(3-11-25)

2025 bkgnd notes on cross-sections & lifetimes

2025-bkgnd.pdf

(1)

Cross-sections

Dimensionally, $\sigma \sim \frac{1}{[E]^2}$

2→2 scattering $$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} |A|^2}$$

$\Rightarrow |A|$ dimensionless

High energy limit $\Rightarrow p_f \approx p_i \Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{\hbar^2 |A|^2}{(8\pi)^2 s}$

Isotropic $\Rightarrow$ $$\boxed{\sigma = \frac{\hbar^2 |A|^2}{16\pi s}}$$

Decay rates

Dimensionally, $\Gamma \sim [E]$

$$\Gamma = \frac{1}{2m} \int (\text{LIPS}) |A|^2$$

1→2 decay: $(\text{LIPS})_2 = \frac{p_f}{(4\pi)^2 m} d\Omega \Rightarrow |A| \sim [E]$

massless products $\Rightarrow \int (\text{LIPS})_2 = \frac{1}{2(4\pi)^2} \int d\Omega = \frac{1}{8\pi} \Rightarrow \boxed{\Gamma = \frac{|A|^2}{16\pi m}}$

1→3 decay $(\text{LIPS})_3 = \frac{1}{32\pi^3} \underbrace{dE_1 dE_2}_{m^2/8 \text{ for massless final state}} \Rightarrow |A| \sim$ dimensionless

(Coleman, p. 257)

$$\boxed{\Gamma = \frac{|A|^2 m}{512\pi^3}}$$

$$t = -\frac{s}{2}(1-\cos\theta) \Rightarrow \cos\theta = \frac{2t}{s}+1$$

$$d\Omega = 2\pi\, d(\cos\theta) = \frac{4\pi}{s}\, dt$$

$$d\sigma = \frac{d\sigma}{d\Omega}\, d\Omega = \frac{d\sigma}{dt}\, dt$$

$$\frac{d\sigma}{dt} = \frac{d\Omega}{dt}\frac{d\sigma}{d\Omega} = \frac{4\pi}{s}\frac{d\sigma}{d\Omega} = \frac{4\pi}{s}\left(\frac{\hbar}{8\pi E_{cm}}\right)^2 |A|^2$$

$$\frac{d\sigma}{dt} = \frac{\hbar^2}{16\pi s^2}|A|^2$$

t runs from $-s$ to 0

$$\sigma = \int_{t=-s}^{0} \frac{d\sigma}{dt}\, dt$$

Dimensionally, $\frac{d\sigma}{dt} \sim \frac{1}{[E]^4}$

(3-12-25)

2

3-particle phase space

Coleman QFT notes, & textbook

Griffiths (2e), §9.2, p. 311

Also my notes on kinematics & phase space & 2210 background notes (2017)

$$d(LIPS)_3 = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3p_3}{(2\pi)^3 2E_3} (2\pi)^4 \delta^{(3)}(\vec{p}_1+\vec{p}_2+\vec{p}_3)\, \delta(m-E_1-E_2-E_3)$$

Do $d^3\vec{p}_3$ using $\delta^{(3)}(\Sigma \vec{p}) \Rightarrow |\vec{p}_3| = |\vec{p}_1+\vec{p}_2|$

Let θ_{12} be $\measuredangle$ btw $\vec{p}_2$ & $\vec{p}_1$ and ϕ_{12} be azimuth of $\vec{p}_2$ around $\vec{p}_1$

$$d(LIPS)_3 = \frac{1}{(2\pi)^5} \frac{1}{8} \int \frac{p_1^2\, dp_1\, d\Omega_1}{E_1} \frac{p_2^2\, dp_2}{E_2} d(\cos\theta_{12})\, d\phi_{12} \frac{1}{E_3} \delta(m-E_1-E_2-E_3)$$

Assume all 3 final state particles are massless.

