α-decay

$$ {}^{A+4}_{Z+2}X \longrightarrow {}^{A}_{Z}Y + {}^{4}_{2}He $$

$$ Q = \Delta(X) - \Delta(Y) - \Delta({}^{4}He) $$

Q typically ~ 5 MeV

2-body decay $\Rightarrow$ final state kinetic energies are fixed

[problem] $$ K_\alpha = \frac{m_X + m_Y - m_\alpha}{2m_X} Q $$

$$ \simeq \frac{(A+4) + A - 4}{2(A+4)} Q $$

$$ = \frac{A}{A+4} Q $$

$\Rightarrow$ α gets most of kinetic energy

[$Q \sim 5$ MeV, $m_\alpha c^2 \sim 4$ GeV $\Rightarrow$ non rel]

non rel $\Rightarrow$ $$ \frac{K_\alpha}{K_Y} = \frac{\frac{1}{2} m_\alpha v_\alpha^2}{\frac{1}{2} m_Y v_Y^2} = \frac{m_Y}{m_\alpha}\left(\frac{m_\alpha v_\alpha}{m_Y v_Y}\right)^2 = \frac{m_Y}{m_\alpha} $$

K_α determines range of α in air ($\sim$ cm)

Asimov's neutrinos

Bragg 1904

^{232}Th	2.8cm
^{226}Ra	3.3cm
^{212}Po	8.6cm

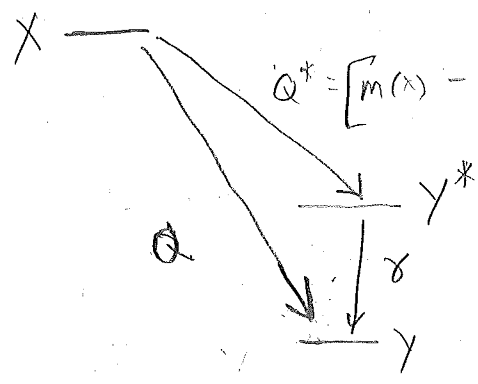
$^{227}Th \rightarrow {}^{223}Ra + {}^{4}He$

$$Q = 6.15 \text{ MeV} \quad \left[\begin{array}{l} \Delta = 25.806 \\ \Delta = 17.235 \\ Q = 6.146 \end{array} \right]$$

$$K_\alpha = \frac{223}{227} Q = 6.04 \text{ MeV}$$

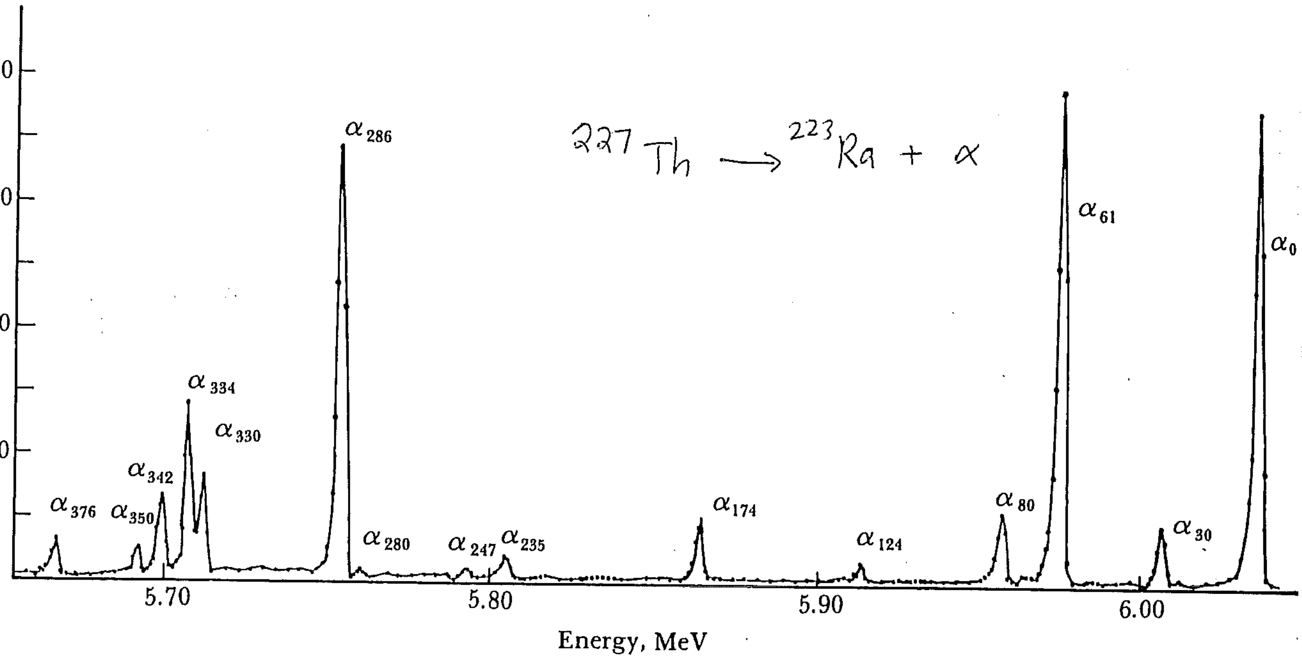
[→ see data]

