \ldots)$$

w/ introduction of color, baryon decuplet becomes totally antisymmetric (obeys Pauli exclusion)

baryon octet? $(\underset{\sim}{8}_S$ or $\underset{\sim}{8}_A$, $\underset{\sim}{2}_S$ or $\underset{\sim}{2}_A$, $\underset{\sim}{1}_A)$

$(8_S, 2_S, 1_A)$ antisymm'c in 1st 2 entries

$(8_A, 2_A, 1_A)$ " " "

Need find linear comb. that is antisymm'c in all 3 entries.

The flavor and spin wavefunctions

$$8_s = \frac{1}{\sqrt{6}}(2uud - duu - udu) \quad \text{of } SU(3)_{\text{flavor}}$$
$$2_s = \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow) \quad \text{of } SU(2)_{\text{spin}}$$

are both symmetric under exchange of the first two entries. Therefore,

$$\begin{aligned}(8_s, 2_s) = \frac{1}{6}(&4u\uparrow u\uparrow d\downarrow - 2d\uparrow u\uparrow u\downarrow - 2u\uparrow d\uparrow u\downarrow \\ &-2u\downarrow u\uparrow d\uparrow + d\downarrow u\uparrow u\uparrow + u\downarrow d\uparrow u\uparrow \\ &-2u\uparrow u\downarrow d\uparrow + d\uparrow u\downarrow u\uparrow + u\uparrow d\downarrow u\uparrow)\end{aligned}$$

is symmetric under exchange of the first two entries. The flavor and spin wavefunctions

$$8_a = \frac{1}{\sqrt{2}}(udu - duu) \quad \text{of } SU(3)_{\text{flavor}}$$
$$2_a = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \quad \text{of } SU(2)_{\text{spin}}$$

are both antisymmetric under exchange of the first two entries. Therefore,

$$(8_a, 2_a) = \frac{1}{2}(u\uparrow d\downarrow u\uparrow - d\uparrow u\downarrow u\uparrow - u\downarrow d\uparrow u\uparrow + d\downarrow u\uparrow u\uparrow)$$

is symmetric under exchange of the first two entries. The linear combination

$$\begin{aligned}\frac{1}{\sqrt{2}}\left[(8_s, 2_s) + (8_a, 2_a)\right] = \frac{1}{6\sqrt{2}}(&4u\uparrow u\uparrow d\downarrow - 2d\uparrow u\uparrow u\downarrow - 2u\uparrow d\uparrow u\downarrow \\ &-2u\downarrow u\uparrow d\uparrow + 4d\downarrow u\uparrow u\uparrow - 2u\downarrow d\uparrow u\uparrow \\ &-2u\uparrow u\downarrow d\uparrow - 2d\uparrow u\downarrow u\uparrow + 4u\uparrow d\downarrow u\uparrow) \\ = \frac{\sqrt{2}}{3}&(u\uparrow u\uparrow d\downarrow + \text{cyclic permutations}) \\ -\frac{1}{3\sqrt{2}}&(d\uparrow u\uparrow u\downarrow + \text{all permutations})\end{aligned}$$

is symmetric under exchange of *any* two entries. Finally,

$$1_a = \frac{1}{\sqrt{6}}(RGB - GRB + GBR - BGR + BRG - RBG) \quad \text{of } SU(3)_{\text{color}}$$

is antisymmetric under exchange of any two entries. Therefore, the complete wavefunction

$$\frac{1}{\sqrt{2}}\left[(8_s, 2_s, 1_a) + (8_a, 2_a, 1_a)\right] \quad \text{of } SU(3)_{\text{flavor}} \times SU(2)_{\text{spin}} \times SU(3)_{\text{color}}$$

is antisymmetric under exchange of any two entries, thus obeying the Pauli exclusion principle for fermions.