Now change variables from θ_{12} to E_3:

$E_3^2 = |\vec{p}_3|^2 = |\vec{p}_1+\vec{p}_2|^2 = E_1^2+E_2^2+2E_1E_2\cos\theta_{12}$

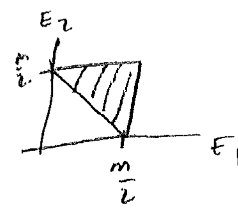
Holding E_1 & E_2 fixed, we have $E_3\, dE_3 = E_1E_2\, d(\cos\theta_{12})$ so

$$d(LIPS)_3 = \frac{1}{(2\pi)^5} \frac{1}{8} \int d\Omega_1\, d\phi_{12}\, dE_1\, dE_2\, dE_3\, \delta(m-E_1-E_2-E_3)$$

where range of integration is $E_1: 0 \to \infty$, $E_2: 0 \to \infty$ and

$E_3: |E_1-E_2|$ to E_1+E_2 (as θ_{12} goes from 0 to π)

Thus $\int_{|E_1-E_2|}^{E_1+E_2} dE_3\, \delta(m-E_1-E_2-E_3) = \begin{cases} 1 & \text{if } |E_1-E_2| \le m-E_1-E_2 \le E_1+E_2 \\ 0 & \text{otherwise} \end{cases}$



Inequalities hold if $E_1, E_2 \le \frac{m}{2}$ and $E_1+E_2 \ge \frac{m}{2}$

$$\boxed{\begin{aligned} d(LIPS)_3 &= \frac{1}{8(2\pi)^5} \underbrace{\int d\Omega_1\, d\phi_{12}}_{(4\pi)(2\pi)} \int_0^{\frac{m}{2}} dE_1 \int_{\frac{m}{2}-E_1}^{\frac{m}{2}} dE_2 \\ &= \frac{1}{4(2\pi)^3} \int_0^{\frac{m}{2}} dE_1 \int_{\frac{m}{2}-E_1}^{\frac{m}{2}} dE_2 \end{aligned}}$$

← cf Griffiths (2e), (9.31)

If $|A|^2 =$ const the integral over energies gives $\frac{1}{2}\left(\frac{m}{2}\right)^2$.

$$\int d(LIPS)_3 = \frac{m^2}{32(2\pi)^3} \Rightarrow \Gamma = \frac{1}{2m} \int d(LIPS)_3 |A|^2 = \boxed{\frac{m|A|^2}{512\pi^3}}$$

strong

Hadronic processes: geometric approach

Size of nuclei $R = A^{\frac{1}{3}} r_0$ where $r_0 = 1.2$ fm

geometric cross-section $\sigma = \pi R^2 = \boxed{A^{2/3} \pi r_0^2}$

proton $\sigma = \pi r_0^2 = 4.5\ \text{fm}^2 = 45$ mb (TDLee: $\sigma = 30$ mb)

nitrogen $\sigma = 0.25$ b

uranium $\sigma = 1.7$ b

- experimentally (K. McDonald) $\left.\begin{array}{l}(\sigma_{pp})_{tot} = 40\ \text{mb} \\ (\sigma_{\pi p})_{tot} = 25\ \text{mb}\end{array}\right\}$ consistent w/ 45 mb

- nuclear path length in air $l = \frac{1}{n\sigma_{nitrogen}} = 0.7$ km

$n = \frac{\rho N_0}{A}$, $\rho = 1.25E\text{-}3\ \frac{g}{cm^3}$, $n = 5.38E19\ cm^{-3}$

- neutron path length in uranium $l \approx 11$ cm

- size of nuclei related to range of strong force
- if mediated by pion exchange, range $\sim c\Delta t \sim \frac{\hbar}{m_\pi c} \sim 1.4$ fm
- thus $\sigma \sim \pi \left(\frac{\hbar}{m_\pi c}\right)^2$

Halzen Martin
K. McDonald

(Strong)

Alternatively, consider $\pi p \to \pi p$ scattering as analogous to Compton scattering $\gamma p \to \gamma p$

