Representations of Lie algebras

[an algebra is a vector space of objects w/ a product. Lie algebra is a certain kind of algebra related to a Lie group. This is a whole field of mathematics & I can't give you a course on group theory, so we'll just jump in w/ a specific example]

Def: the Lie algebra $SU(N)$ consists of all complex traceless hermitian $N \times N$ matrices T.

[complex = matrix w/ complex entries]

traceless = sum of diagonal entries vanishes: $Tr(T) = 0$

hermitian: $T^\dagger = T$

$T^\dagger$ = hermitian conjugate of T (transpose complex conjugate)

The Lie algebra product is defined as the commutator of matrices $[T_1, T_2] \equiv T_1 T_2 - T_2 T_1$

If T_1, T_2 are hermitian & traceless, then so is $i[T_1, T_2]$

<u>SU(2)</u> $T = \begin{pmatrix} z_1 & z_2 \\ z_3 & z_4 \end{pmatrix}$ [8 d.o.f]

$T^\dagger = \begin{pmatrix} z_1^* & z_3^* \\ z_2^* & z_4^* \end{pmatrix}$

[4 d.o.f]

$T^\dagger = T \Rightarrow z_1, z_4 \in \mathbb{R}$

$z_2 = z_3^*, \; z_3 = z_2^*$

[3 d.o.f]

$\mathrm{Tr}(T) = 0 \Rightarrow z_1 + z_4 = 0$

Thus $T = \begin{pmatrix} a & b - ic \\ b + ic & -a \end{pmatrix}$ where $a, b, c \in \mathbb{R}$

$= a \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{\sigma_3} + b \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\sigma_1} + c \underbrace{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}_{\sigma_2}$

Pauli spin matrices

All elements of su(2) Lie algebra are linear combinations of σ_i.
We say σ_i <u>generate</u> the Lie algebra

dim. SU(N) $\equiv$ # of generators

$\Rightarrow$ dim SU(2) = 3

[HW: Write down a set of independent generators for SU(3). What is dim SU(N)?]

Conventionally we define SU(2) <u>generators</u>: $I_i = \frac{1}{2}\sigma_i$ $(i = 1, 2, 3)$

These generators obey $[I_i, I_j] = \sum_{k=1}^{3} i\epsilon_{ijk} I_k$ (SU(2) commutation relations)

$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1, \; \epsilon_{213} = \epsilon_{132} = \epsilon_{321} = -1,$ rest $= 0$.

The subset of generators w/ only diagonal entries
is the Cartan subalgebra (CSA).

rank $SU(N)$ = # of elements of CSA

CSA of $SU(2)$ $\Rightarrow$ $I_3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$

rank $SU(2) = 1$

CSA of $SU(3)$ $\Rightarrow$ $I_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$Y = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}$ "hypercharge"

other choices are possible

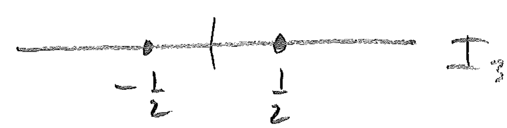
rank $SU(3) = 2$

The fundamental representation of SU(N) is N-dimensional, $\underset{\sim}{N}$

It consists of a multiplet of states whose weights are the diagonal elements of the CSA.

(These states are eigenvectors of the CSA and the weights are the eigenvalues)

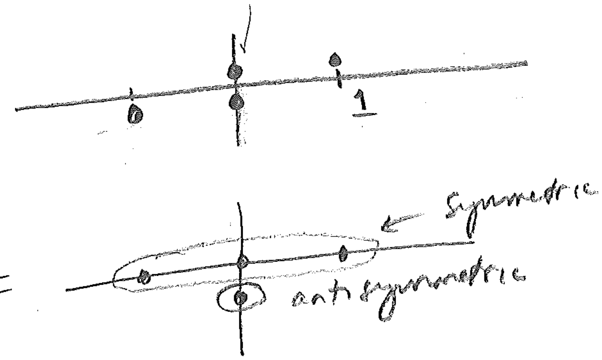
eg SU(2). $I_3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$



This is the (spin-$\frac{1}{2}$) multiplet of spin/isospin!

