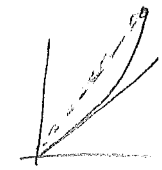
Nuclear fission

[Heaviest naturally occurring element ⇒ on verge of instability since there are no heavier ones]

$^{238}_{92}U \longrightarrow X + Y +$ (several additional particles)

Q? additional particles are
(1) p
(2) n
(3) equal #s of p + n



$\frac{B}{A}$ for U ~ 7.5 MeV

$\frac{B}{A}$ for X, Y ~ 8.4 MeV

difference ~ 1 MeV per nucleon ⇒ about 200 MeV released in each fission. → 80% into kinetic energy of X and Y

chemical reaction ⇒ < 1 eV released for each reaction

factor of ~ 10,000,000 [xkcd cartoon "log scale"]

[matter-antimatter ~ 1 GeV per nucleon!]

[Put Ohanian in Gedankenlab]

$$^{238}_{92}U \longrightarrow {}^{143}_{56}Ba + {}^{93}_{36}Kr + 2n$$

[really Mc^2]

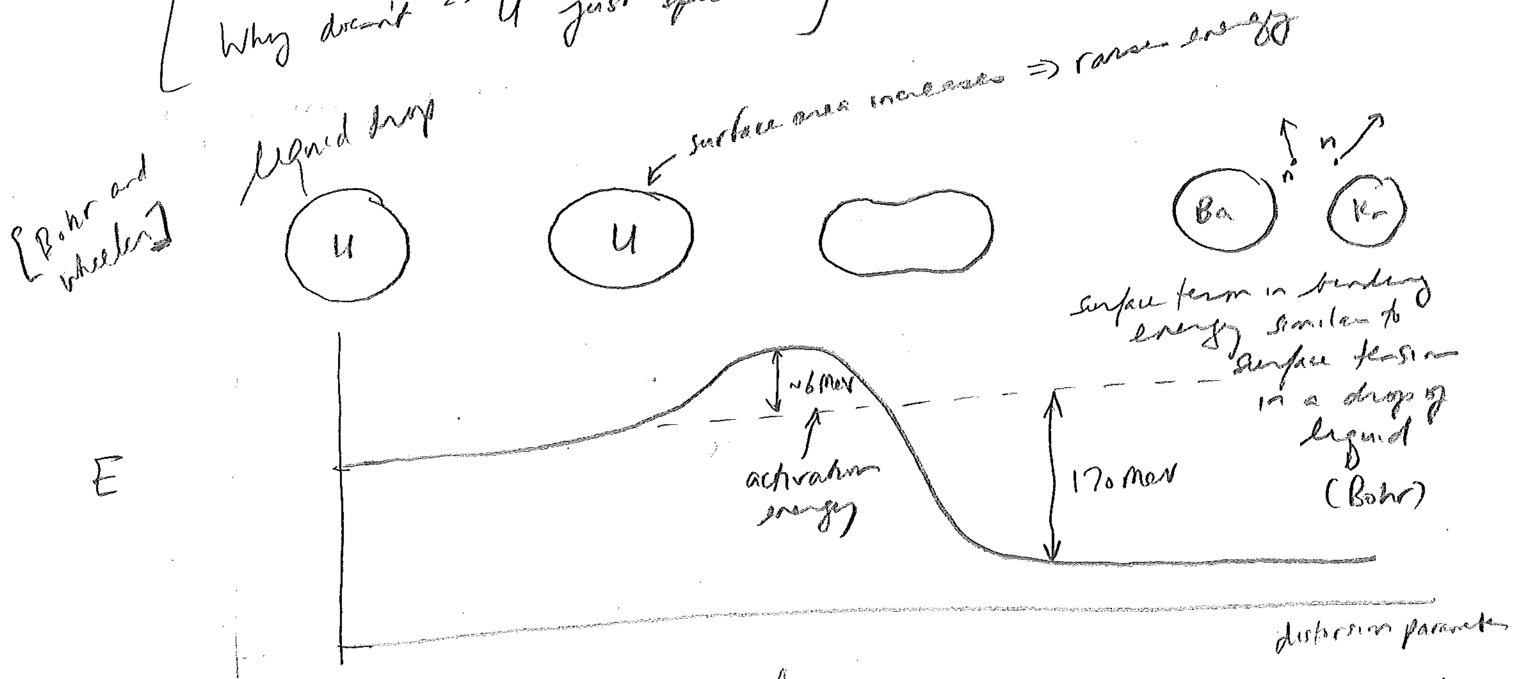
$$Q = M(^{238}U) - M(^{143}Ba) - M(^{93}Kr) - 2m(n)$$

$$= \underset{\sim}{238\,u} + \Delta(^{238}U) - \underset{\sim}{143\,u} - \Delta(^{143}Ba) - \underset{\sim}{93\,u} - \Delta(^{93}Kr) - \underset{\sim}{2u} - 2\Delta(n)$$

$$= \Delta(^{238}U) - \Delta(^{143}Ba) - \Delta(^{93}Kr) - 2\Delta(n) = 169.4 \text{ MeV}$$

$$[= 47.306 - (-74.070) - (-64.150) - 2(8.071) = 169.384 \text{ MeV}]$$

[If ^{238}U is unstable
Why doesn't ^{238}U just split?]