When α particle has less than expected K,
it is accompanied by γ, whose energy is
the difference



$$Q^* = [m(X) - \underbrace{m(Y^*)}_{\text{mass of excited state}} - m(\alpha)]c^2$$

Tipler, p. 390-1



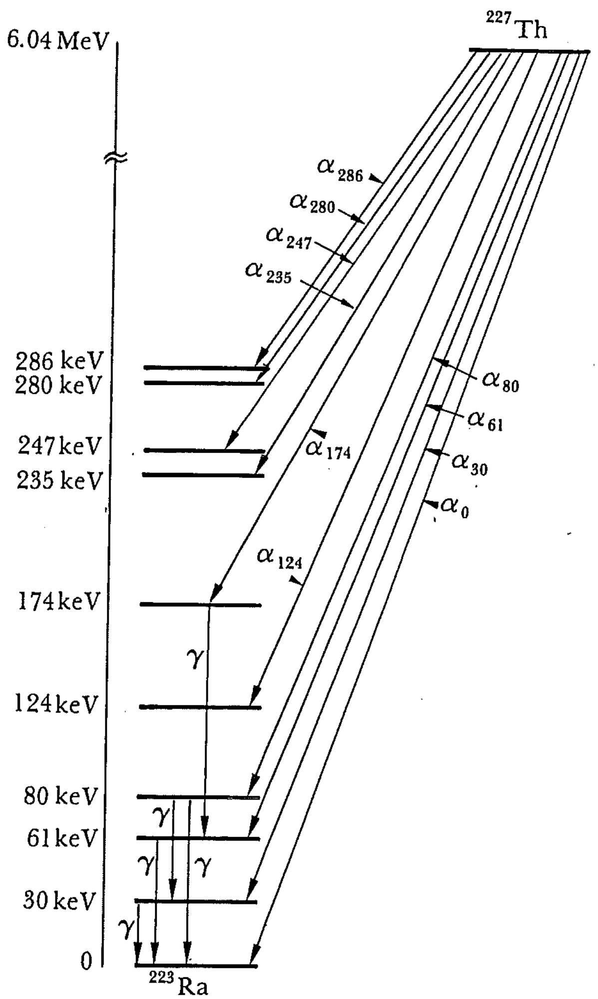


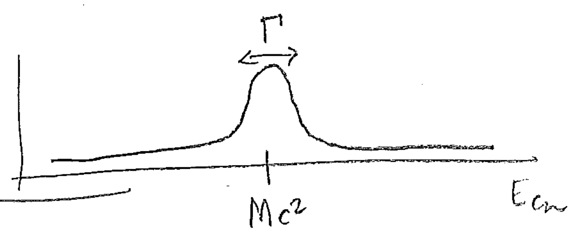
Figure 11-13
Energy levels of ^{223}Ra determined by measurement of α-particle energies from ^{227}Th, as shown in Figure 11-12. Only the lowest-lying levels and some of the γ-ray transitions are shown.

Need to write this.

Explain width of γ-decay lines as due to ΔE ~ ~~fast decays~~ finite lifetime

Heisenberg uncertainty $\Delta E \, \Delta t \geq \frac{\hbar}{2}$

$\Rightarrow$ The energy (mass) of a particle w/ mean life τ is uncertain by $\Delta E \sim \frac{\hbar}{\tau}$. Define the width Γ of an unstable particle as



$$\Gamma = \frac{\hbar}{\tau} = \frac{(6.6 \times 10^{-22}\ \mathrm{MeV \cdot s})}{\tau}$$

