2. Mass m, or rest energy mc^2

[masses of atoms measured in terms of]

unified atomic mass unit $u = \frac{1}{12}$ (mass of ^{12}C atom)

[i.e. $m(^{12}C) \equiv 12\,u$]

1 mole of u = 1 g $\Rightarrow$ $1\,u = \frac{1\,g}{N_A} = \frac{10^{-3}\,kg}{6.022 \times 10^{23}} \sim 1.7 \times 10^{-27}\,kg$

[roughly mass of proton]

A particle of mass 1 u has rest energy

$mc^2 = (1.7 \times 10^{-27}\,kg)(3.0 \times 10^{8}\,\frac{m}{s})^2 = 1.5 \times 10^{-10}\,J$

$1\,eV = (1.6 \times 10^{-19}\,C)\,V = 1.6 \times 10^{-19}\,J$

$1\,MeV = 1.6 \times 10^{-13}\,J$

$\Rightarrow \boxed{1\,u \Longrightarrow 931.494\ MeV}$

[we'll use rest energy throughout the course]

Rest energies of particles

Baryons ["heavy"]

proton 938.3 MeV ~ 1 GeV

neutron 939.6 MeV

[nearly the same. Why? uud, udd. Strong force acts same on u + d quarks (even though emc. does not). Isospin symmetry.]

Leptons ["light"]

electron 0.511 MeV [about $\frac{1}{2000}$ of proton]

muon 105.6 MeV

tau 1776 MeV

neutrinos < 1 eV

[why are these masses what they are? why increase?]

Force carriers

all massless except

$W^{\pm}$ 80.4 GeV

Z^0 91.2 GeV

range of weak force $= \dfrac{\hbar}{M_W c} = \dfrac{\hbar c}{M_W c^2} = \dfrac{(197 \text{ MeV fm})}{(80.4 \text{ GeV})} \sim 2.5 \times 10^{-18}$ fm

Higgs 125 GeV

Energy of a free particle

$$E = \underbrace{mc^2}_{\text{rest}} + \underbrace{K}_{\text{kinetic}}$$

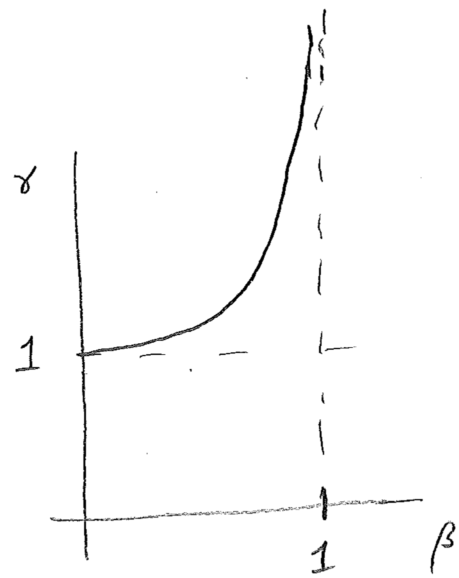
$$= mc^2 + \Big(\underbrace{\tfrac{1}{2} m v^2}_{\text{valid for } v \ll c} + \tfrac{3}{8} m \frac{v^4}{c^2} + \cdots \Big)$$

$$= mc^2 \left(1 + \tfrac{1}{2}\left(\tfrac{v}{c}\right)^2 + \tfrac{3}{8}\left(\tfrac{v}{c}\right)^4 + \cdots \right)$$

$$= \frac{mc^2}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

$$= \gamma mc^2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}$$



$\gamma \to \infty$ as $\beta \to 1$ so all particles of mass must obey $\beta < 1$.

$$K = E - mc^2 = (\gamma - 1) mc^2$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

ultrarel. limit $\gamma \to \infty$

$$\beta \approx 1 - \frac{1}{2\gamma^2}$$

protons in LHC have $E = 7$ TeV

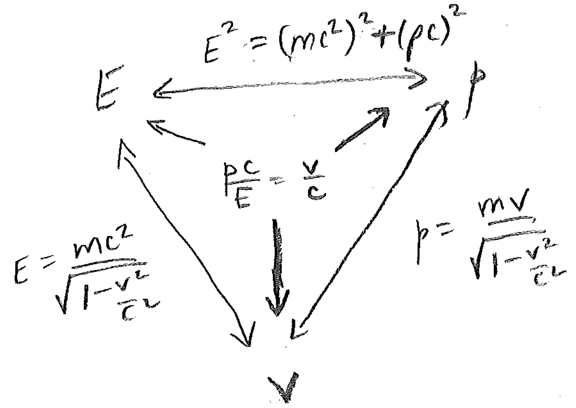
$$\Rightarrow \gamma = \frac{E}{mc^2} = 7000$$

$$\beta = 1 - (10^{-8})$$

momentum

Non relativistic approximation $\vec{p} = m\vec{v}$

Exact $\vec{p} = \gamma m \vec{v}$ [so $p \to \infty$ as $v \to c$]



[verify $E^2 = (mc^2)^2 + (pc)^2$]

$E \sim mc^2$ as $v \to 0$

$E \sim pc$ as $v \to c$

Non relativistic expansion
$$E = \sqrt{(mc^2)^2 + (pc)^2}$$
$$= mc^2 \sqrt{1 + \left(\frac{p}{mc}\right)^2}$$
$$= mc^2 \left[1 + \frac{1}{2}\left(\frac{p}{mc}\right)^2 - \frac{1}{8}\left(\frac{p}{mc}\right)^4 + \ldots\right]$$
$$= mc^2 + \underbrace{\frac{p^2}{2m} - \frac{p^4}{8m^3c^2} + \ldots}_{K}$$

Massless particles

$$E = mc^2 + K$$

massless particles have no rest energy; all their energy is kinetic ($E = K$).

They are never at rest.

[How can this be? Not relativistically invariant statement. Boost into frame of moving particle but in that frame they have no energy $\Rightarrow$ cannot exist. How can exist in 1 frame & not another? Loophole: if travelling at $v = c$ cannot boost into that frame!]

Massless particles always travel at $v = c$.

[cf PPB: γ, g, graviton]

$$E^2 = (pc)^2 + \cancelto{0}{(mc^2)^2} \quad \Rightarrow \quad E = pc$$

$$\text{But} \quad \frac{v}{c} = \frac{pc}{E} = 1 \quad \Rightarrow \quad v = c$$

Photon energy depends not on speed but on frequency

$$E = K = hf$$

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

$$\begin{tikzcd}
E \arrow[r, leftrightarrow, "E = pc"] \arrow[d, leftrightarrow, "E = hf"'] & p \arrow[d, leftrightarrow, "p = \frac{h}{\lambda}"] \\
f \arrow[r, leftrightarrow, "f = \frac{c}{\lambda}"'] & \lambda
\end{tikzcd}$$

[why are we so interested in E & $\vec{p}$? Because they are conserved]

In any interaction, the total energy & momentum of a system of particles is conserved

$$\vec{p}_{tot,i} = \vec{p}_{tot,f}$$
$$E_{tot,i} = E_{tot,f}$$

$$\sum_{init} (mc^2 + K) = \sum_{final} (mc^2 + K)$$

Neither mass nor kinetic energy is separately conserved but only their sum

Define $Q = \sum_{init} mc^2 - \sum_{final} mc^2$

= amt of rest energy transformed into kinetic

If $Q > 0$, then decay can occur (hence most elementary ptls are unstable)
Q = kinetic energy released (e.g. fission)

If $Q < 0$, then initial particles must have enough kinetic energy to create the more massive particle in the final state

(e.g. cosmic ray collisions, particle accelerators)