All other representations of SU(N) can be built from products of the fundamental rep.

eg SU(2): $\underset{\sim}{2} \otimes \underset{\sim}{2} = \underset{\sim}{3}_S \oplus \underset{\sim}{1}_A$



[we've already done all this in addition of angular momentum]

Now let's try SU(3)

$\underset{\sim}{2} \otimes \underset{\sim}{2} \otimes \underset{\sim}{2} = \underset{\sim}{4} \oplus \underset{\sim}{2}_S \oplus \underset{\sim}{2}_A$

completely sym — sym in 1+2 — anti in 1+2

[Now let's try for SU(3)]

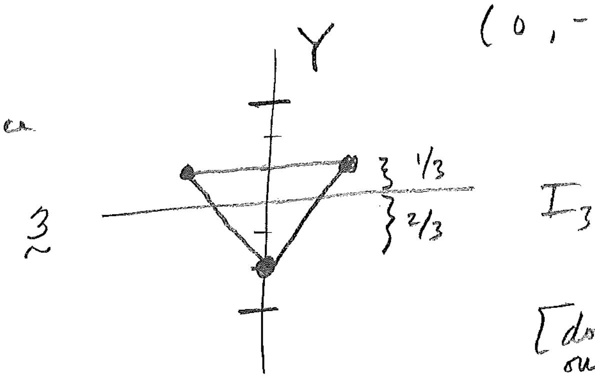
Recall CSA $\quad I_3 = \begin{pmatrix} \frac{1}{2} & & \\ & -\frac{1}{2} & \\ & & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} \frac{1}{3} & & \\ & \frac{1}{3} & \\ & & -\frac{2}{3} \end{pmatrix}$

[Observe SU(2) is a subalgebra of SU(3)]

Fund. rep of SU(3) is $\underset{\sim}{3}$.

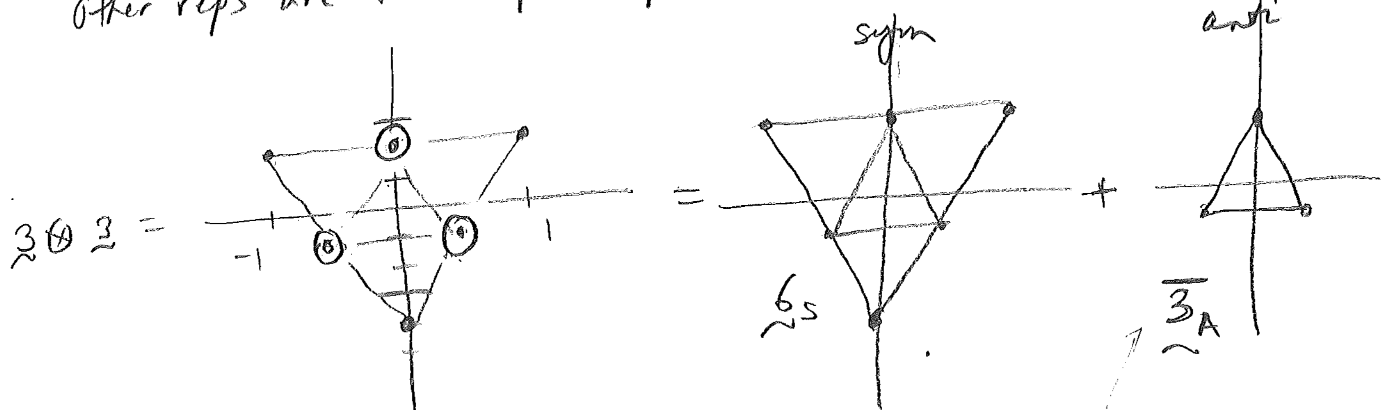
weights of the fund. multiplet are $\quad (\frac{1}{2}, \frac{1}{3})$, $(-\frac{1}{2}, \frac{1}{3})$, $(0, -\frac{2}{3})$