Recall $\sigma = \dfrac{\hbar^2 |A|^2}{16\pi s} = \dfrac{\hbar^2 (4\pi\alpha)^2}{16\pi s} = \pi \left(\dfrac{\alpha\hbar}{m_p}\right)^2$

(So here range given not by Compton wavelength of target but by its "classical radius")

More precisely (since $|A|^2 = 2 + 2\cos^2\theta$) $\sigma = \dfrac{8\pi}{3}\left(\dfrac{\alpha\hbar}{m_p}\right)^2$

Halzen, Martin + K McDonald argue that $\frac{8\pi}{3} \to 4\pi$ because γ only have 2 of 3 polarizations

By analogy $\underbrace{\sigma_{\pi p \to \pi p}}_{\text{elastic}} = 4\pi \left(\dfrac{\alpha_s \hbar}{m_p}\right)^2 = \alpha_H^2 \,(5.5\,mb)$

smaller than geometric because $m_p >> m_\pi$

McDonald says $\sigma_{elastic} \sim 3\,mb$ so this is comparable.

Halzen Martin argue $\alpha_H \sim 1$ to 10 (+ later $\alpha_H \sim 15$)

[McDonald argues to replace $m_p \to m_\pi$ (pion cloud)

$\sigma = \alpha_s^2 \, 4\pi \left(\dfrac{\hbar}{m_\pi}\right)^2 \implies \alpha_s \sim 0.1$

but this is rather dodgy]

TD Lee says $\alpha_H \sim 1$, so $\sigma = \pi\left(\dfrac{\hbar}{m_\pi}\right)^2 \sim 60\,mb$

strong

Naive arguments for strong force

2 → 2

$$\sigma = \frac{\hbar^2}{16\pi s}|A|^2$$

only differs from previous estimate by fact of $\frac{8}{3}$ or 4

If $A \sim 4\pi\alpha_H \Rightarrow \boxed{\sigma = \alpha_H^2 \pi \left(\frac{\hbar}{m}\right)^2}$

$= \alpha_H^2 (1\,mb)$ for πp proton

$(\sigma_{elastic} = 3\,mb)$

so $\alpha_H \sim 1$

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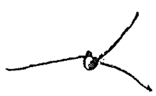
1 → 2



$$\Gamma = \frac{|A|^2}{16\pi m}$$

If $A \sim \sqrt{4\pi\alpha_H}\, m \Rightarrow \boxed{\Gamma = \frac{m}{4}\alpha_H}$

$\rho \to \pi\pi \Rightarrow \frac{\Gamma}{m} \sim \frac{1}{5}$ so $\alpha_H \sim 1$

$K^* \to K\pi \Rightarrow \frac{\Gamma}{m} \sim \frac{1}{20}$

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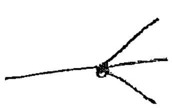
1 → 3

$$\Gamma = \frac{|A|^2 m}{512\pi^3}$$



If $A = 4\pi\alpha_H \Rightarrow \Gamma = \frac{m\alpha_H^2}{32\pi} \approx \boxed{\frac{m}{100}\alpha_H^2}$

$\omega \to \pi\pi\pi \Rightarrow \frac{\Gamma}{m} \approx \frac{1}{100}$ so $\alpha_H \sim 1$

EM

QED processes

$\boxed{2 \to 2}$

$\gamma e^- \to \gamma e^-$

$e^- e^+ \to \mu^- \mu^+$

$e^- p \to e^- p$

$e^- e^+ \to \gamma\gamma$

All have $A \sim e^2 = 4\pi\alpha$.