~~expt $\Rightarrow \Gamma_Z = 2.5$ GeV~~

~~$\Rightarrow \tau = 3 \times 10^{-25}$ s~~

$$\text{so} \quad \lambda = \frac{1}{\tau} = \frac{\Gamma}{\hbar}$$

		Q	$T_{\frac{1}{2}}$
[6.803]	^{228}U	6.8 MeV	550 s
[6.146]	^{227}Th	6.15 MeV	18.7 days
[4.274]	^{238}U	4.3 MeV	4.5×10^{9} yrs

ratio $\sim 4\times10^{-15}$

$T_{\frac{1}{2}}$ increases rapidly w/ decreasing Q

empirical Geiger-Nuttall rule (1911)

$$\log T_{\frac{1}{2}} = c_1 \frac{Z}{\sqrt{Q}} + c_2$$

explained by QM tunnelling
Gamow (1929)
Gurney + Condon

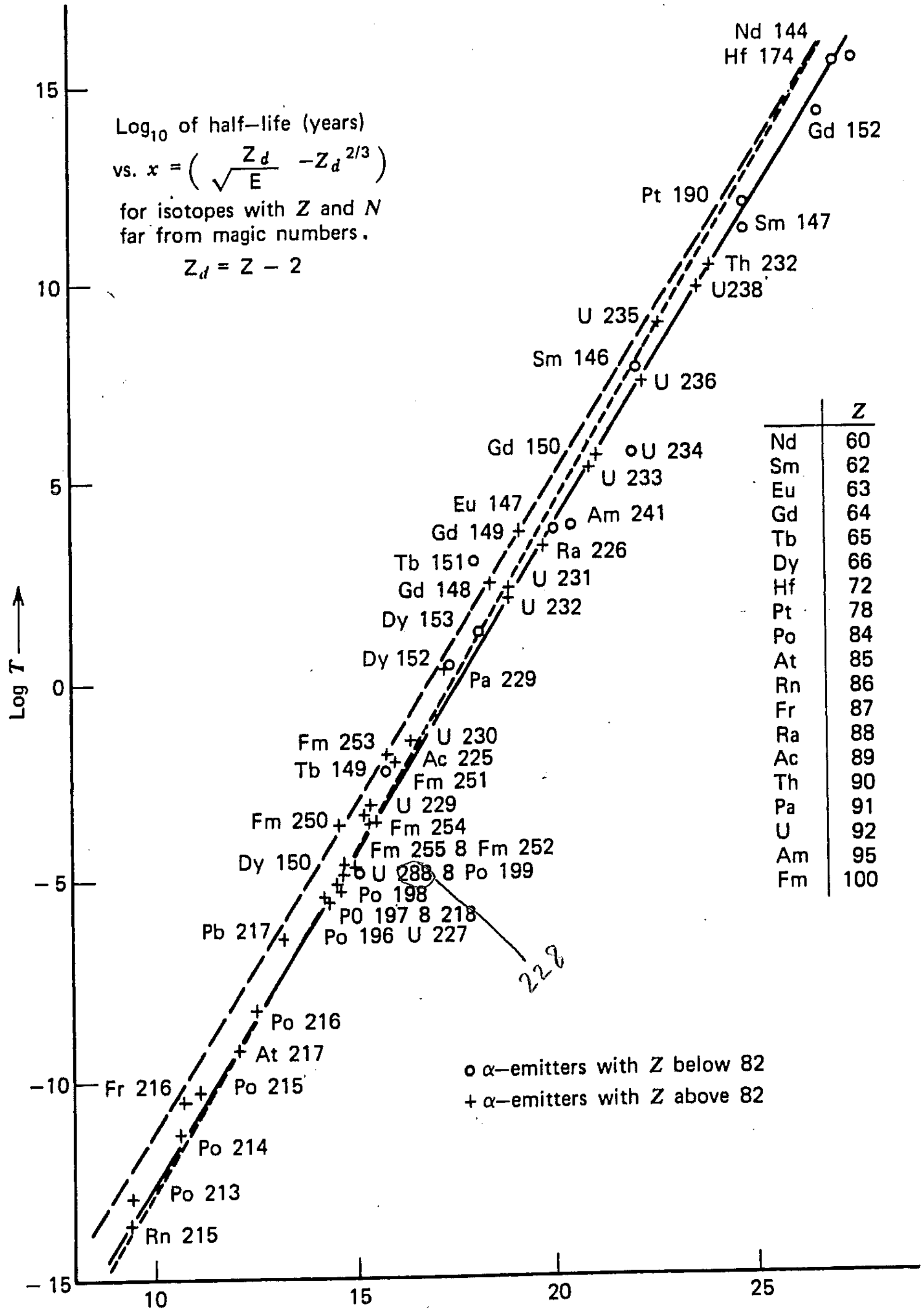


Figure 5-9. Plot of $\log_{10} 1/\tau$ versus $C_2 - C_1 Z_1/\sqrt{E}$ with $C_1 = 1.61$ and a slowly varying $C_2 = 28.9 + 1.6 Z_1^{2/3}$. (From E. K. Hyde, I. Perlman, and G. T. Seaborg, *The Nuclear Properties of the Heavy Elements,* Vol. 1, Prentice-Hall, Englewood Cliffs, N.J. (1964), reprinted by permission.)