PHYSICS 280, Spring 2005 *Wavefunction for a spin-up proton in the quark model*

2005

The flavor and spin wavefunctions

$$8_s = \frac{1}{\sqrt{6}}(duu + udu - 2uud) \qquad \text{of } \ SU(3)_{\text{flavor}}$$

$$2_s = \frac{1}{\sqrt{6}}(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow - 2\uparrow\uparrow\downarrow) \qquad \text{of } SU(2)_{\text{spin}}$$

are both symmetric under exchange of the first two entries. Therefore,

$$\begin{aligned}(8_s, 2_s) = \frac{1}{6}(&d\downarrow u\uparrow u\uparrow + u\downarrow d\uparrow u\uparrow - 2u\downarrow u\uparrow d\uparrow \\ &+ d\uparrow u\downarrow u\uparrow + u\uparrow d\downarrow u\uparrow - 2u\uparrow u\downarrow d\uparrow \\ &- 2d\uparrow u\uparrow u\downarrow - 2u\uparrow d\uparrow u\downarrow + 4u\uparrow u\uparrow d\downarrow\end{aligned}$$

is symmetric under exchange of the first two entries. The flavor and spin wavefunctions

$$8_a = \frac{1}{\sqrt{2}}(duu - udu) \qquad \text{of } \ SU(3)_{\text{flavor}}$$

$$2_a = \frac{1}{\sqrt{2}}(\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow) \qquad \text{of } SU(2)_{\text{spin}}$$

are both antisymmetric under exchange of the first two entries. Therefore,

$$(8_a, 2_a) = \frac{1}{2}(d\downarrow u\uparrow u\uparrow - u\downarrow d\uparrow u\uparrow - d\uparrow u\downarrow u\uparrow + u\uparrow d\downarrow u\uparrow)$$

is symmetric under exchange of the first two entries. The linear combination

$$\begin{aligned}\frac{1}{\sqrt{2}}[(8_s, 2_s) + (8_a, 2_a)] = \frac{1}{6\sqrt{2}}(&4d\downarrow u\uparrow u\uparrow - 2u\downarrow d\uparrow u\uparrow - 2u\downarrow u\uparrow d\uparrow \\ &- 2d\uparrow u\downarrow u\uparrow + 4u\uparrow d\downarrow u\uparrow - 2u\uparrow u\downarrow d\uparrow \\ &- 2d\uparrow u\uparrow u\downarrow - 2u\uparrow d\uparrow u\downarrow + 4u\uparrow u\uparrow d\downarrow \\ = \frac{\sqrt{2}}{3}(&d\downarrow u\uparrow u\uparrow + \text{cyclic permutations}) \\ -\frac{1}{3\sqrt{2}}(&u\downarrow d\uparrow u\uparrow + \text{all permutations})\end{aligned}$$

is symmetric under exchange of *any* two entries. Finally,

$$1_a = \frac{1}{\sqrt{6}}(RGB - GRB + GBR - BGR + BRG - RBG) \qquad \text{of } SU(3)_{\text{color}}$$

is antisymmetric under exchange of any two entries. Therefore, the complete wavefunction

$$\frac{1}{\sqrt{2}}[(8_s, 2_s, 1_a) + (8_a, 2_a, 1_a)] \qquad \text{of } SU(3)_{\text{flavor}} \times SU(2)_{\text{spin}} \times SU(3)_{\text{color}}$$

is antisymmetric under exchange of any two entries, thus obeying the Pauli exclusion principle for fermions.

$$3 \otimes 3 \otimes 3 = \underset{\text{decuplet}}{10_S} \oplus \underbrace{8_S \oplus 8_A}_{\text{octet}} \oplus \underset{\text{what about this?}}{1_A}$$

baryon singlet? $\left(\underset{\sim}{1}_A, \overset{\text{flavor}}{?}, \overset{\text{color}}{\underset{\sim}{1}_A} \right)$

would need to be totally antisymmetric $\Rightarrow$ not possible for 3 spin $\frac{1}{2}$ particles (can't all be different)

[Not enough info given to determine octet wavefunctions for Λ^0, Σ^0, so don't assign such a problem]

Magnetic moments of baryons

[Recall]
$$\vec{\mu} = g \frac{q}{2m} \vec{J}$$

Dirac eqn predicts $g=2$ for point like spin $\frac{1}{2}$ particles

$$|\mu| = 2 \cdot \frac{q}{2m} \cdot \frac{\hbar}{2} = \frac{q\hbar}{2m}$$

If p + n were elementary then $\begin{cases} \mu_p = \mu_N \\ \mu_n = 0 \end{cases}$ $\quad \mu_N \equiv \frac{e\hbar}{2m_p}$

Experimental values $\begin{cases} \mu_p = 2.79\,\mu_N & [g = 5.58] \\ \mu_n = -1.91\,\mu_N & [g = -3.82] \end{cases}$

Can we explain these?

