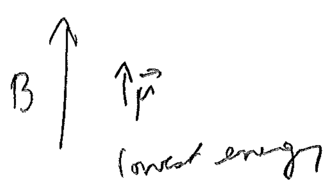
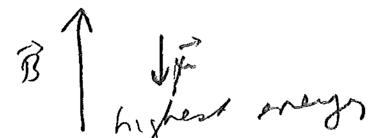
5. Magnetic dipole moment μ

μ = strength of magnetic

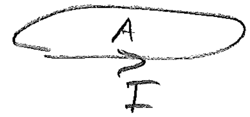
A magnet tries to align w/ an external magnetic field $\vec{B}$





$$E_{dipole} = -\vec{\mu} \cdot \vec{B}$$

electromagnet = loop of current



$$\mu = IA$$

point charge q in a circular orbit of period T

$$I = \frac{q}{T}$$

Can show that μ is proportional to orbital angular momentum

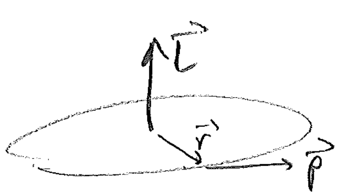
$$\mu = \frac{q}{2m} L$$

[problem]

[$dA = \frac{1}{2} r^2 d\theta$

$\frac{dA}{dt} = \frac{1}{2} \omega r^2 = \frac{L}{2m}$

$\mu = IA = \frac{q}{T} A = q \frac{A}{T} = \frac{qL}{2m}$]



$$\boxed{\vec{\mu} = \frac{q}{2m} \vec{L}}$$

$$\vec{\mu} = \frac{q}{2m}\vec{L}$$

$|\vec{\mu}|$ quantized because $\vec{L}$ quantized

Electron in Bohr model has $L = l\hbar$ so $\mu = \underbrace{\left(\frac{e\hbar}{2m_e}\right)}_{\text{minimum dipole moment}} l$

Bohr magneton $\mu_B \equiv \dfrac{e\hbar}{2m_e}$ $\left[= 5.788E{-11}\ \frac{\text{MeV}}{\text{T}}\right]$

$\left[\text{Gaussian}: \mu_B = \frac{e\hbar}{2mc}\right]$

A particle of spin $\vec{J}$ (intrinsic angular momentum) also has an intrinsic magnetic dipole moment (not associated w/ motion)

[would expect $\vec{\mu} = \frac{q}{2m}\vec{J}$ but here qu. mech spin differs from classical]

$$\vec{\mu} = g\left(\frac{q}{2m}\right)\vec{J}$$

"g-factor" (Landé) not necessarily 1.

Dirac eqn. (1928) predicts that $g=2$ for fundamental spin-$\frac{1}{2}$ particle [deep elegant result, relativistic]

$$\Rightarrow \mu_{\text{electron}} = 2\left(\frac{e}{2m_e}\right)\left(\frac{\hbar}{2}\right) = \frac{e\hbar}{2m_e} = \mu_B$$

Dirac eqn is not exact; there are QED corrections

exp. ⇒ $g_e = 2.002\ 319\ 304\ 361\ 4(6)$

[experimental result: very accurate: 13 sig figs
agrees w/ theoretical calculation to accuracy measured]
Feynman calls this our "crown jewel"]

usually quoted as fractional difference from Dirac prediction

$$\frac{\mu - \mu_B}{\mu_B} = \frac{g_e - 2}{2} = 0.001\ 159\ 652\ 180\ 76\ (27)$$ (exp)

"electron magnetic moment anomaly"

[PDB 2012]

agrees w/ theoretical prediction $\left[\frac{g-2}{2} = \frac{\alpha}{2\pi} + \ldots\right]$

Currently there is some discrepancy in muon magnetic moment

$$\left(\frac{g_\mu - 2}{2}\right)_{exp.} = 0.001\ 165\ 920\ 9\ (6)$$ (exp)

(Standard model prediction)

$$\left(\frac{g_\mu - 2}{2}\right)_{th} = 0.001\ 165\ 918\ 0\ (6)$$

[RPP 2012, p584]

$$\text{Difference} = 0.000\ 000\ \underbrace{002\ 9\ (8)}_{}$$

$\sim 3\sigma$ effect [new physics? susy?]

$$\left(\frac{g_\nu - 2}{2}\right)_{exp} = 0.$$

Of course ν have no magnetic moment, being neutral

Proton (spin ½ but not fundamental)

$$\mu = g_p\left(\frac{e}{2m_p}\right)\left(\frac{\hbar}{2}\right)$$

expt ⇒ $g_p = 2(2.793\ldots) \Rightarrow \mu = 2.793\,\mu_N$

where the nuclear magneton $\mu_N \equiv \dfrac{e\hbar}{2m_p}$; $(\mu_N \ll \mu_B)$

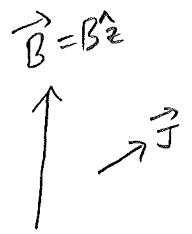
[we can't predict g_p as accurately as g_e
but quark model comes within 10% → later in course]

neutron

expt ⇒ $\mu = -1.913\,\mu_N$ due to charged quarks inside neutral neutron.

Recall: the $2j+1$ spin states of a spin-j particle are degenerate in a rotationally-symmetric background

An external magnetic field breaks the symmetry & splits the degeneracy

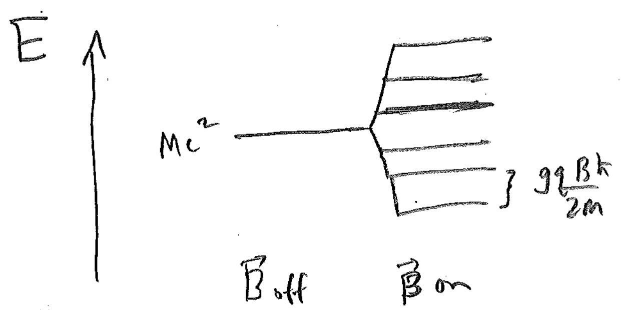


$$E = Mc^2 - \vec{\mu}\cdot\vec{B}$$

$$= Mc^2 - \frac{gq}{2M}\vec{J}\cdot\vec{B}$$

$$\vec{B}\cdot\vec{J} = BJ_z = Bm\hbar$$

$$E = Mc^2 - \left(\frac{gq\,B\hbar}{2M}\right)m \qquad m = -j, \dots, j$$



different energy levels
$\Rightarrow$ multiple spectral lines

[depends on B & on g]

Zeeman effect

In atomic physics