$\ln \frac{1}{\tau} \approx -3.71 \frac{Z_1}{\sqrt{E}}$

Gasiorowicz

α-decay lifetime

Recall $N(t) = N_0 e^{-\lambda t}$

decay const $\lambda = \dfrac{\text{probability of decay}}{\text{time}}$

[inherently quantum mechanical ⇒ random, nondeterministic ⇒ can only predict probabilities]

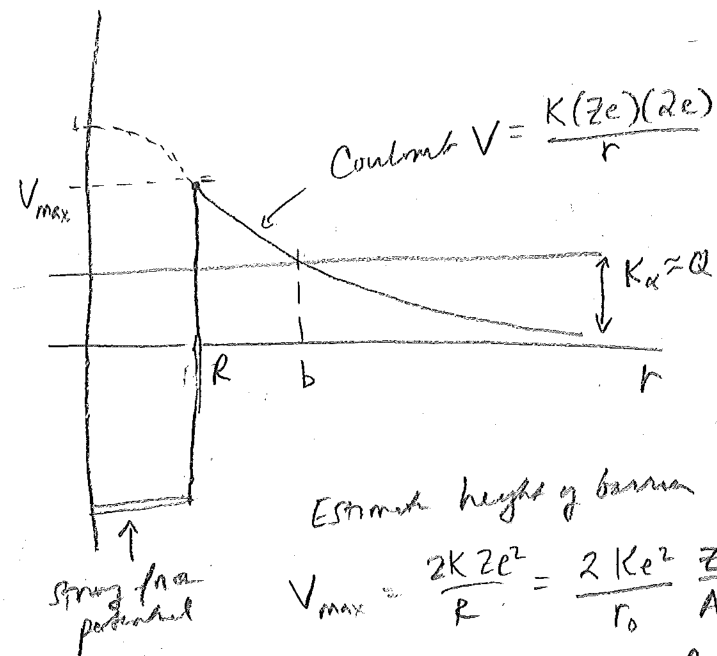
$\sim |\text{tunneling amplitude}|^2$

[λ depends on strength of interaction;
α-decay occurs via strong interaction
so why is α-decay so slow?]

α particle has to <u>tunnel</u> through potential barrier to escape the nucleus.

~~Gamow 1928~~
~~Gurney & Condon~~

[consider reverse process : $\alpha + {}^{A}Y \rightarrow {}^{A+4}X$]



[Saw this before in semi-empirical]

Estimate height of barrier V_{max}

$$V_{max} = \frac{2KZe^2}{R} = \frac{2Ke^2}{r_0}\frac{Z}{A^{1/3}} = 2\left(\frac{Ke^2}{\hbar c}\right)\frac{\hbar c}{r_0}\frac{Z}{A^{1/3}} = 2\alpha\frac{\hbar c}{r_0}\frac{Z}{A^{1/3}}$$

$$= 2\left(\frac{1}{137}\right)\left(\frac{197\,\text{MeV fm}}{1.2\,\text{fm}}\right)\frac{Z}{A^{1/3}} = (2.4\,\text{MeV})\frac{Z}{A^{1/3}}$$

For $A = 234$, $Z = 90$, $V_{max} \sim 35\,\text{MeV}$ [so $V_{max} >> Q$]

[actually $V_{max} \sim 25\,\text{MeV}$]

[from 2140] Tunneling probability

$$P = e^{-\frac{2\sqrt{2m}}{\hbar}\int_R^b dr\,\sqrt{V(r)-E}}$$

[HW: calculate this to find]

$$\text{For } R \rightarrow 0, \quad P \approx e^{-4(\text{MeV}^{\frac{1}{2}})\frac{Z}{\sqrt{Q}}}$$

[HW in problem, calc exponent to be 3.96
Geiger-Nuttall ⇒ + 3.68 empirically]

Now $\lambda \propto P$

$\tau \propto \frac{1}{P}$

$\ln \tau = -\ln P + \text{const}$

$\ln T_{\frac{1}{2}} = -\ln P + \text{const}$

$\approx +4\,(\text{MeV}^{\frac{1}{2}})\frac{Z}{\sqrt{Q}} + \text{const}$ (Geiger-Nuttall rule!)

2?4