Proton consisting 3 quarks: Spin-flavor wavefunction for spin up proton

$$\frac{\sqrt{2}}{3}(\underbrace{u\uparrow u\uparrow d\downarrow}_{} + 2 \text{ cyclic}) - \frac{1}{3\sqrt{2}}(\underbrace{d\uparrow u\uparrow u\downarrow}_{} + 5 \text{ perms})$$

$$\vec{\mu}_p = \sum \vec{\mu}_i \longrightarrow 2\mu_u - \mu_d \qquad\qquad \mu_d$$

probability

$$\Rightarrow \mu_p = \left(\frac{2}{9}\right)(2\mu_u - \mu_d) \cdot 3 + \frac{1}{18}(\mu_d) \cdot 6$$
$$= \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$$

Neutron wave function: ($u \leftrightarrow d$ everywhere) $\Rightarrow \mu_n = -\frac{1}{3}\mu_u + \frac{4}{3}\mu_d$

Quarks are pointlike spin $\frac{1}{2}$ particles

$$\mu_u = 2\frac{(\frac{2}{3}e)}{(2m_u)}\frac{\hbar}{2} = \frac{2e\hbar}{3m_u}$$

$$\mu_d = 2\frac{(-\frac{1}{3}e)}{(2m_d)}\frac{\hbar}{2} = -\frac{e\hbar}{6m_d}$$

$$\Rightarrow \mu_p = \frac{4e\hbar}{9m_u} + \frac{e\hbar}{18m_d}$$

$$\mu_n = -\frac{e\hbar}{9m_u} - \frac{2e\hbar}{9m_d}$$

What are the masses of quarks? Not observed.

<u>Naive model</u>: $m_u \approx m_d \approx \frac{1}{3}M_p$.

"constituent masses"

then $$\mu_p = \frac{4e\hbar}{3m_p} + \frac{e\hbar}{6m_p} = \frac{3e\hbar}{2m_p} = 3\mu_N$$

$$\mu_n = -\frac{e\hbar}{3m_p} - \frac{2e\hbar}{3m_p} = -\frac{e\hbar}{m_p} = -2\mu_N$$

[Not too bad!]

[HW: calc Ω^- mag moment]

Table 4.4 Quark masses (MeV/c^2)

Quark flavor	Bare mass	Effective mass
u	2	336
d	5	340
s	95	486
c	1300	1550
b	4200	4730
t	174 000	177 000

Warning: These numbers are somewhat speculative and model dependent [12].

Table 5.5 Magnetic dipole moments of octet baryons

Baryon	Moment	Prediction	Experiment
p	$(\frac{4}{3})\mu_u - (\frac{1}{3})\mu_d$	2.79	2.793
n	$(\frac{4}{3})\mu_d - (\frac{1}{3})\mu_u$	−1.86	−1.913
Λ	μ_s	−0.58	−0.613
Σ^+	$(\frac{4}{3})\mu_u - (\frac{1}{3})\mu_s$	2.68	2.458
Σ^0	$(\frac{2}{3})(\mu_u + \mu_d) - (\frac{1}{3})\mu_s$	0.82	?
Σ^-	$(\frac{4}{3})\mu_d - (\frac{1}{3})\mu_s$	−1.05	−1.160
Ξ^0	$(\frac{4}{3})\mu_s - (\frac{1}{3})\mu_u$	−1.40	−1.250
Ξ^-	$(\frac{4}{3})\mu_s - (\frac{1}{3})\mu_d$	−0.47	−0.651

The numerical values are given as multiples of the nuclear magneton, $e\hbar/2m_pc$. *Source*: *Particle Physics Booklet* (2006).

Table 5.3 Pseudoscalar and vector meson masses. (MeV/c^2)

Meson	Calculated	Observed
π	139	138
K	487	496
η	561	548
ρ	775	776
ω	775	783
K^*	892	894
ϕ	1031	1020

Table 5.6 Baryon octet and decuplet masses. (MeV/c^2)

Baryon	Calculated	Observed
N	939	939
Λ	1114	1116
Σ	1179	1193
Ξ	1327	1318
Δ	1239	1232
Σ^*	1381	1385
Ξ^*	1529	1533
Ω	1682	1672