Spontaneous fission → tunneling [$T_{\frac{1}{2}}(^{235}U) = 3.5\times 10^{17}$ yrs due to fission
HRK p. 1169]

How supply energy? neutron bombardment

$$n + {}^{238}U \longrightarrow {}^{239}U$$

$$Q = \Delta(^{238}U) + \Delta(n) - \Delta(^{239}U) = 4.8 \text{ MeV}$$

$$[= 47.306 \text{ MeV} + 8.071 - 50.571 = 4.806 \text{ MeV}]$$

Thus ^{239}U is formed in an excited state (w/ 4.8 MeV energy above ground state)

^{239}U has an activation energy of 6.6 MeV [? 5.7 MeV in HRK, p. 1156]

so neutrons must have additional kinetic energy of 1.8 MeV to cause fission

<u>"fast neutron fission" of ^{238}U</u>

$$\frac{\frac{1}{2}mv^2}{mc^2} \cong \frac{1.8}{939} \Rightarrow \frac{v^2}{c^2} \cong \frac{1}{300}, \quad \frac{v}{c} \sim \frac{1}{17}$$

1938 O. Hahn
F. Strassmann (Berlin)

chemists

Fission first observed 1938 Hahn, Strassman (Berlin)

by bombarding U w/ neutrons.

after Chadwick discovered n,
Fermi used that because no
Coulomb barrier...

Found radioactive Ba; precipitated barium out of the solution.
Actually, thought it was radium [cf Periodic Table]
& wrote paper — new isotope of Ra.
Consternation: tried to separate out Ra from Ba, couldn't.

Hahn wrote to Lise Meitner, former colleague.
She was an austrian physicist but also Jewish, she was
& when Germany annexed austria in 1938, lost her job.
subject to the German racial laws &
Also in danger of her life so she left for Sweden.

L. Meitner
O. Frisch
"fission"

She discussed this w/ her nephew Otto Frisch (working w/ Bohr in Copenhagen)
& they realized that U was splitting into two smaller
nuclei ⇒ named it fission. (after cell division)

[cf What Little I Remember by Otto Frisch]

[cf Meitner: A Life in Physics by Ruth Sime]

[mysterious: Hahn + Strassmann obtained fission of U
by thermal neutrons (KE ~ 0.04 eV)
(whereas ^{238}U required fast neutrons)

1939 N. Bohr — Bohr realized $U = \begin{cases} 99.3\% & ^{238}U \\ 0.7\% & ^{235}U \end{cases}$

$$n + {}^{235}U \rightarrow {}^{236}U$$

$$Q = \Delta({}^{235}U) + \Delta(n) - \Delta({}^{236}U) = 6.5 \text{ MeV}$$

$$[= 40.916 + 8.071 - 42.442 = 6.545 \text{ MeV}]$$

[^{236}U has activation energy of 6.2 MeV] [5.2 MeV in HRK?]

No additional KE required. → why the difference in ϕ in energy?

$^{238}U \rightarrow {}^{239}U$ even → odd

$^{235}U \rightarrow {}^{236}U$ odd → even

"slow neutron fission of ^{235}U"

omit → [In fact, slow neutron fission more effective because higher cross-section for absorption]

neutrons released from fission can initiate new fissions ⟹ chain reaction! controlled → fission reactor, uncontrolled → bomb

---

Serway 14.6 $\quad {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 2n$

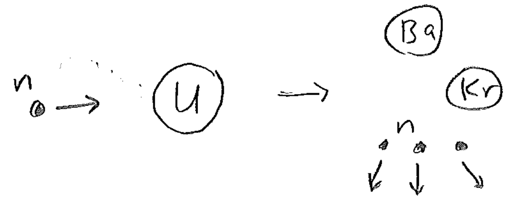
Serway 435 [$\Delta = 40.916$ $\quad \Delta = -79.771$ $\quad \Delta = -68.680$ $\quad \Delta = 2(8.071)$ $\quad \Rightarrow Q = 173.225$]

