3. Spin j

[i.e. the angular momentum it has at rest]

Angular momentum is quantized.

Orbital angular momentum $\vec{L} = \vec{r} \times \vec{p}$ is quantized $\Rightarrow$ integer multiples of $\hbar$ (Bohr)

$$L = l\hbar, \quad l \in \mathbb{Z}$$

[observe $L = r \times p$ and $\frac{\hbar}{2} \leq \Delta x \Delta p$ have same units]

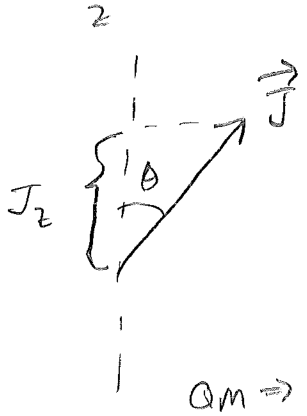
Spin J (intrinsic ang. mom) is quantized in multiples of $\frac{1}{2}\hbar$

$$J = j\hbar, \quad j \in \tfrac{1}{2}\mathbb{Z}$$

Each type of particle has a specific value of j

$j = 0$	scalar	(eg Higgs)
$j = \frac{1}{2}$	spinor	(eg quarks, leptons)
$j = 1$	vector	(eg $W^{\pm}$, Z^0, γ, gluons)
$j = \frac{3}{2}$	spinor-vector	(e.g. gravitino? supersymmetric partner of graviton)
$j = 2$	tensor	(e.g. graviton)

[Angular momentum, of course, is a vector and has direction as well as magnitude. If we choose a z-axis, $\vec{J}$ makes some angle θ with respect to it]



$J_z = J \cos\theta$ where $J = |\vec{J}|$

QM ⇒ $J = j\hbar$

Each component of $\vec{J}$ is also quantized

$J_z = m\hbar$ where $\begin{cases} m = \text{integer if } j = \text{integer} \\ m = \text{half-integer if } j = \text{half integer} \\ |m| \leq j \end{cases}$

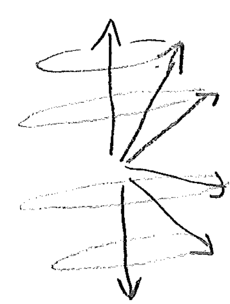
$$\Rightarrow \quad m = \underbrace{-j, -j+1, -j+2, \ldots, j-1, j}_{2j+1 \text{ possible values}}$$

m determines the direction of the spin

e.g. $j = \frac{1}{2} \Rightarrow m = -\frac{1}{2}$ or $\frac{1}{2}$

$-\frac{1}{2}$: spin down; $\frac{1}{2}$: spin up

In general $\theta = \arccos\left(\frac{J_z}{J}\right) = \arccos\left(\frac{m}{j}\right)$



[what is j in this example?]

"spatial quantization"

[don't take picture too seriously]

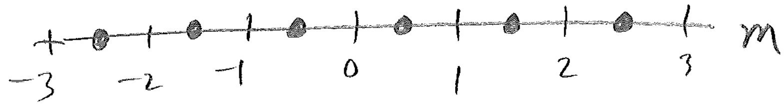
A particle w/ spin $j > 0$ can exist in $(2j+1)$ different states (# of states = multiplicity) differing only in direction.

In an isotropic (rotationally-symmetric) environment, these states all have same energy (mass).

They are "degenerate states".

We represent the possible spin states as points on a weight diagram

For $j = \frac{5}{2}$



Denote the probability amplitude of the spin states by $|j, m\rangle$

[we used ψ before; now using ket notation]

$$\left.\begin{array}{l} |j, j\rangle \\ |j, j-1\rangle \\ |j, j-2\rangle \\ \vdots \\ |j, -j\rangle \end{array}\right\}$$

The set of states is called a (spin-j) multiplet.

Denote it by $\underset{\sim}{2j+1}$

(Spin $\frac{1}{2}$)-particle

$$\left.\begin{array}{ll} \text{Spin up state} & |\tfrac{1}{2},\tfrac{1}{2}\rangle = |+\rangle = \uparrow \\ \text{Spin down state} & |\tfrac{1}{2},-\tfrac{1}{2}\rangle = |-\rangle = \downarrow \end{array}\right\} \ \underline{\text{doublet}},\ \underset{\sim}{2}$$

These states have definite values of J_z viz $\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$

they are called [ask them] eigenstates of J_z

More general states are possible: linear combinations of eigenstates

$|\psi\rangle = \alpha|+\rangle + \beta|-\rangle$ α, β are complex numbers

$|\psi\rangle$ has an indeterminate value of J_z

If measure J_z, will obtain for a result either

$\frac{\hbar}{2}$ w/ prob. $|\alpha|^2$

$-\frac{\hbar}{2}$ w/ prob $|\beta|^2$

$|\alpha|^2 + |\beta|^2 = 1$

After measurement, particle will have a definite value of J_z

$|\psi\rangle \xrightarrow[\text{measure}]{} |+\rangle$ or $|-\rangle$

"collapse of wavefunction"

A general state of spin-j is

$|\psi\rangle = \sum c_m |j,m\rangle$ where $\sum_{m=-j}^{j} |c_m|^2 = 1$

Massive particles of spin j possess $(2j+1)$ spin states

but Massless particles can only have $J_z = \pm j\hbar$

$\Rightarrow$ only 2 spin states

gluons, photons : 2 polarizations
gravitons : 2 polarizations

Spin-statistics theorem

All bosons have integer spin
All fermions have half-integer spin

Also works for composites

2 spin-$\frac{1}{2}$ particles can have spin 1 ($\uparrow\uparrow$) or spin 0 ($\uparrow\downarrow$) $\Rightarrow$ boson
(ρ^+ meson $= u\bar{d}$) (π^+ meson $= u\bar{d}$)

3 spin-$\frac{1}{2}$ particles can have spin $\frac{3}{2}$ ($\uparrow\uparrow\uparrow$) or spin $\frac{1}{2}$ ($\uparrow\uparrow\downarrow$) $\Rightarrow$ fermion
(Δ^+ baryon $= uud$) ($p = uud$)

4 spin-$\frac{1}{2}$ particles can have spin 2, spin 1, or spin 0 ($\uparrow\uparrow\downarrow\downarrow$) $\Rightarrow$ boson

($^4He = ppnn$)

[Bose-Einstein Cond.
Superfluid]

Experiments on ^{14}N showed that it was a boson and ~~has~~ $J=1$. Conflict w/ pre-neutron model of $14p + 7e$.