Let $A = 4\pi\alpha M$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} |A|^2 = \left(\frac{\hbar\alpha}{2E_{cm}}\right)^2 \frac{p_f}{p_i} |M|^2$$

If elastic, then $p_f = p_i$. If isotropic, $\sigma = 4\pi\left(\frac{d\sigma}{d\Omega}\right)$

$$\boxed{\sigma = \pi\left(\frac{\alpha\hbar}{E_{cm}}\right)^2 |m|^2}$$

Compton $\gamma e^- \to \gamma e^-$

low energy: $E_{cm} \simeq m_e$, $|M|^2 = 2 + 2\cos^2\theta \Rightarrow \frac{8}{3}$

$$\sigma = \frac{8}{3}\pi \underbrace{\left(\frac{\alpha\hbar}{m_e}\right)}_{r_e}^2 = 0.666 \text{ barns} \quad \text{(Thomson)}$$

high energy: $E_{cm}^2 = 2m_e E_\gamma$, but not isotropic

$$\sigma \sim \frac{\alpha^2\hbar^2}{m_e E_\gamma} \ln\left(\frac{E_\gamma}{m}\right)$$ (Klein-Nishina) (cf K. McDonald notes)

see also Perkins p. 22

$e^- e^+ \to \mu^- \mu^+$

high energy $\sigma = \frac{\pi\alpha^2\hbar^2}{s} |M|^2$

exact $\sigma = \frac{4}{3}\pi \frac{\alpha^2\hbar^2}{s}$

EM

$e^- p \rightarrow e^- p$ at low energy

$$A = 4\pi\alpha\left(\frac{E_1 E_2}{p^2 \sin^2\frac{\theta}{2}}\right) = 4\pi\alpha\left(\frac{m_2}{m_1 v_1^2 \sin^2\frac{\theta}{2}}\right)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar\alpha}{2E_{cm}}\right)^2\left(\frac{m_2}{m_1 v_1^2 \sin^2\frac{\theta}{2}}\right)^2$$

$$= \left(\frac{\hbar\alpha}{2 m_1 v_1^2 \sin^2\frac{\theta}{2}}\right)^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\beta^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

$$\Rightarrow \sigma(\theta > \tfrac{\pi}{2}) = \pi\beta^2$$

[Set $\sin^2\frac{\theta}{2} = 1$, multiply by 4π to get $\sigma(\theta > \frac{\pi}{2})$]

$$\sigma = \pi\left(\frac{\hbar\alpha}{m_1 v_1^2}\right)^2$$

Thus: $\boxed{\sigma = \frac{\pi\alpha^2}{(2T_1)^2}}$ or $\pi\left(\frac{\hbar\alpha Z_1 Z_2}{2T_1}\right)^2$, $T_1 = 5$ MeV, $Z_1 = 2$, $Z_2 = 79$ $\Rightarrow \sigma = 17$ barns

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$e^- e^+ \rightarrow \gamma\gamma$ at low energy

(Griffiths)

$$A = 4\,(4\pi\alpha)$$

But $\frac{p_f}{p_i} = \frac{c}{v}$

$$\sigma = \frac{\pi\alpha^2}{E_{cm}^2}\frac{c}{v}|M|^2 = \frac{\pi\alpha^2 (4)^2}{(2m_e)^2}\frac{c}{v} = \boxed{\frac{4\pi\alpha^2}{m_e^2}\left(\frac{c}{v}\right)}$$

$\boxed{2 \rightarrow 3}$ $\gamma X \rightarrow \gamma e^+ e^-$

TDLee (ch 9) $\rightarrow$ $\sigma \sim \left(\frac{28}{9}\right)\frac{Z^2\alpha^3}{m_e^2}\ln\left(\frac{E_\gamma}{m_e}\right)$ (Bethe Heitler)

screening $\Rightarrow \ln\left(\frac{183}{Z^{1/3}}\right)$ $\Rightarrow \sigma \sim 0.4\,b$ in air

$\Rightarrow$ mean free path $\sim$ 460 m in air
13 cm in Al
7 mm in Pb

see my cosmic ray calculation for verification

EM

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p. 22

Perkins: $e^+e^- \to \gamma\gamma$

$\gamma\gamma \to e^+e^-$

$\gamma e \to \gamma e$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} (4\pi\alpha)^2$$