~~However that dep Q is a Coulomb barrier~~, then $Q \approx (0.72 \text{ MeV}) \frac{Z^2}{A^{1/3}}$

First controlled nuclear reaction

12/2/1942 U of Chicago squash courts (Fermi)

e.g. $n + {}^{235}U \longrightarrow {}^{236}U \longrightarrow {}^{141}Ba + {}^{92}Kr + 3n$



Secondary neutrons, avg # ~ 2.47/fission
avg E ~ 2 MeV

- escape through surface w/o causing fission
  - ↓ solution
  - increase volume to surface ratio (critical mass)
- fast n fission of ^{238}U [possible since $\bar{E} > 1.8$ MeV]
  - occurs < 20% of time because of low cross-sect for fast n absorption
  - → not sustainable [need at least 40%]
- slowed by inelastic collisions w/ ^{238}U (> 80% of time)
  - ↓ slow n fission of ^{235}U (high cross section for slow neutrons)
  - slow neutrons absorbed by $^{238}U \to {}^{239}U$ w/o subsequent fission (between 1-100 eV)
- slowed by a moderator (water, heavy water, ultrapure graphite) below 1 eV
  - ↓ slow n fission of ^{235}U w/ thermal neutrons (0.04 eV)

Def k = multiplication factor = # secondary neutrons that go on to produce subsequent fissions

k < 1 subcritical
k = 1 critical (self-sustaining)
k > 1 supercritical

$$P(t) = P_0\, k^{\frac{t}{t_{gen}}}$$

Use neutron absorbers to keep reaction from going supercritical
eg Cadmium control rods

[Richard Rhodes, <u>Making of the Atomic Bomb</u>]

Just talk about this:

[What if want a supercritical rxn, ie a bomb?]

- slow n fission not suitable, energy release causes U to expand & ∴ n's will all escape
- fast n fission of ^{238}U impossible because $k<1$.
- fast n fission of ^{235}U __is__ possible
- enrichment: electromagnetic separation } Oak Ridge
  gaseous diffusion } ($2 Billion)

Little Boy ~ 80% ^{235}U (about 50 kg)

(critical mass ~10 cm)

[do Pu after β decay]

chemical vs nuclear energy release

$$\left(x\ \frac{MJ}{kg}\right)\left(\frac{10^6 J}{MJ}\right)\left(\frac{kg}{10^3 g}\right)\left(\frac{1\,eV}{1.602E{-}19\,J}\right)\left(\frac{g}{6.022E23\ nucleons}\right) = 0.01036\ \frac{eV}{nucleon}$$

	$\frac{MJ}{kg}$	$\frac{eV}{nucleon}$
TNT	4.2	0.043
Sugar	19	0.20
Coal	24	0.25
Fat	39	0.40
gasoline	46	0.48
Uranium	7.6E7	790,000

Also $\frac{185\,MeV}{^{235}U} \sim \frac{185}{235} \sim 0.79\ \frac{MeV}{nucleon}$

$\sim$ about $1\ \frac{MeV}{nucleon}$

this column: Xkcd cartoon

ratio $\left(\frac{\text{uranium}}{\text{gasoline}}\right) \sim 1,600,000$

ratio $\left(\frac{\text{uranium}}{\text{TNT}}\right) \sim \boxed{18,000,000}$

1 kg U = 18,000,000 kg TNT = 18,000 metric tons = 18 kilotons

Hiroshima ~ 15 kilotons TNT ~ 0.83 kg U fissioned
Little Boy ~ 50 kg ^{235}U $\Rightarrow$ 0.017 efficiency

Kinetic energy of fragments: 80 MeV = (2E9)(0.04 eV) = (2E9)(300 K) = 6E11 K
10% efficiency $\Rightarrow$ 6E10 K (primer, p. 11, say 10^{10} K)

Fast fission (Los Alamos Primer)

- Avogadro's # = 2^{79} so if $\nu = 2$, 79 generations would fission 1 mole = 235 g of uranium.
- mean free path of a 1 MeV neutron is 13 cm (experimental)

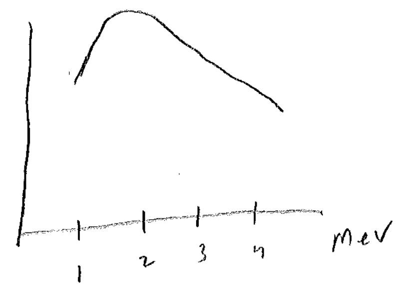
$\therefore$ mean free time per generation $= \frac{13\text{ cm}}{v} = \frac{0.13\text{ m}}{1.4E7\text{ m/s}} = 10^{-8}\text{ s} = 10\text{ ns}$