2-dim'l weight space



[doesn't look like our mesons or baryons]

Other reps are built from products

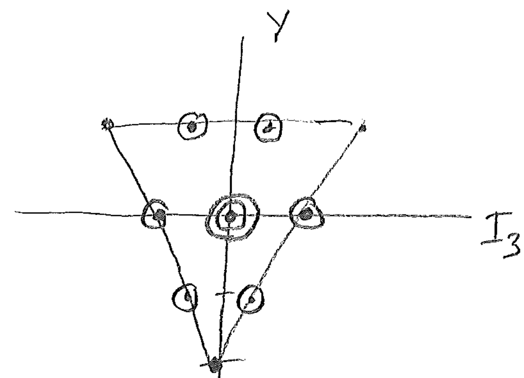


[still doesn't look like mesons or baryons]

[stop writing tildes]

$$3 \otimes 3 = 6_S \oplus \bar{3}_A$$

$6_S \otimes 3$ =



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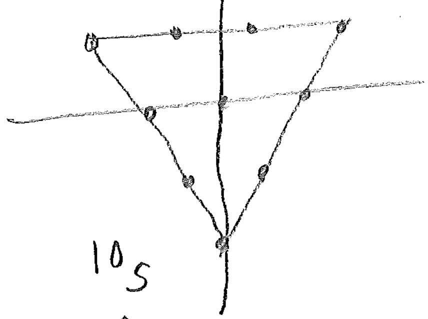

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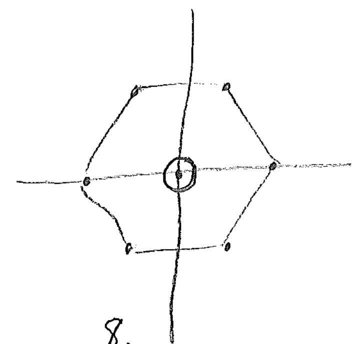


10_S

↑

Completely Symm

8_S

↑

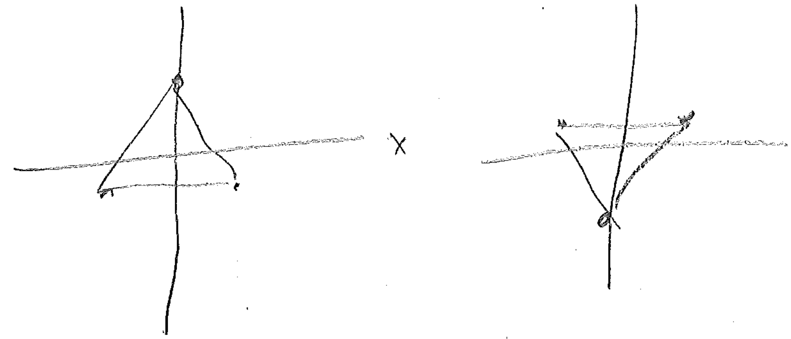
In 1↔2

<u>Baryon decuplet!</u>

<u>Baryon octet</u>

where $Y = S+1$

$\bar{3}_A \times 3$



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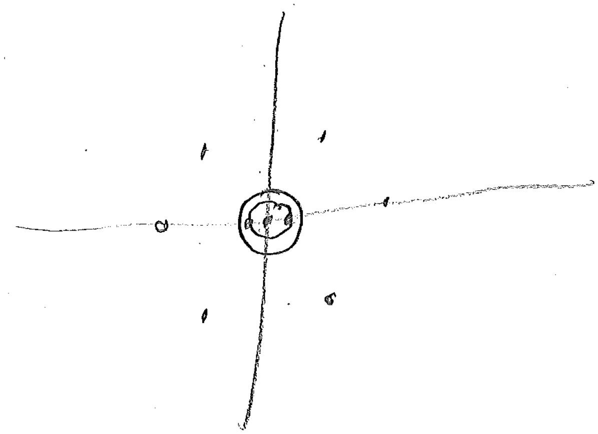