At high energies

$$\sigma = \frac{\pi\alpha^2\hbar^2}{s}|M|^2$$

but all are enhanced by $\ln\left(\frac{s}{m^2}\right)$
due to electron propagator

$$\sigma(e^+e^- \to \gamma\gamma) = \frac{2\pi\alpha^2}{s}\left[\ln\left(\frac{s}{m^2}\right) - 1\right]$$

$$\sigma(\gamma\gamma \to e^+e^-) = \frac{4\pi\alpha^2}{s}\left(\ln\left(\frac{s}{m^2}\right) - 1\right)$$

$$\sigma(\gamma e \to \gamma e) = \frac{2\pi\alpha^2}{s}\left[\ln\left(\frac{s}{m^2}\right) + \frac{1}{2}\right]$$

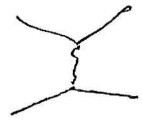
Inverse Compton (where $E_f^\gamma > E_i^\gamma$) produces
high energy γ from stars

Low energies

$$\sigma(\gamma e \to \gamma e) = \frac{8\pi\alpha^2}{3m_e^2} = \frac{2}{3}\text{ barn}$$

$\sigma(\gamma\gamma \to e^+e^-)$: threshold at $s = 4m^2$
maximum of $\frac{1}{4}\sigma_{\text{Thomson}}$ at $s \sim 8m^2$
falls off as $\frac{1}{s}$

EM

Coulomb 

$$A = \frac{e^2 s}{t} = \frac{4\pi\alpha s}{t}$$

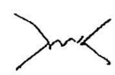
$$\frac{d\sigma}{dt} = \frac{\hbar^2}{16\pi s^2}\left(\frac{4\pi\alpha s}{t}\right)^2 = \frac{\pi\hbar^2\alpha^2}{t^2}$$

$$\sigma = \int_{-s}^{0} \frac{\pi\hbar^2\alpha^2}{t^2}\,dt = \infty$$

$$\sigma_{\substack{\text{back}\\ \text{scatter}}} = \int_{-s}^{-\frac{s}{2}} \frac{\pi\hbar^2\alpha^2}{t^2}\,dt = \pi\hbar^2\alpha^2\left[\frac{2}{s} - \frac{1}{s}\right] = \frac{\pi\hbar^2\alpha^2}{s}$$

$\theta > \frac{\pi}{2}$

$$= \pi\left(\frac{\hbar\alpha}{E_{cm}}\right)^2$$

$e^- e^+ \to \mu^- \mu^+$ 

$$A = \frac{e^2 s}{s} = 4\pi\alpha$$

$$\frac{d\sigma}{dt} = \frac{\hbar^2}{16\pi s^2}(4\pi\alpha)^2 = \frac{\pi\hbar^2\alpha^2}{s^2}$$

$$\sigma = \int_{-s}^{0} \frac{\pi\hbar^2\alpha^2}{s^2}\,dt = \frac{\pi\hbar^2\alpha^2}{s}$$

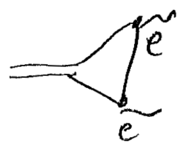
EM

QED Naive

$1 \to 2$: $\Gamma = \frac{|A|^2}{16\pi m}$

If $A = \underbrace{\sqrt{4\pi\alpha}}_{e}\, m \Rightarrow \Gamma = \frac{m}{4}\alpha$

But this isn't right because

$\pi^0 \to \gamma\gamma$  $A = 4\pi\alpha\, m \Rightarrow \Gamma = \pi\alpha^2 m$

(or perhaps divide by 2 because $\gamma\gamma$ final state)

$\tau = \frac{\hbar}{\pi\alpha^2 m_\pi} = 3\,E{-}20$ sec [naive approach not so good]

P+S 19.119 $\Rightarrow \Gamma = \alpha^2 \frac{m_\pi}{64\pi^3}\left(\frac{m_\pi}{f_\pi}\right)^2$

$= \pi\alpha^2 m_\pi \underbrace{\underbrace{\frac{1}{64\pi^4}}_{1.6E-4} \underbrace{\left(\frac{m_\pi}{f_\pi}\right)^2}_{2.1}}_{3.4E-4} = 8E{-}17$ sec

weak

Naive approach

$G_F = \frac{g^2}{m_W}$

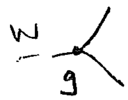
9

weak int.