(where $\frac{v}{c} = \sqrt{\frac{2E_{kin}}{m_n c^2}} = \sqrt{\frac{2}{940}} = 0.046 \Rightarrow v = 1.4 \times 10^{7}\ \frac{m}{s}$)

- 80 generations $\Rightarrow$ 0.8 μs

Neutrons emitted by fission

	ν
^{235}U	2.52
^{239}Pu	2.95



$\langle E_{kin} \rangle \sim 2$ MeV

(fraction > 1 MeV) $\sim 3/4$

Neutron cross-section for $E_{kin} > 1$ MeV

First, estimate geometrical cross-section $\pi R^2 = A^{2/3} \pi r_0^2$

$\pi r_0^2 = 4.5\text{ fm}^2$ for $r_0 = 1.2$ fm

$A = 235 \Rightarrow A^{2/3} = 38 \Rightarrow \pi R^2 = 170\text{ fm}^2 = 1.7$ barns

(1 barn $= 10^{-28}\text{ m}^2 = 100\text{ fm}^2$).

This is about right for fast neutrons

	σ	
^{235}U	1.22 barns	50% fission, 50% re-emit slower neutron (< 1 MeV)
^{239}Pu	1.73 barns	100% fission
^{238}U	~0.7 barns	25% fission, 75% re-emit slower neutron (< 1 MeV) (<10% γ-emission, followed by β to Np + Pu)

$\Rightarrow$ ^{238}U does not undergo fast n fission because $\nu \cdot$ (fraction > 1 MeV) $\cdot$ (fraction $\to$ fission) $= (2.2)(0.75)(0.25) = 0.4 < 1$

Note to self...

$^{235}U \rightarrow {}^{141}Ba + {}^{92}Kr + 2n$

Q using binding energy formula $= 780.9 + 1170.6 - 1787.2$

$= \underline{164.3}$ MeV as Meitner did

Q using measured masses (p. 435, Serway) $= \underline{173.9}$ MeV

Q: using Coulomb energies alone (as Frisch did)

$$E_{coul} = (0.71 \text{ MeV}) \frac{Z(Z-1)}{A^{1/3}}$$

^{235}Uranium $\Rightarrow$ 963.2

^{141}Ba $\Rightarrow$ 420.1

^{92}Kr $\Rightarrow$ 198.2

345 MeV released

Fission physics

- Fission releases ~ 200 MeV per nucleus but requires an input of ~ 6 MeV (i.e. nucleus must have 6 MeV K.E. above rest mass to be roiled enough to split)
- Absorption of a n adds 5.3 MeV; ~ 1 MeV still needed
- Fast neutrons (~ 1 MeV kinetic energy) provide this extra energy and will fission ^{238}U and ^{232}Th. (and ^{235}U)
- ^{235}U, however, on absorbing a n, goes from odd to even which adds an additional 1 ~ 2 MeV; thus neutrons of any energy will fission ^{235}U. (similarly ^{239}Pu) [i.e. ≳ 6 MeV K.E. is released in transmutation]

fission σ at any energy

- But ^{238}U (also ^{232}Th) absorbs n at ~ 25 eV (capture resonance) to form $^{239}U \xrightarrow{\beta} {}^{239}Np \xrightarrow{\beta} {}^{239}Pu$.

Thus, E ≳ 1 MeV, fission σ
E ~ 25 eV, capture σ
E < 25 eV, elastic

- Thus slow (thermal ⇒ 0.04 eV) ~~(thermal)~~ n's are captured by ^{238}U rather than ~~[illegible]~~ slow fissioning ^{235}U.
- Unless a moderator (H, C) is used to slow n's down to energies below the capture resonance
- H also absorbs neutrons, but C absorbs less (unless there ~~a lot~~ are boron impurities). ^2H doesn't absorb n's.
- Slow n fission σ can be ↑ by purifying ^{235}U. ⇒ helps chain rxn; ~~cut~~ size of reactor reduced. Slow n fission ⇒ time btw fissions ~ 10^{-3} s ⇒ $\boxed{\text{too slow for weapon}}$ (it would vaporize) but suitable for reactors (requires O(tons))

(of course ∄ slow n fission of ^{238}U)

- Fast n fission ~~of~~ of ^{238}U $\boxed{\text{hindered}}$ because many n's scattered & then captured.
- ^{235}U doesn't capture n's, and has a higher fast n cross-section than ^{238}U (I think) (^{235}U ~ 10^{-23} cm^2, ^{238}U ~ 10^{-24} cm^2) ⇓ smaller critical mass ~ O(kgs)

Also, time btw fissions is ~10^{-6} s ⇒ bomb!