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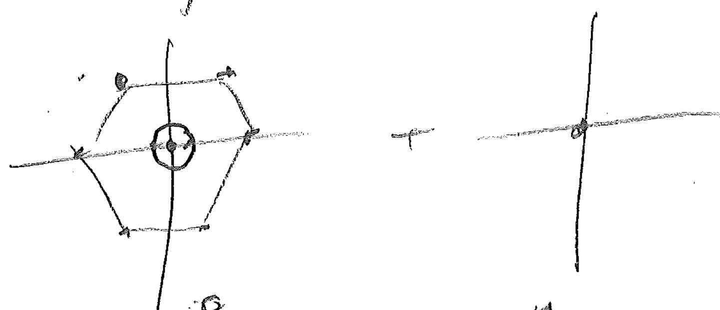

$\underbrace{8_A \quad + \quad 1_A}_{\text{nonet}}$ when $Y = S$

$$3\otimes3\otimes3 = (6\oplus\bar{3})\otimes3 = (6\otimes3) + \bar{3}\otimes3 = 10\oplus8_S\oplus8_A\oplus1$$

Baryons + mesons are ~~[illegible]~~ multiplets of $SU(3)_{\text{flavor}}$ Lie algebra: $\left\{\begin{array}{l}\text{Gell-Mann}\\ \text{Ne'eman}\end{array}\right.$

"eight-fold way"

$Y = S + A$

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The Noble Eightfold Path

1. Right View 2. Right Intention	Wisdom
3. Right Speech 4. Right Action 5. Right Livelihood	Ethical Conduct
6. Right Effort 7. Right Mindfulness 8. Right Concentration	Mental Development

The Noble Eightfold Path describes the way to the end of suffering, as it was laid out by Siddhartha Gautama. It is a practical guideline to ethical and mental development with the goal of freeing the individual from attachments and delusions; and it finally leads to understanding the truth about all things. Together with the Four Noble Truths it constitutes the gist of Buddhism. Great emphasis is put on the practical aspect, because it is only through practice that one can attain a higher level of existence and finally reach Nirvana. The eight aspects of the path are not to be understood as a sequence of single steps, instead they are highly interdependent principles that have to be seen in relationship with each other.

1. Right View

Right view is the beginning and the end of the path, it simply means to see and to understand things as they really are and to realise the Four Noble Truth. As such, right view is the cognitive aspect of wisdom. It means to see things through, to grasp the impermanent and imperfect nature of worldly objects and ideas, and to understand the law of karma and karmic conditioning. Right view is not necessarily an intellectual capacity, just as wisdom is not just a matter of intelligence. Instead, right view is attained, sustained, and enhanced through all capacities of mind. It begins with the intuitive insight that all beings are subject to suffering and it ends with complete understanding of the true nature of all things. Since our view of the world forms our thoughts and our actions, right view yields right thoughts and right actions.

2. Right Intention

While right view refers to the cognitive aspect of wisdom, right intention refers to the volitional aspect, i.e. the kind of mental energy that controls our actions. Right intention can be described best as commitment to ethical and mental self-improvement. Buddha distinguishes three types of right intentions: 1. the intention of renunciation, which means resistance to the pull of desire, 2. the intention of good will, meaning resistance to feelings of anger and aversion, and 3. the intention of harmlessness, meaning not to think or act cruelly, violently, or aggressively, and to develop compassion.

3. Right Speech

Right speech is the first principle of ethical conduct in the eightfold path.

Charles Sommerfield, Murray Gell-Mann and Barton Zwiebach