2 → 2 scattering: 

$A = G_F s$ to be dimensionless

$$\sigma = \frac{\hbar^2 |A|^2}{16\pi s} \overset{?}{=} \frac{\hbar^2 G_F^2 s}{16\pi}$$

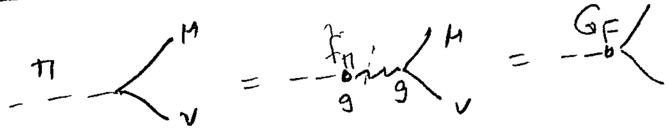
1 → 2 decay 

$$A = g m_W = \sqrt{G_F}\, m_W^2$$

$$\Gamma = \frac{|A|^2}{16\pi m_W} = \frac{G_F m_W^3}{16\pi}$$

$G_F = \frac{g^2}{m_W^2}$

[actual: $\frac{G_F m_W^2}{6\pi\sqrt{2}}$]

1 → 2 decay 

$$A = G_F m_\pi^3$$

$$\Gamma = \frac{G_F^2 m_\pi^5}{16\pi}$$

Griffiths (actual: $\Gamma = \frac{G_F^2 m_\pi^5}{16\pi}(0.2)$)

1 → 3 decay 

$$A = G_F m^2$$

$$\Gamma = \frac{|A|^2 m}{512\pi^3} = \frac{G_F^2 m^5}{512\pi^3} =$$

(actual: $\frac{G_F^2 m^5}{192\pi^3}$)

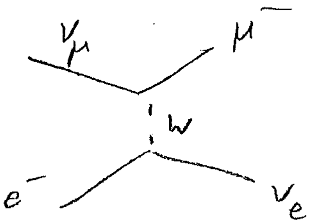
$\pi^- \to \pi^0 e^- \bar{\nu}_e$ [Segre 631]

(3-30-25) week

10

High energy ν scattering

[Perkins, Pcl Astro, prob 1.9., solution in back]



$$A = \frac{g^2 s}{t - m^2} M \qquad G_F = \frac{g^2}{m^2}$$

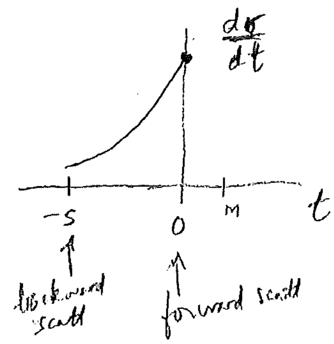
$$= \frac{G_F m^2 s}{t - m^2} M$$

[for fundamental spin ½ scattering $M \to H$ at high energies.. see my ν scatt notes of Griffiths]

$$\frac{d\sigma}{dt} = \frac{\hbar^2}{16\pi s^2} \left(\frac{G_F m^2 s}{t - m^2} M \right)^2$$

$$\sigma = \int_{-s}^{0} \frac{d\sigma}{dt} dt = \frac{1}{\pi} \left(\hbar G_F m^2 \frac{M}{4} \right)^2 \int_{-s}^{0} \frac{dt}{(t-m)^2}$$

$$= \frac{1}{\pi} \left(\hbar G_F m^2 \frac{M}{4} \right)^2 \left[\frac{1}{m^2} - \frac{1}{s+m^2} \right]$$



$$= \frac{1}{\pi} \left(\hbar G_F \frac{M}{4} \right)^2 \left[\frac{m^2 s}{s + m^2} \right]$$

$$\longrightarrow \begin{cases} s \ll m^2 & \dfrac{\hbar^2 G_F^2 s}{\pi} \left(\dfrac{M}{4} \right)^2 \\ \\ s \gg m^2 & \underbrace{\dfrac{\hbar^2 G_F^2 m^2}{\pi}}_{} \left(\dfrac{M}{4} \right)^2 \end{cases}$$

$m = 80.377$ GeV

$G_F = 1.166\text{E-}5 \text{ GeV}^{-2}$

$G_F^2 m^2 = 8.78\text{E-}7 \text{ GeV}^{-2}$

$\hbar = 0.197$ GeV fm

$\underbrace{1.1 \times 10^{-8} \text{ fm}^2}$

$\underbrace{1.1 \times 10^{-10} \text{ barns}}$

0.11 nb

weak

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$

Griffiths (2e), 9.20 gives

$$|A|^2 = 32\, G_F^2 m_\mu^2 E_1 (m_\mu - 2E_1)$$

$$\Gamma = \frac{1}{2m} \int d(LIPS)_3 \, |A|^2$$

$$= \frac{1}{8m(2\pi)^3} \int_0^{\frac{m}{2}} dE_1 \; 32 G_F^2 m^2 E_1 (m - 2E_1) \underbrace{\int_{\frac{m}{2}-E_1}^{\frac{m}{2}} dE_2}_{E_1}$$

$$= \frac{4m G_F^2}{(2\pi)^3} \underbrace{\int_0^{\frac{m}{2}} dx \; x^2 (m - 2x)}_{\frac{m^4}{96}}$$

$$\Gamma = \frac{G_F^2 m^5}{192\pi^3}$$

whereas naive assumption $|A| = G_F m^2 M$, $|A|^2 = G_F^2 m^4 |M|^2$

$$\Gamma = \frac{G_F^2 m^5}{512\pi^3} |M|^2 \qquad \text{so} \quad |M|^2 = \frac{8}{3}, \quad M = 1.6$$

weak

Perkins Pcl Astrophysics

prob 1.6

Examples of $1 \to 3$ weak decay

(a) $\tau^+ \to e^+ \nu_e \bar{\nu}_\tau$ $Q = 1775$ MeV $R = 6.1E11\ s^{-1}$

(b) $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ $Q = 105$ MeV $R = 4.6E5\ s^{-1}$

(c) $\pi^+ \to \pi^0 e^+ \nu_e$ $Q = 4.1$ MeV $R = 0.39\ s^{-1}$ (because BR ~ 10^{-8}) $\Rightarrow |m|^2 = 3.0$

(d) ${}^{14}O \to {}^{14}N\ e^+ \nu_e$ $Q = 1.8$ MeV $R = 5.1E-3\ s^{-1}$ ($\tau = 3$ min) $\Rightarrow |m|^2 = 2.4$

(e) $n \to p e^- \bar{\nu}_e$ $Q = 0.78$ MeV $R = 1.13E-3\ s^{-1}$ ($\tau = 15$ min) $\Rightarrow |m|^2 = 35$

$|m|^2 = (2.4)^2$ + factor of 5.8 error from $m_e = 0$ assumption

First 2 examples: $\Gamma = \dfrac{G_F^2 Q^5}{192\pi^3}$ to within accuracy given

Last 3 examples: $\Gamma = \dfrac{G_F^2 Q^5}{60\pi^3} |m|^2$

$$R = \frac{(1.166E{-}11\ \text{MeV}^{-2})^2\ Q^5\ |m|^2}{60\pi^3\,(6.58E{-}22\ \text{MeV}\cdot s)}$$

$$= \frac{Q^5\,|m|^2}{9001\ \text{MeV}^5}$$

$$|m|^2 = \frac{9001\ \text{MeV}^5}{Q^5} R$$

Stone 2023 notes)

weak decay.

$\boxed{W^\pm}$: $\Gamma \cdot \frac{G_F m_W^3 f^2}{16\pi}$ → $\frac{G_F m_W^2}{6\pi\sqrt{2}}$ f^2 1.8856 $= \frac{8}{3\sqrt{2}}$

2025: What is a?

$|A|^2 - G_F m_W^4 f^2$

$|a|^2 \simeq G_F m_W^2 f^2 \approx 0.14$

$a \approx \sqrt{G_F} m_W f \approx 0.27 f = 0.37$

$\Gamma \approx \frac{m_W}{16\pi} |a|^2 \approx 0.0028\, m_W \approx 220$ MeV

(actual full width ~ 2 GeV)

$\tau = \frac{1}{3}(10^{-23} s)$

partial

$\boxed{\pi \to \mu \nu_\mu}$: $\Gamma = \frac{G_F^2 f_\pi^2}{8\pi m_\pi^3}(m_\pi^2 - m_\mu^2)^2 m_\mu^2$

$G_F m_W^2 = 0.0754$

$= \frac{G_F^2}{8\pi} m_\pi^5 \underbrace{\left(\frac{f_\pi}{m_\pi}\right)^2}_{\approx 1} \underbrace{\left(\frac{m_\mu}{m_\pi}\right)^2\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2}_{\approx 0.1}$

$\left(\frac{m_\pi}{m_W}\right)^2 = 3 \times 10^{-6}$

$G_F m_\pi^2 = 2.27 \times 10^{-7}$

This gets squared!

$= \frac{m_\pi}{8\pi} \underbrace{(G_F m_\pi^2)^2}_{5E-14} (0.1)$

5×10^{-15}

$\Gamma \approx (2 \cdot 10^{-16}) m_\pi \Rightarrow \tau \sim \frac{66\,\text{meV}(10^{-22})}{140\,\text{MeV}(2 \cdot 10^{-16})} \sim \frac{1}{4} \cdot 10^{-7}$ sec)

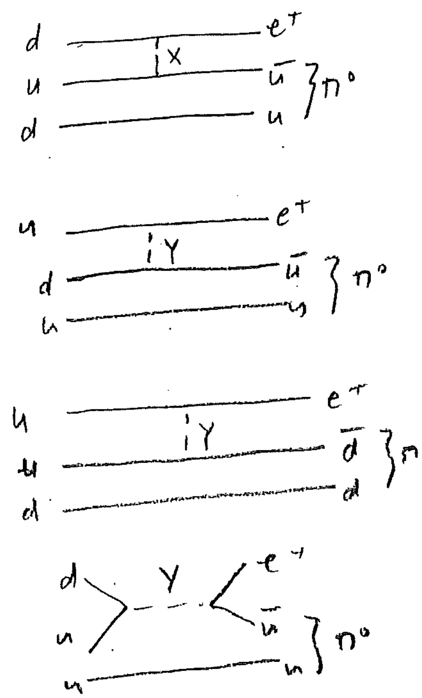
$a \approx 0.2\, G_F m_\pi^2 \approx 5 \times 10^{-8}$

GUT

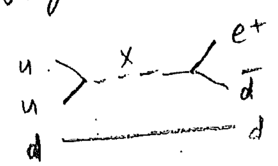
Proton decay: $p \to e^+ \pi^0$ in SU(5)

Perkins PA p. 85

$1 \to 2$ decay



why not?



$$A \sim \frac{g^2}{m_X^2} \cdot (f m_p)^3, \qquad g^2 = 4\pi\alpha$$

$(\alpha \approx \frac{1}{42})$

Perkins p. 86

so that $[A] \sim E$

f = fudge factor

$$\Gamma = \frac{|A|^2}{16\pi m_p} = \frac{(4\pi\alpha)^2}{16\pi} \frac{f^6 m_p^5}{m_X^4}$$

$$\tau = \frac{1}{\Gamma} = \frac{M_X^4}{\pi f^6 \alpha^2 m_p^5}$$

see Perkins PA, p. 85

$$T = \frac{m_X^4}{A \alpha^2 m_p^5} = \frac{4.3E29}{A} \text{ years}$$

w/ $\alpha = \frac{1}{42}$, $m_X = 3E14$ GeV